AMERICAN MACHINIST GEAR BOOK

American Machinist Gear Book

Simplified Tables and Formulas
for Designing, and Practical Points in
Cutting All Commercial Types of Gears

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PREFACE

This book has been written to fill a pressing want; to give practical data for cutting, molding, and designing all commercial types, and to present these subjects in the plainest possible manner by the use of simple rules, diagrams, and tables arranged for ready reference. In other words, to make it a book for "the man behind the machine," who, when he desires information on a subject, wants it accurate and wants it quick, without dropping his work to make a general study of the subject. At the same time a general outline of the underlying principles is given for the student, who desires to know not only how it is made, but what is made. Controversies and doubtful theories are avoided. Tables and formulas commonly accepted are given without comment. A great deal of this matter has previously been published in the columns of the AMERICAN MACHINIST, but is revised to make the subject more complete. Credit is given in all cases when the author is known; there may be cases however, where record of the original source of information has been lost, as is often the case when data are in daily use and the authority is obscure. Obviously in such cases the author's name cannot be given.

CHARLES H. LOGUE.

May 1, 1910.

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SECTION I

Tooth gearing furnishes an efficient and simple means of transmitting power at a constant speed ratio, making it possible to time the movements of machine parts positively.

Owing to refinements in the tooth form, the introduction of generating machines and facilities to cut gears of the largest size accurately, loads may now be transmitted at speeds which a comparatively short time ago were considered prohibitive. The need for gears that would answer the exacting requirements of automobile construction has done much to bring this about. Designing automobile gears, however, is a case of fitting the gears to the machine; it is a question of securing material that will stand the strain; the gear dimensions are practically self determined; however, this is not the only kind of gearing that has been designed after this fashion.

We have an excellent formula for the strength of gear teeth, but it contains a variable factor—the allowance to be made on account of impact—concerning which very little is known. The most important question of all, that of wear, has heretofore been left practically untouched. The best data obtainable has been given. Few records have been kept of actual performances, and nothing whatever has been found relative to the abrasion of different materials in tooth contact.

The various ways in which gears are mounted is responsible for the apparent contradiction of what few data are at hand, as a gear driver which is entirely satisfactory on one machine will be worthless on another at the same load and the same speed.

The circumferential speed that may be allowed for gears of different types is another neglected factor, and last, but not least, what do we know of gear efficiency? In fact the most important information relative to gear transmissions has been entirely a matter of guesswork. It is hardly to be assumed that this will ever be reduced to an exact equation, but there should be some basis from which to form our conclusions.

Gears may be roughly divided into three general classes: Gears connecting parallel shafts; gears connecting shafts at any angle in the same plane; gears connecting shafts at any angle not in the same plane.

In the first class are included spur, helical, herringbone, and internal gears. The second class covers bevel gears only. The third class includes worm, spiral and skew bevel gears.

Gears connecting parallel shafts are the most efficient, and from a point of efficiency may be graded into herringbone, internal, spur, and, lastly, helical gears.

The efficiency of the second class, bevel gears, varies with the shaft angle, increasing as the angle approaches zero.

As a general thing the third class should be avoided wherever possible, although worm gears have their peculiar uses; for instance, where a quiet, self-locking drive is required without reference to the loss of power.

Spiral gears are employed where the load is light and the gear ratio is low, say under 10 to 1; worm gears are often employed for low ratios down to 1 to 1, but are extremely difficult to cut and therefore expensive. When the worm is made much coarser than quadruple thread there is generally trouble.

Skew bevel gears are used where the distance between the shafts is not great enough to employ worm or spiral gears. Skew bevel gears are simpler, and easier to cut than has been generally supposed, but are still things to avoid.

These three general classifications are commercially subdivided as follows:

KIND	RELATION OF AXES	PITCH SURFACES	NOTES
Spur	Parallel	Cylinders	
Bevel	Intersecting at any angle in the same plane..........	Cones	
Helical	Parallel	Cylinders	
Herringbone	Parallel	Cylinders	Double Helical
Spiral	At any angle not in same plane..................	Cylinders	For small ratios
Worm	At any angle not in same plane..................	Cylinders	For large ratios
Skew Bevel	At any angle not in same plane..................	Hyperboloids	Where shaft centers are close
Internal	Parallel or at any angle in same plane............	Cylinders or cones	With teeth cut on inner surface
Elliptical	Parallel or at any angle in same plane............	Elliptical cylinders or elliptical cones	
Irregular	Parallel or at right angles in same plane............	Any	Irregular pitch lines
Intermittent	Parallel or at right angles in same plane............	Cylinders	To give driven gear a period, or periods of rest during one revolution of driver
Friction	Parallel or at any angle in same plane............	Cylinders or cones	Contact surfaces representing the pitch surfaces of a toothed gear

COMMERCIAL CLASSIFICATION OF GEARS

TOOTH PARTS

Fixed axes are connected by imaginary pitch surfaces, which roll upon each other and transmit uniform motion without slipping. The object in toothed gearing is to provide these imaginary surfaces with teeth, the action of which will make the uniform motion of the pitch surfaces positive; not depending upon friction produced by direct pressure as in friction gears, which are an excellent representation of pitch surfaces.

If the teeth are not so formed that this condition is fulfilled the movement of the driven gear will be made up of accelerations and retardations which will not only absorb a large percentage of the power but disintegrate the material of which the gear is constructed and seriously affect the operation of the machine. Tool marks on planer and boring mill work corresponding to the teeth in the driven gear may be traced directly to this.

There is but one form of tooth in common use—the involute; the cycloidal form has practically disappeared. For a thorough understanding of tooth contact, however, it must be included.

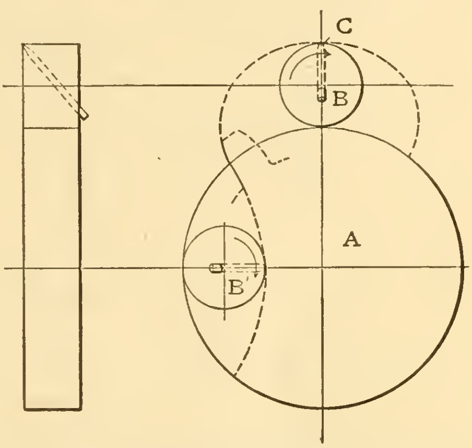


FIG. I. GENERATING THE CYCLOIDAL TOOTH.

CYCLOIDAL

Generated by rolling a circle above and below the pitch circle of gear; a point on its circumference describing the tooth outline. See Fig. 1.

INVOLUTE

Generated by rolling a straight line on the base circle of gear, any point on this line describing the involute curve. See Fig. 2. The same result is obtained by unwinding a string from the base circle. See Fig. 3.

OCTOID

Conjugated by a tool representing a flat sided crown gear tooth; a modification of the involute. Used only on bevel gear generating machines. See Fig. 33.

THE CYCLOID

An illustration of the manner in which the cycloidal tooth is generated is illustrated by Fig. 1; the wheel *A* being the pitch circle and *B* and *B'* the describing circles which are of the same diameter. The point *C* will describe the face of the tooth as the circle *B* is rolled on the pitch circle, and the flank of the tooth as the circle *B'* is rolled inside the pitch circle. In other words,

the exterior cycloid is formed by rolling the describing circle on the outside of the pitch circle, this exterior cycloid engaging the interior cycloid, which is formed by rolling the describing circle on the inside of the pitch circle.

The describing circle is commonly made equal to the pitch radius of a 15-tooth pinion of the same pitch as the gear being drawn.

According to J. Howard Cromwell: "Roomer, a celebrated Danish astronomer, is said to have been the first to demonstrate the value of these curves for tooth profiles." But De la Hire is credited with demonstrating that it was

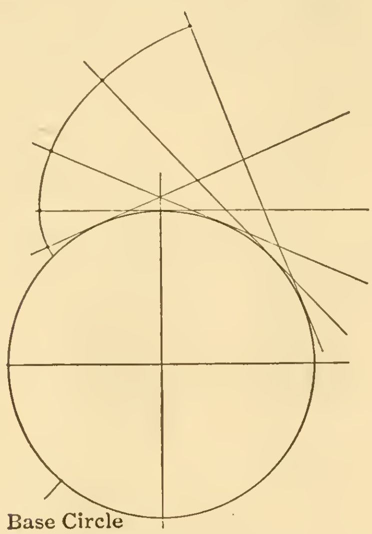


FIG. 2. THE INVOLUTE GENERATED BY A STRAIGHT LINE.

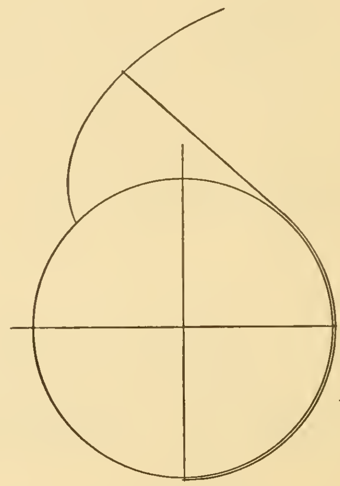


FIG. 3. THE INVOLUTE GENERATED BY A STRING.

possible to form both the face and flanks of any number of gears with the same describing circle.

The pressure angle of the teeth is not constant in one direction, but varies from zero at the pitch point to about 22 degrees at the end of the contact with a rack tooth. The contact points of all the teeth engaged intersect the line of action, which is a segment of the describing circle drawn from the line of centers. See Fig. 34.

Wilfred Lewis has said: "The practical consideration of cost demands the formation of gear teeth upon some interchangeable system.

"The cycloidal system cannot compete with the involute, because its cutters are formed with greater difficulty and less accuracy, and a further expense is entailed by the necessity for more accurate center distances. Cycloidal teeth must not only be accurately spaced and shaped but their wheel centers must be fixed with equal care to obtain satisfactory results. Cut gears are not only more expensive in this system, but also when patterns are made for castings

the double curved faces require far more time and care in chiseling. An involute tooth can be shaped with a straight-edged tool, such as a chisel or a plane, while the flanks of cycloidal teeth require special tools, approximating in curvature the outline desired. It is, therefore, hardly necessary to argue any further against the cycloidal gear teeth, which have been declining in popularity for many years, and the question now to be considered is the angle of obliquity most desirable for interchangeable involute teeth."

In this same connection George B. Grant, of the Philadelphia Gear Works, wrote: "There is no more need of two different kinds of tooth curves for gears of the same pitch than there is need of two different threads for standard screws, or of two different coins of the same value, and the cycloidal tooth would never be missed if it were dropped altogether. But it was first in the field, is simple in theory, is easily drawn, has the recommendation of many well-meaning teachers and holds its position by means of 'human inertia,' or the natural reluctance of the average human mind to adopt a change, particularly a change for the better."

THE INVOLUTE

The pressure on the teeth of involute gears is constantly in the direction of the line of action. The line of action is drawn through the pitch point at an angle from the horizontal equal to the angle of obliquity. All contact between the teeth is along this line. The base circle is drawn inside the pitch circle and tangent to the line of action.

The action of a pair of involute gears is the same as if their base circles were connected by a cross belt; the point at which the belt crosses being the pitch point *P*; the straight portion of the belt not touching the base circles representing the lines of action. See Fig. 4. At the pitch point the velocities of both gears are equal. To show that the involute is but a limiting case of the cycloidal system, consider the describing line as a curve of infinite radius, which is rolled upon the pitch circle. As this describing line cannot be rolled inside the pitch circle to form the interior cycloid that will engage the exterior cycloid formed by rolling the describing line outside the pitch circle of the mating gear, the pitch circles upon which the cycloids are formed must be separated so as to allow the exterior cycloids to engage each other. The original pitch circles becoming the base circles. See Fig. 5.

The distance between the pitch circle and the base circle, and therefore, the angle of obliquity, depends upon the proportionate length of tooth to be used and the smallest number of teeth in the system. To obtain contact for the full length of the tooth, the base circle must fall below the lowest point reached

by the teeth of the mating gear. Below the base line there can be no contact of any value.

There is such a difference between the largest possible gear and the rack that it is at first a little difficult to see the application of the methods used to describe the involute to the rack tooth.

As the diameter of the gear is increased, the radii used to draw the

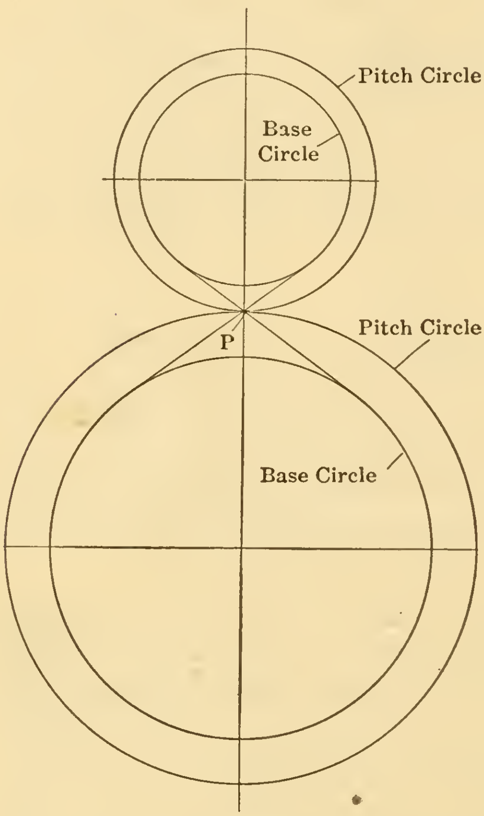


FIG. 4. THE ACTION OF INVOLUTE TEETH ILLUSTRATED BY A CROSSED BELT CONNECTING THE BASE CIRCLES.

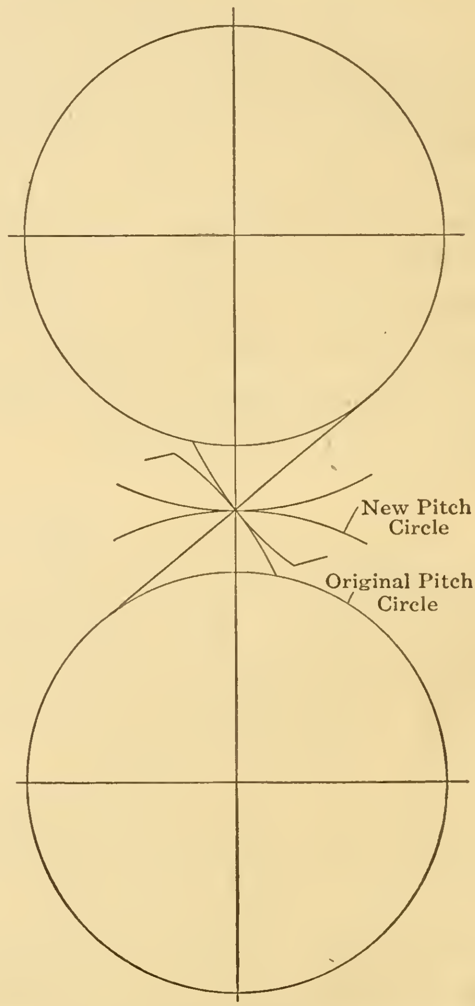


FIG. 5. SEPARATING THE PITCH CIRCLES TO ALLOW THE EXTERIOR CYCLOIDS TO ENGAGE.

involute curve are lengthened, and the teeth have less curvature. Until finally, when the radius of the pitch circle is of infinite length, the tooth radii are also infinite, and the involute is a straight line, drawn at right angles to the line of action.

The theoretical rack tooth, therefore, has perfectly flat sides, each side being inclined toward the center of the tooth to an angle equaling the angle of obliquity. See Fig. 6.

ORIGIN OF THE INVOLUTE TOOTH

The origin of the involute curve as applied to the teeth of gears is credited to De la Hire, a French scientist, a complete description and explanation of its use being published about 1694 in Paris. The first English translation of this work was published in London in 1696 by Mandy.* Professor Robinson, of Edinburgh, later describes this theory, references being made to his work in "An Essay on Teeth of Wheels," by Robertson Buchannan, edited by Peter Nicholson and published in 1808. In this essay the involute as applied to the teeth of gears is fully described, Fig. 7 being a copy of a cut used therein for illustration. That the principal advantage of the involute system was then well understood will be shown in the following paragraph, referring to Fig. 7:

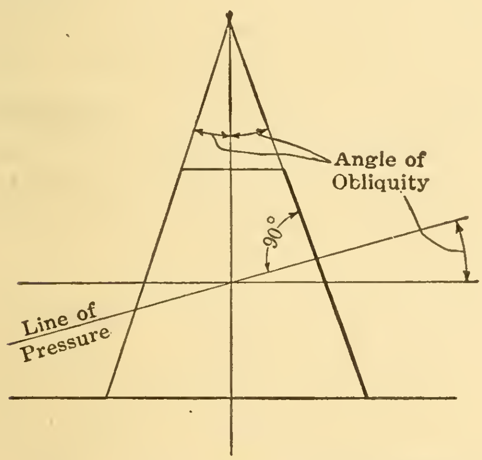


FIG. 6. THE INVOLUTE RACK TOOTH.

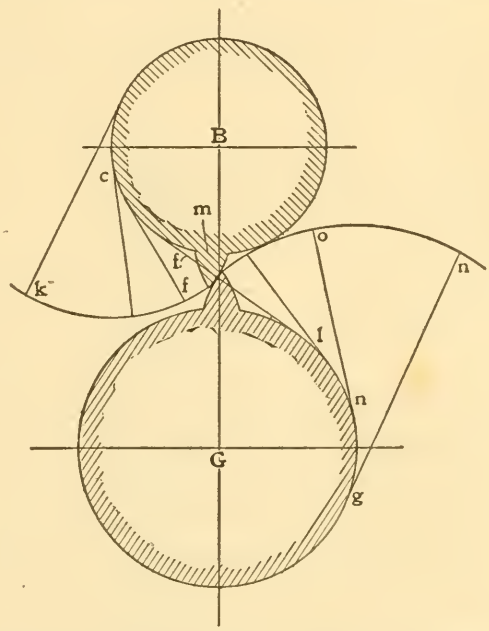


FIG. 7. ACTION OF THE INVOLUTE TOOTH.

"It is obvious that these teeth will work both before and after passing the line of centers, they will work with equal truth, whether pitched deep or shallow, a quality peculiar to them and of very great importance."

The theory of the involute gear tooth is also described by Sir. David Brewster, Dr. Thomas Young, Mr. Thomas Reid and others.

Professor Robert Willis, gives a very complete description of this form of tooth in his "Principles of Mechanism," 1841. Up to this period the involute tooth was not seriously considered, the cyclodial being the favorite. The in-

* However, the origin of the involute gear tooth is surrounded by mystery, no two authorities agreeing upon the subject. According to Robert Willis, in his "Principles of Mechanism," the involute was first suggested for this purpose by Euler, in his second paper on the Teeth of Wheels. N.C. Petr XI. 209.

volute tooth was objected to on account of the great thrust supposed to be put on the bearings by the oblique action of the teeth.

In an 1842 edition of M. Camus' work, "A treatise on the Teeth of Wheels," edited by John I. Hawkins, a series of experiments with wooden models was made to demonstrate the actual thrust occasioned by different angles of obliquity. The result of these experiments is given as follows:

"These experiments, tried with the most scrupulous attention to every circumstance that might affect their result, elicit this important fact—that the teeth of wheels in which the tangent of the surfaces in contact makes a less angle than 20 degrees with the line of centers, possess no tendency to cause a separation of their axes: consequently, there can be no strain thrown upon the bearings by such an obliquity of tooth."

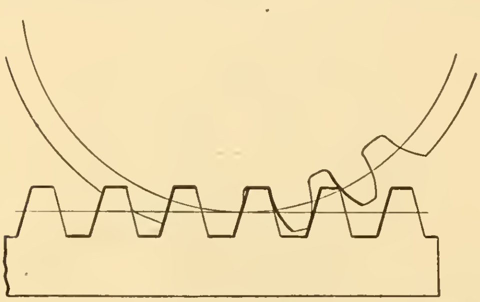


FIG. 8. THE MOLDING PROCESS.

J. Howard Cromwell, in his treatise on Tooth Gearing, 1901, says: "Such an obliquity as 20 degrees must, unless counteracted by an opposing force, tend to separate the axes; and, as suggested by Mr. Hawkins, this opposing force is most probably the friction between the teeth, which would tend to drag the axes together with as much force as that tending to separate them."

That the involute system is closely connected to the cycloidal system is shown by Dr. Brewster in his reference to De la Hire's work.

"De la Hire considered the involute of a circle as the last of the exterior epicycloids; which it may be proved to be, if we consider the generating straight line (see Fig. 2) as a curve of infinite radius."

The 14½ degree angle of obliquity, as proposed by Professor Robert Willis in his "Principles of Mechanics," was adopted by the Brown & Sharpe Company some forty years ago. Since that time this system has come into general use.

THE MOLDING PROCESS

If a gear blank made of some pliable material is forced into contact with a rack, as shown in Fig. 8, the rack tooth would conjugate teeth in the blank.

It does not matter what form is given the conjugating tooth, as long as it has a regular line of action; all gears formed by it will interchange.

The Bilgram spur and spiral gear generating machine operates upon this principle. See Fig. 9. The cutter *A*, which is a reciprocating or planing tool having the profile of a correct rack tooth—namely, a truncated, straight-sided

wedge. While this tool reciprocates, it also travels slowly to the right, the blank meanwhile turning under it, the motion being that which would exist were the tool a rack tooth and the blank a gear. During this combined move-

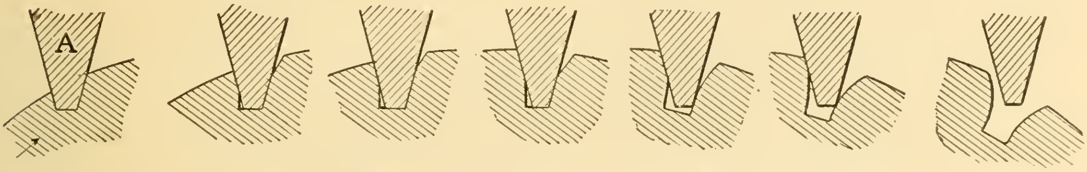


FIG. 9. ACTION OF THE TOOL IN GENERATING A TOOTH.

ment the tool cuts the tooth space in the manner indicated. In the Bilgram bevel-gear machine the tool does not move sidewise, the blank being rolled upon it as a complete gear might be rolled on a stationary rack, but in the spur-gear machine this action is reversed—the blank turning on a fixed center, while the tool moves over it, as it would be turned by a moving rack.

The Fellows' gear shaper is designed on the same principle, but instead of a rack tooth as a planing tool, a gear of from 12 to 60 teeth is used, the motion of cutter and blank being the same as between gears in mesh. See Fig. 10. These cutters are ground to shape after being hardened as shown in Fig. 11, in which the emery wheel is shaped as the planing tool in Fig. 9. The cutter being ground taking the place of the gear.

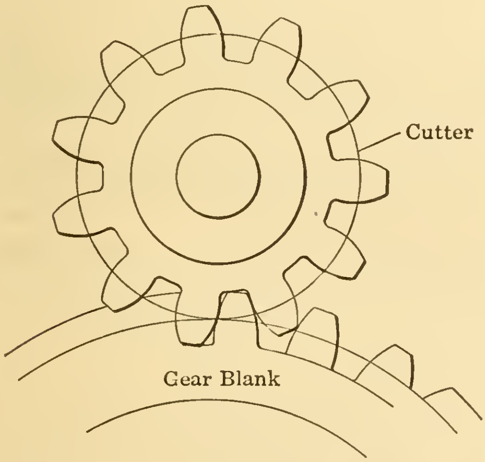


FIG. 10. ACTION OF THE FELLOWS' GEAR CUTTER.

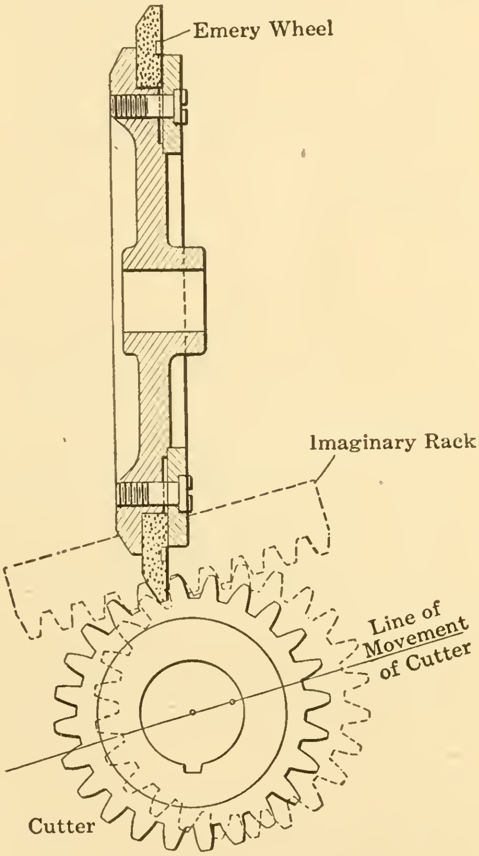


FIG. 11. GENERATION OF THE FELLOWS' GEAR-CUTTER TEETH.

TO DRAW THE INVOLUTE CURVE

The involute curve is constructed on the base circle as follows: Draw the pitch circle and through pitch point *P*, Fig. 12, draw the line of action at the required angle of obliquity. Tangent to this line draw the base circle.

Divide the base circle into any number of equal spaces, 1′, 2′, 3′, 4′, 5′, 6′, as shown in Fig. 13. From each of these points draw lines intersecting at center *O*. Draw lines 1′-1, 2′-2, 3′-3, etc., tangent to base circle and at right angles with lines extending to center. Make the length of line 1′-1 equal to one of the divisions of base circle: line 2′-2 equal to two divisions, line 3′-3 equal to three divisions, and so on. Then through points 1, 2, 3, 4, 5, 6, etc., trace the involute curve. Find a convenient radius, not necessarily on base circle, from which to draw the balance of the teeth, several radii sometimes being necessary to get the proper curve, especially for a small number of teeth. The involute curve does not extend below the base circle.

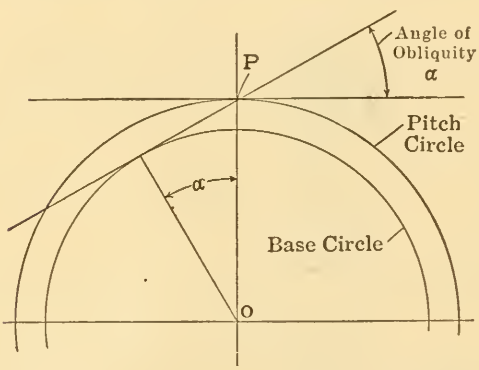


FIG. 12. LOCATING THE BASE CIRCLE.

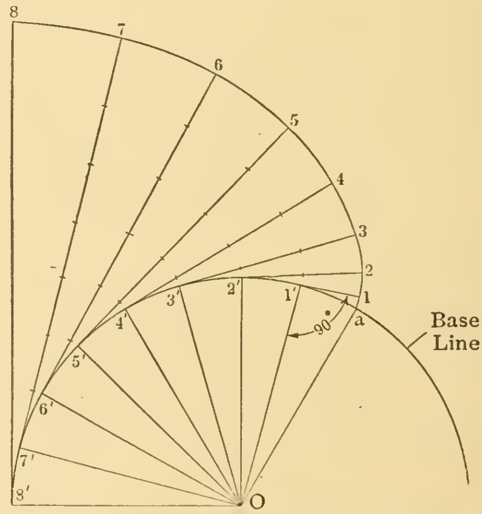


FIG. 13. DRAWING THE INVOLUTE.

Below the base circle drawing the teeth is simply a matter of obtaining sufficient clearance to avoid interference with the teeth of the mating gear.

SINGLE CURVE TEETH

This method of drawing gear teeth should be used only when the gear is to be pictured, not for templets. It is approximately correct only for 14½ degree teeth and for 30 teeth and over, although it may always be used for the curve between the base circle and the pitch circle.

Referring to Fig. 14, draw the pitch diameter and locate addendum, dedendum, and tooth spaces. With a radius of one half the radius of the pitch circle draw semicircle *A* from the center to the pitch line with the point of dividers

located on the center line midway between these points. Take one half of this radius or one quarter the radius of pitch circle and, with point of

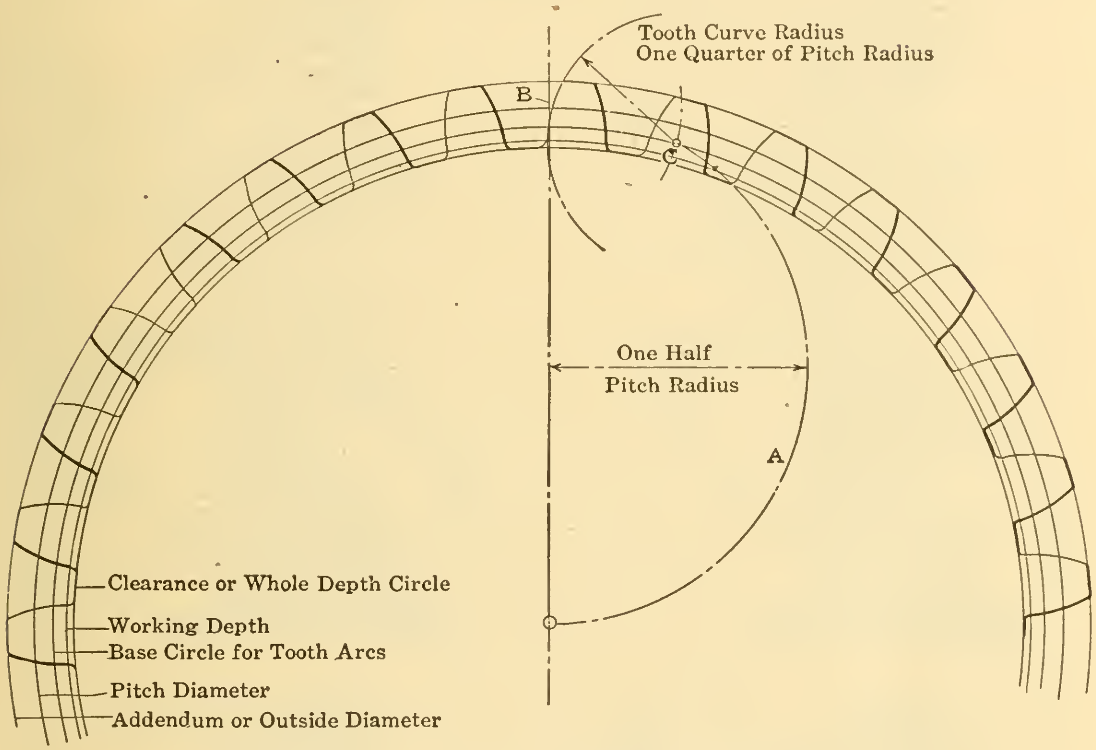


FIG. 14. LAYING OUT A SINGLE CURVE TOOTH.

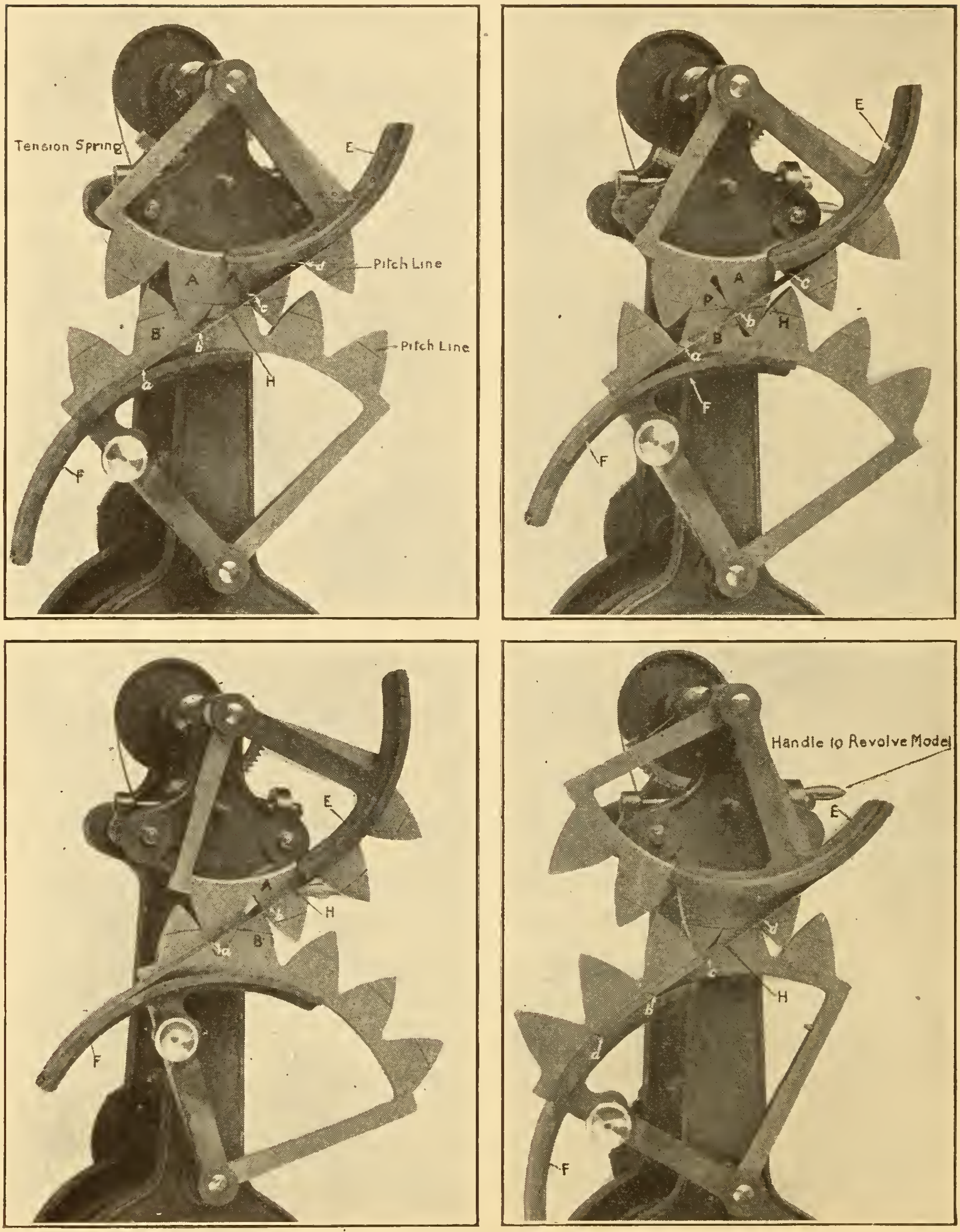
dividers at B on pitch circle draw an arc cutting semicircle A at point C. This is the center for the first tooth curve and locates the base circle for all tooth arcs.

DEMONSTRATION OF INVOLUTE PRINCIPLE BY A MODEL

An excellent Reuleaux model for demonstrating the principle of the involute system was loaned by Cornell University and is shown in Figs. 15 to 22.

The segments in the model represent gears of 21 and 17 teeth, about 1¾ inch circular pitch. The angle of obliquity is 30 degrees, which is sufficiently great to drop the base circle slightly below the bottom of the teeth in the smallest gear of the pair, thereby securing a theoretical tooth free from undercut or correction for interference. The teeth are carried to a point to show all the tooth action possible.

The base circles upon which the involute curve is constructed are represented in this model by rims E and F, upon which is tightly wrapped the band H, which, when wound from the base circle of the gear to the base circle of the pinion, represents the line of action, also the angle of obliquity, or pressure, the



FIGS. 15 AND 17. FIGS. 16 AND 18.

INVOLUTE GEAR TOOTH MODEL.

thrust of the teeth in contact being constantly in the direction of this band, which intersects all points of contact between the teeth.

Referring to Fig. 2, this band represents a line rolled on the base circle, also, as is self-evident, a string unwound from its base circle, as in Fig. 3, any given point on which will describe the involute curve.

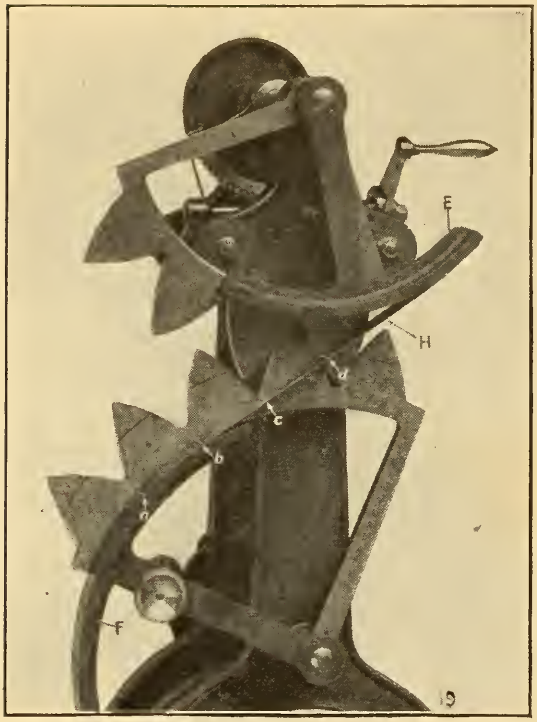
The points describing teeth in the gear segments are shown on the model by the lines *a*, *b*, *c* and *d* on the band *H* connecting the base circles, any of which will follow the contour of both teeth engaged from top to bottom as the gears are rotated, as well as those not yet in action. In fact, the generating of the involute curve is begun just as soon as any point on the band is raised from the rim representing the base circles and continues until the movement of the gear is stopped, the tooth outline being described by one of the points crossing the pitch line. The amount of this curve that is used above and below the pitch line depends upon the proportionate length of the tooth.

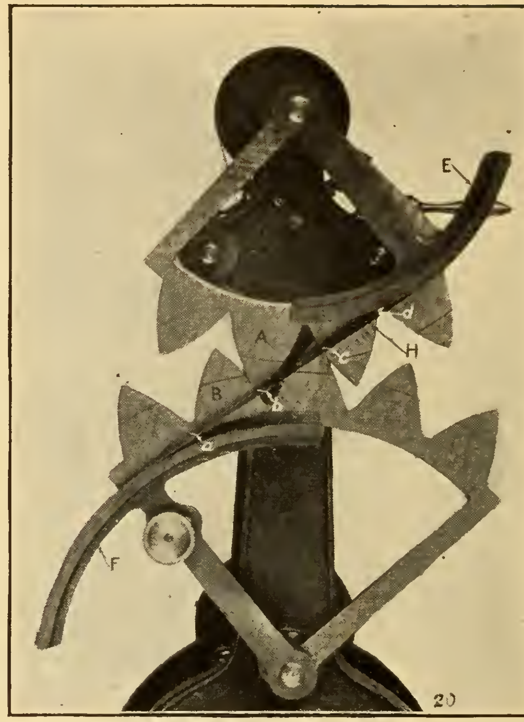
The location of points *a*, *b*, *c*, *d* on band *H* have no significance in the model; they are placed to correspond with the location of the teeth, being projected on a radial line drawn from the bottom of the tooth curve. If the pitch of gear segments had been coarser these points would simply have been farther apart.

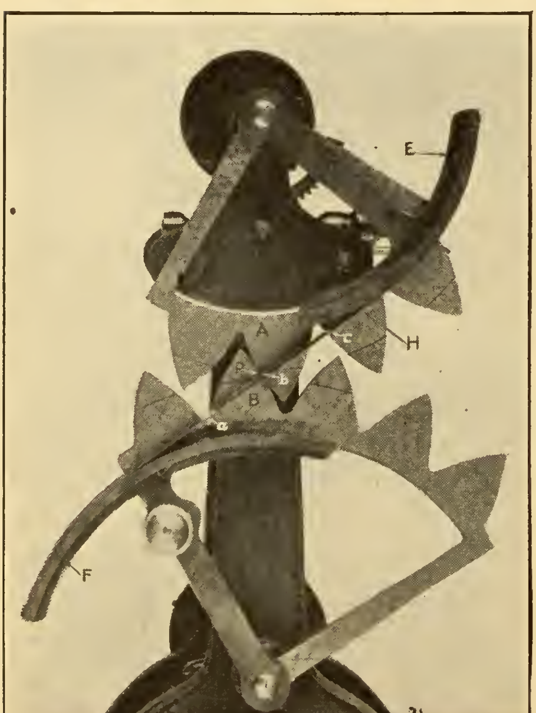
In the model, the length of the tooth is restricted only by the meeting of the curves describing the opposite sides of the tooth. The tooth is carried below the pitch line the same distance as above, plus a sufficient distance for clearance.

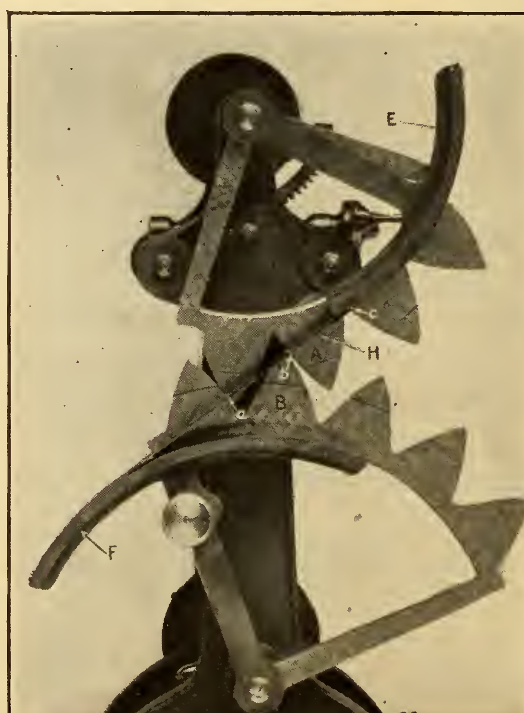
In case the gear segments were taken off, the model would simply represent the base circles of two gears, connected by a band, the angle of which, from the horizontal, would indicate the angle of obliquity. The driven shaft is propelled by the band, acting as a belt; any point upon it will describe the proper tooth outline from the base line up, the pitch point being at the intersection of this band and the line of centers. See Fig. 4.

Another important point is shown. The contact point of any two teeth engaged is followed by one of the points *a*, *b*, *c*, etc., as the gears are rotated and the band or line upon which these points are marked is always tangent to the two base circles. This illustrates the law of contact for involute teeth defined in connection with Fig. 35. The action between two involute teeth is that of two cylinders rolling and slipping upon each other. The diameter of these cylinders is constantly changing, one becoming larger and the other becoming smaller as the teeth enter and leave contact. The impulse given the driven gear will be variable if these conditions are not fulfilled during the entire action. This illustrates the importance of having the tooth curves theoretically correct.









FIGS. 19 AND 21. FIGS. 20 AND 22.

INVOLUTE GEAR TOOTH MODEL.

If the base circles E and F in the model were brought closer together it would reduce the pitch circles of both gears proportionately, also reduce the angle of obliquity, as the band or line representing the angle of obliquity must always be tangent to both base circles and pass through the pitch point, where the velocities of gears are equal. Drawing the base circles apart increases the pitch circles, also the obliquity, although the action of the teeth remains correct as long as they are engaged. See Figs. 20, 21, and 22.

In Fig. 15, the point b on band H is just touching the point of the tooth A as it enters into contact with tooth B. In Fig. 16, it has followed the contact, and therefore the outline of both engaging teeth to the pitch point P, and in Fig. 17, it is just leaving the point of the tooth B at the end of its contact with the tooth A. The point b will continue toward the upper tooth A until it completes the involute and comes to rest on the base circle E.

Figs. 18 and 19 show relation of points c and d with other teeth in the segments as the gears are revolving to the left, and affords a better opportunity to study the entire action of the model.

In Figs. 20, 21, and 22 the centers have been widened ¼ inch. Fig. 20 shows tooth A just entering contact with tooth B. In Fig. 21 the point b and band has followed the contact to the pitch point, which is now midway between the two pitch circles as marked on segments.

Fig. 22 shows the tooth A leaving contact, giving the entire range of action. This illustrates a peculiarity of the involute system, and explains how it is possible to obtain correct tooth action if a pair of gears are moved from their proper centers.

It will be noticed that the points a, b, c, and d follow the tooth outlines and points of contact just as accurately as when on proper centers, although the angle of obliquity is changed. The involute curve is always the same for a given *base diameter* and, as the pitch diameter, and not the base diameter is altered, a change in the center distance will make no change in the action of the teeth. This is illustrated by Fig. 23.

There will be new pitch diameters automatically established at the pitch point as the centers are moved, simply a different portion of the involute curve is used for the tooth. It is apparent that the farther out the tooth is placed the greater will be the distance between the pitch and the base circles and the greater the angle of obliquity. With the model in this position, if the lines drawn on gear segments to represent the pitch diameter were moved out until they were rolled together at the pitch point and the teeth made heavier at the pitch line to take up backlash, we would have gears of increased obliquity, which in turn, could be still farther apart as long as there were any teeth left, for with any increase of angle the length of tooth is necessarily shorter.

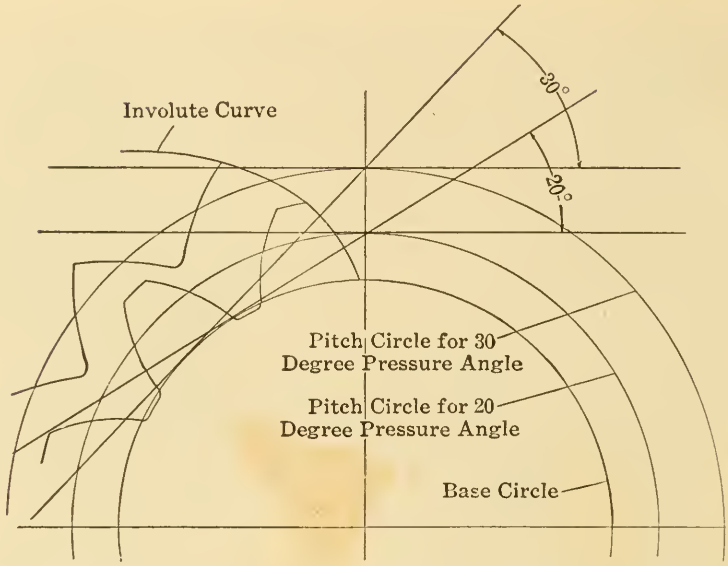


FIG. 23. DIAGRAM SHOWING HOW PROPER ACTION IS MAINTAINED AS THE GEAR AXES ARE SEPARATED.

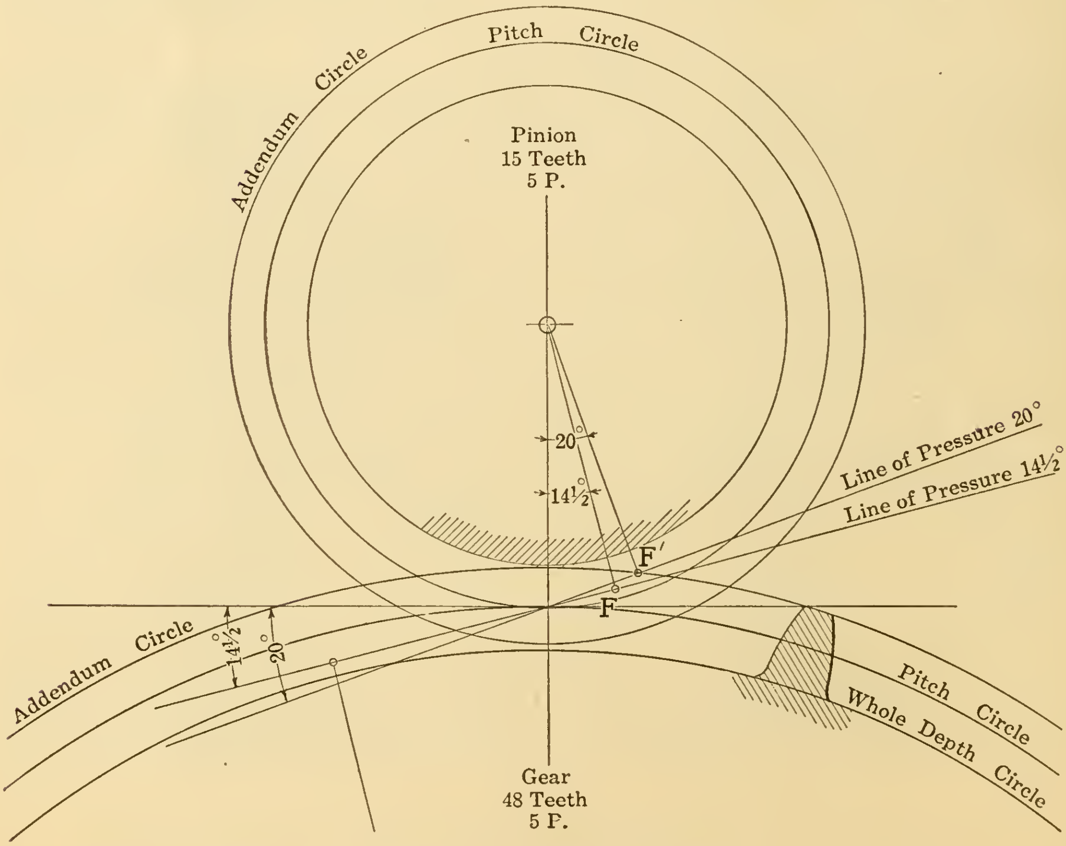


FIG. 24. GRAPHICAL DEMONSTRATION FOR INTERFERENCE OF SPUR GEARS.

INTERFERENCE IN INVOLUTE GEARS

The limitations and inaccuracies of the involute system are well explained in the following paragraphs by C. C. Stutz:

While the general principles governing the interference of involute gears are well known, the following graphical demonstrations, formulas, and plotted diagrams may place this general information in more efficient form for the use of many.

Fig. 24 shows a graphical demonstration of the interference of a 5-pitch, 15-tooth true involute form spur pinion and a 5-pitch, 48-tooth mating gear. The point F is the right-angled intersection of a line drawn from the center of the pinion, and at an angle of 14½ degrees with the common center line of the pinion and gear, with the line of pressure which is drawn through the point of tangency of the two pitch circles and at an angle of 14½ degrees to the common tangent at that point. If this point falls within the addendum circle of the meshing gear, the tooth of the meshing gear will interfere from this point up to its addendum circle. Therefore the tooth from this point on the curve must be corrected to overcome it.

If the point F falls on or outside of the addendum circle of the meshing gear no interference will result. The point F' for an angle of obliquity of 20 degrees falls on the addendum circle and thus the gear and pinion indicated in the illustration would mesh without interference for this angle.

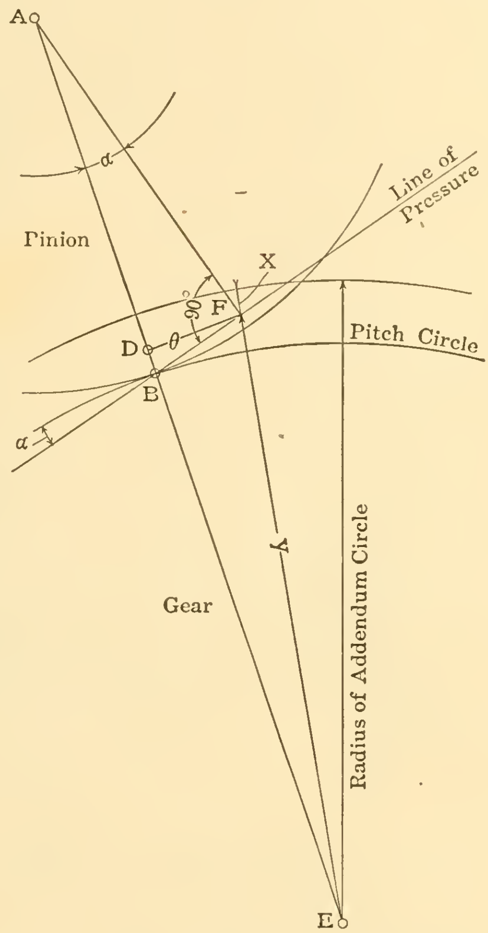


FIG. 25. INTERFERENCE OF SPUR GEARS.

FORMULA FOR LOCATING THE POINT OF INTERFERENCE OF SPUR GEARS

Referring to Fig. 25:

Let $AF=c$. $AB=r_2$. $AD=d$. $BE=r_1$. $DE=f$.

α = the angle of tooth pressure.

y = the distance from the center of the gear to the point at which interference begins.

x = the distance from the point at which interference begins to the addendum circle of the gear measured along a radius.

O = the perpendicular distance from the point at which interference begins to the center line of the pinion and gear.

Then

r_1 = the pitch radius of the gear.

r_2 = the pitch radius of the pinion.

D' = the pitch diameter of the gear.

D = the outside diameter of the gear.

Then

$$c = r_2 \cos \alpha, \text{ and}$$

$$d = c \cos \alpha = r_2 \cos^2 \alpha.$$

Now

$$f = r_1 + r_2 - d.$$

$$= r_1 + r_2 (1 - \cos^2 \alpha),$$

and

$$O = c \sin \alpha = r_2 \sin \alpha \cos \alpha.$$

Now

$$y^2 = f^2 + O^2 \text{ and } y = \sqrt{f^2 + O^2}.$$

Then by substituting

$$y = \sqrt{[r_1 + r_2 (1 - \cos^2 \alpha)]^2 + (r_2 \sin \alpha \cos \alpha)^2}$$

For a pressure angle of 14½ degrees

$$y = \sqrt{(r_1 + 0.0627\, r_2)^2 + (0.2424\, r_2)^2},$$

and

$$x = \frac{D}{2} - y.$$

For a pressure angle of 20 degrees

$$y = \sqrt{(r_1 + 0.1169\, r_2)^2 + (0.3214\, r_2)^2},$$

and

$$x = \frac{D}{2} - y.$$

Solving for x and y will give the point of interference for any particular case.

DIAGRAM FOR LOCATION OF INTERFERENCE

Fig. 26 shows a diagram giving the location of the point of the beginning of interference for one diametral pitch involute gears from 10 to 135 teeth meshing with a 12-tooth pinion. The ordinates are the distances from the point where interference commences to the addendum circle of the gear measured along the radius. They correspond to the quantity x in the preceding equa-

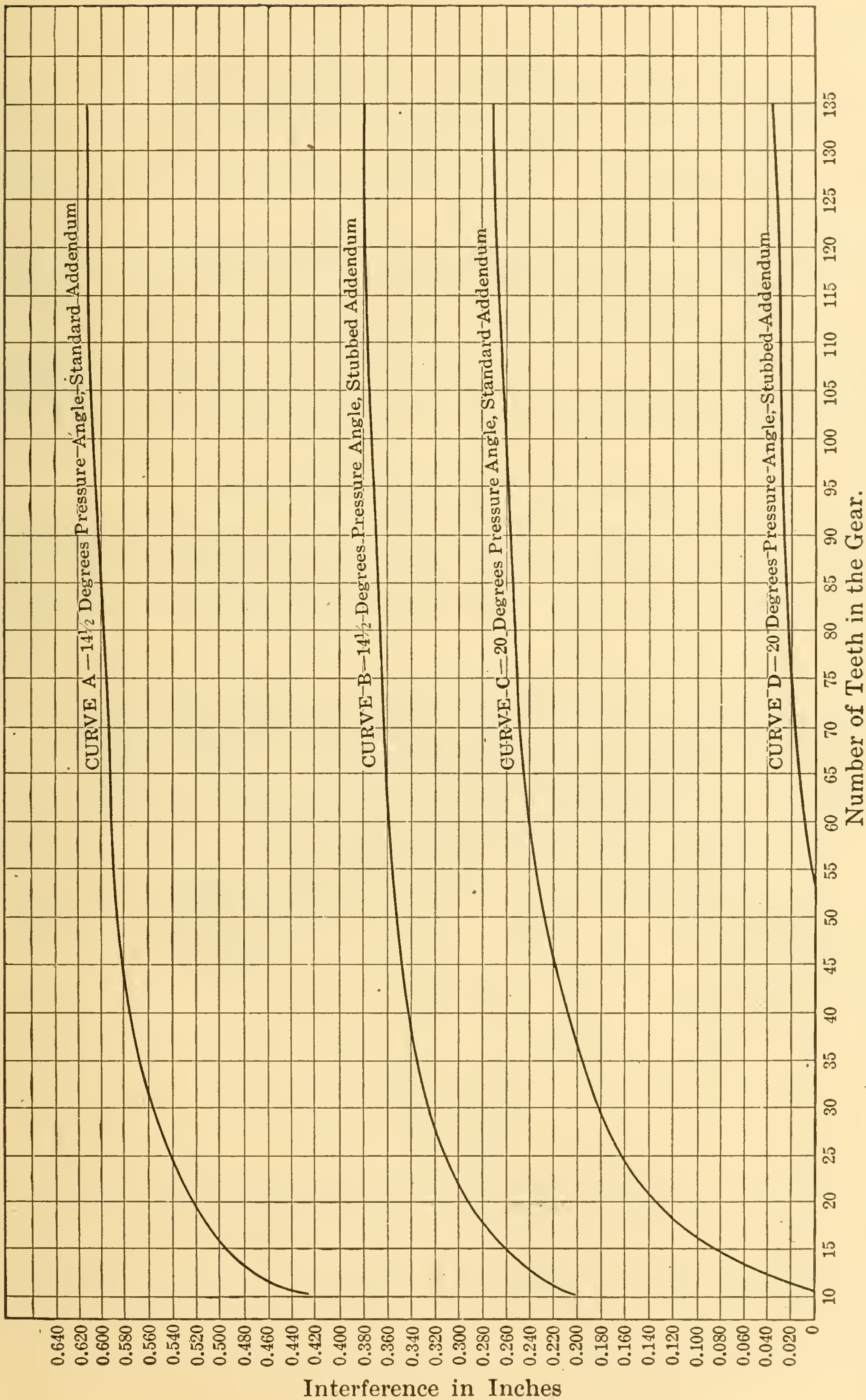


FIG. 26. DIAGRAM SHOWING INTERFERENCE BETWEEN AN INVOLUTE 12-TOOTH, ONE PITCH PINION AND MATING GEARS FROM 10 TO 135 TEETH.

tions. From this point to the addendum circle the tooth outline must be corrected.

The upper curve *A* is for a pressure angle of 14½ degrees and an addendum of 0.3183 × circular pitch. The second, *B*, is for the same pressure angle and a shorter addendum, 0.25 × circular pitch.

This addendum factor is for what is known as the stubbed tooth standard, as proposed by the author on page 23.

The third curve, *C*, is for a pressure angle of 20 degrees and an addendum of 0.3183 × circular pitch, while the lowest one, *D*, is for the 20-degree angle and the stubbed tooth addendum.

The diagram as plotted is for one diametral pitch. To find the corresponding ordinate for any other pitch divide the value given in the diagram by the required pitch. The quotient will be the distance desired.

INTERFERENCE OF RACK AND PINION

Interference will occur between the teeth of a rack and pinion when the point *B*, Fig. 27, which is the intersection of a perpendicular from the point *O* to the line of pressure *A L* falls inside of the rack addendum line *E E*. In the figure

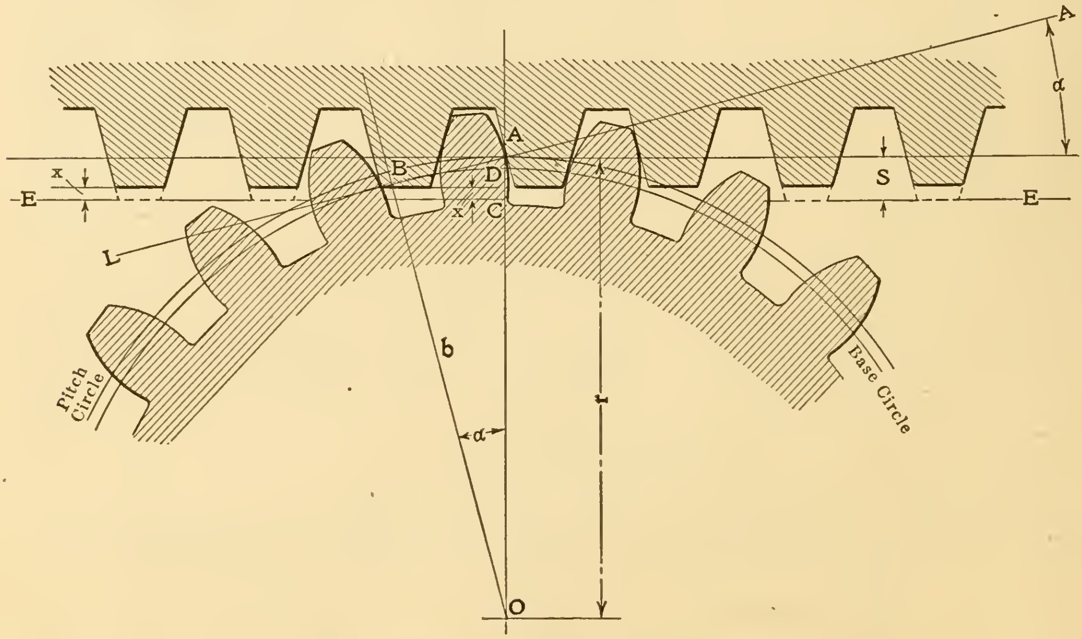


FIG. 27. INTERFERENCE OF GEAR AND RACK.

the distance over which interference takes place is *C D*. It is usual practice to shorten the rack teeth by the amount of this interference and the following equations give an easy method of computing this distance.

Let

N = the number of teeth in the pinion.
p = the diametral pitch.
r = the pitch radius.
b = the radius of the base circle.

Let

α = the pressure angle.
x = the distance necessary to shorten the addendum of the rack tooth and
s = the normal addendum of the rack tooth.

Then

$$s = \frac{1}{p},$$

$$r = \frac{\frac{1}{2}N}{p},$$

$$\begin{aligned}
b &= r \cos \alpha, \\
O\,D &= b \cos \alpha, \\
O\,D &= r \cos^2 \alpha, \\
O\,C &= r - s, \\
x &= O\,D - O\,C, \text{ and substituting} \\
&= r \cos^2 \alpha - (r - s) \\
&= \frac{\frac{1}{2}N}{p}(\cos^2 \alpha - 1) + \frac{1}{p}:
\end{aligned}$$

Whence

$$x = \frac{1 - \frac{1}{2}N\,(1 - \cos^2 \alpha)}{p}.$$

For a pressure angle of 14½ degrees

$$x = \frac{1 - 0.03135\,N}{p}.$$

For a pressure angle of 20 degrees

$$x = \frac{1 - 0.05849\,N}{p}.$$

Solving these equations we find that for the true involute form of tooth and a pressure angle of 14½ degrees interference between the teeth of rack and pinion begins with a pinion of 31 teeth. Similarly for a 20-degree pressure angle the interference begins with a pinion of 17 teeth.

INTERFERENCE OF INTERNAL GEAR AND PINION

The following method of correction and equations are true for all combinations when the pinion has less than 55 teeth.

Referring to Fig. 28:

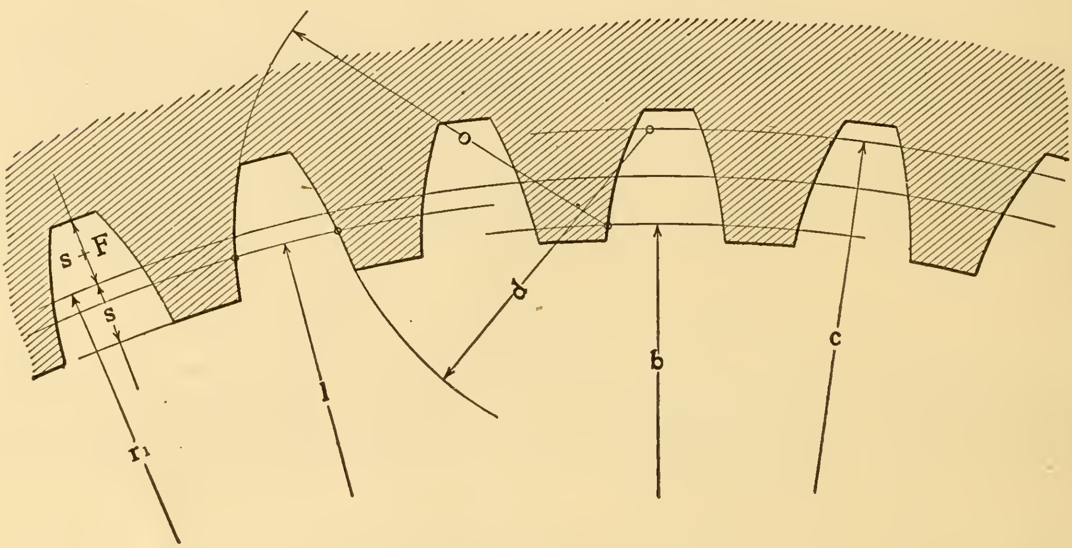


FIG. 28. INTERFERENCE OF INTERNAL GEAR AND PINION.

Let

r_1 = the pitch radius of gear.
r_2 = the pitch radius of pinion.
b = the radius of the base circle.
c = the radius of the correction circle.
d = the radius of the rounding off circle.
e = the radius of the interference circle.
o = the radius of the tooth cutting.
a = the pressure angle.

Then

$$b = r_1 \cos a,$$

$$e = \tfrac{1}{2}(b + r_1),$$

$$c = \frac{r_2}{\cos a},$$

$$o = \tfrac{1}{4} r_1, \text{ and}$$

$$d = \sqrt{\left(\frac{r_1}{4}\right)^2 + \left(\frac{r_2}{4}\right)^2}.$$

EXISTING TOOTH STANDARDS—BROWN & SHARPE'S

The Brown & Sharpe system is perhaps the best known; the angle of obliquity being 14½ degrees.

Addendum, $= 0.3183\ p^1$ or $\dfrac{1}{p}$

Dedendum, $= 0.3683\ p^1$ or $\dfrac{1.157}{p}$

Working depth, $= 0.6366\ p^1$ or $\dfrac{2}{p}$

Whole depth, $= 0.6866\ p^1$ or $\dfrac{2.157}{p}$

Clearance, $= 0.05\ p^1$ or $\dfrac{0.157}{p}$

In which p^1 = circular pitch, and p = diametral pitch.

GRANT'S

The Grant system has an angle of obliquity of 15 degrees, otherwise it is the same as Brown & Sharpe's. This system is used on the Bilgram generator.

SELLERS'

Wm. Sellers & Co. adopted a form of tooth some 32 years ago in which the angle of obliquity was 20 degrees, otherwise the same as Brown & Sharpe's.

HUNT'S

The C. W. Hunt Co. have a standard in which the angle of obliquity is 14½ degrees; the tooth parts being as follows:

Addendum, $= 0.25\ p^1$, or $\dfrac{0.7854}{p}$

Dedendum, $= 0.30\ p^1$, or $\dfrac{0.9424}{p}$

Working depth, $= 0.50\ p^1$, or $\dfrac{1.5708}{p}$

Whole depth, $= 0.55\ p^1$, or $\dfrac{1.7278}{p}$

Clearance, $= 0.05\ p^1$, or $\dfrac{0.157}{p}$

THE AUTHOR'S

This system, presented in connection with a discussion of an interchangeable involute gear-tooth system at the December, 1908, meeting of the American Society of Mechanical Engineers, was originally published in AMERICAN

Machinist, June 6, 1907. Angle of obliquity is 20 degrees. Balance of the tooth parts being the same as the Hunt system described above.

FELLOWS'

The stubbed tooth adopted by the Fellows Gear Shaper Company has an angle of obliquity of 20 degrees. The tooth parts, however, do not bear a definite relation to the pitch; the addendum being made to correspond to a diametral pitch one or two sizes finer, as:

Actual pitch	2	2½	3	4	5	7	8	10	12	14
Pitch depth	2½	3	4	5	7	9	10	12	14	18

The upper figures indicate the diametral pitch for tooth spacing and the lower figures indicate the diametral pitch from which the depth is taken.

In this system the addendum varies from 0.264 to 0.226 of the circular pitch;

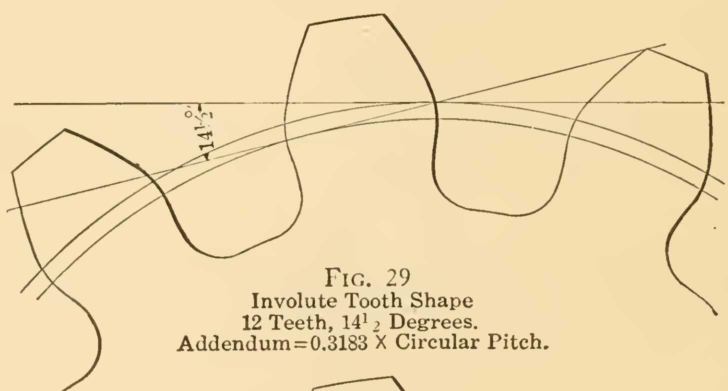


Fig. 29
Involute Tooth Shape
12 Teeth, 14½ Degrees.
Addendum=0.3183 X Circular Pitch.

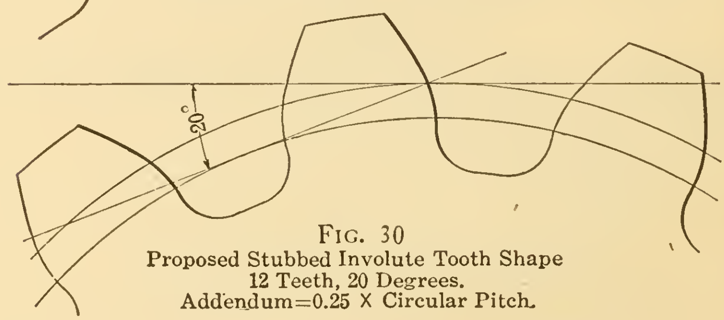


Fig. 30
Proposed Stubbed Involute Tooth Shape
12 Teeth, 20 Degrees.
Addendum=0.25 X Circular Pitch.

COMPARATIVE FORMS OF 14½-DEGREE AND 20-DEGREE STANDARDS.

0.25, which is the addendum for the Hunt and the author's standard, is a rough mean.

The author's standard tooth is shown in Fig. 30 for comparison with the 14½-degree standard in Fig. 29.

Wilfred Lewis discussed tooth standards before the American Society of Mechanical Engineers, 1900, as follows:

"About 30 years ago, when I first began to study the subject, the only system

of gearing that stood in much favor with machine-tool builders was the cycloidal.

"For some time thereafter William Sellers & Co., with whom I was connected, continued to use cylindrical gearing made by cutters of the true cycloidal shape, but the well-known objection to this form of tooth began to be felt, and possibly 25 years ago my attention was turned to the advantages of an involute system. The involute systems in use at that time were the ones here described as standard, having 14½ degrees' obliquity, and another recommended by Willis having an obliquity of 15 degrees. Neither of these satisfied the requirements of an interchangeable system, and with some hesitation I recommended a 20-degree system, which was adopted by William Sellers & Co., and has worked to their satisfaction ever since. I did not at that time have quite the courage of my convictions that the obliquity should be 22½ degrees or one-fourth of a right angle. Possibly, however, the obliquity of 20 degrees may still be justified by reducing the addendum from a value of one to some fraction thereof, but I would not undertake at this time to say which of the two methods I would prefer.

"I brought up the same question nine years ago before the Engineers' Club of Philadelphia, and said at that time that a committee ought to be appointed to investigate and report on an interchangeable system of gearing. We have an interchangeable system of screw threads, of which everybody knows the advantage, and there is no reason why we should not have a standard system of gearing, so that any gear of a given pitch will run with any other gear of the same pitch."

A UNIVERSAL STANDARD

The question of recommending a standard for gear teeth is now in the hands of a special committee appointed by the American Society of Mechanical Engineers. This is a matter of the utmost importance. There are now many different standards in this country alone. Also, owing to the inaccuracies of the present form of teeth, gears for heavy work are generally designed to meet the requirements of that particular pair, changing either the depth or tooth, angle, or both, just enough to avoid any doctoring of the involute; this is expensive. I do not know of any gears heavier than 4½ inch circular pitch that of late years have been cut 14½ degrees. It is common practice to use a modified tooth of some sort for all gears over one diametral pitch. If this is necessary for heavy gears, why not for smaller ones?

To do away with this multiplicity of standards, and bring a universally accepted standard gear-tooth system out of the present chaos, is the work before this committee.

Of course, there will be many natural objections raised to any such change, as there always is, but the amount of expense and trouble entailed in the adoption of such a standard is a small measure of the benefit that will be ultimately derived. Everybody who has studied this matter must admit that a standard tooth of some kind is desired. Putting it off will do no good, unless perchance, some genius discovers a better form than the involute, which is not likely. Therefore, let us hope for a speedy conclusion of the work of this committee and a standard that is a standard.

MODIFIED TEETH

A common method of modifying the involute tooth to avoid either interference, undercutting, or the necessity of departing from the true outline is by shortening the dedendum and lengthening the addendum of the pinion tooth. The opposite treatment is given the gear tooth, the dedendum being made deeper to accommodate the added addendum of the pinion and the addendum of the gear correspondingly short. This method is employed on all bevel-gear generating machines for angles less than 20 degrees to avoid interference, the amount of correction depending, of course, upon both the number of teeth being cut and the number of teeth in the engaging gear, or, in other words, depending upon the position of the base line.

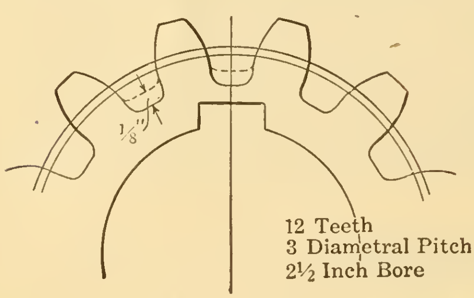


FIG. 31. SHORTENING THE DEDENDUM TO STRENGTHEN KEYWAY.

On bevel-gear generating machines it is the practice to make no modification in the angle for a 20-degree tooth when cutting a depth equal to $0.6866p'$. For this depth of tooth and a pressure angle of 20 degrees interference beginning at 17 teeth, enough roll, however, can be given the blank to allow the generating tool to undercut the flank of the tooth, and avoid interference without any correction of the tooth parts. This is not the case, however, when cutting the standard 14½ or 15 degrees on account of limitation in the movements of the machine. This modification in the tooth parts for bevel gears is accomplished by shifting the face angles and outside diameters, the pinion being enlarged and the gear reduced.

The dedendum of the pinion is sometimes shortened for another reason: Often the bore is so large as to leave insufficient stock between the bottom of the teeth and the keyseat. See Fig. 31. When the pinion cannot be enlarged or the bore reduced the only possible recourse is to shorten the dedendum, taking the amount shortened from the point of the gear tooth. This practice

is not to be recommended although extensively used; it would be much better to apply the short tooth of increased obliquity to such cases.

THE OCTOID

All bevel-gear generating machines operate on the octoid system, and not the involute, as is generally supposed.

An involute spur gear may be generated by the action of a tool representing the rack tooth, as illustrated by Fig. 9. In generating a bevel gear, however, the tool representing the engaging rack tooth must always travel toward the apex of the gear being cut, swinging in a partial circle instead of travelling on

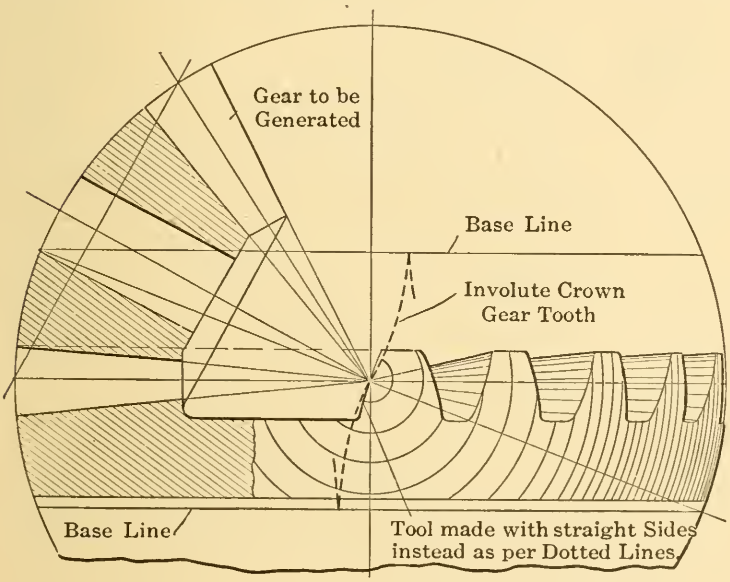


FIG. 32. GENERATING THE OCTOID TOOTH.

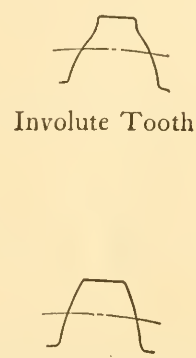


FIG. 33.

American Machinist

a straight line in the direction of the rotation of the gear, as is the case when cutting a spur gear. The base of the bevel-gear tooth is, therefore, a crown gear instead of a rack.

An involute crown gear theoretically correct will have curved instead of straight sides as shown in Fig. 32. As it is not practical to make the generating tools this peculiar shape, they are made straight sided and the octoid tooth is the result.

THE LINE OF ACTION

There is a definite relation between the circle or line which will describe the tooth outline and the line of action. Thus, if the line of action is in the form of a circle, as shown in Fig. 34, that circle of which this line is a segment will describe the tooth outline if rolled upon the pitch circle. The difficulties encoun-

tered in the general application of this law are well illustrated by George B. Grant in section 32 of his "Treatise on Gear Wheels," as follows: "This accidental and occasionally useful feature of the rolled curve has generally been made to serve as a basis for the general theory of the tooth curve, and it is responsible for the usually clumsy and limited treatment of that theory. The general law is simple enough to define, but it is so difficult to apply, that but one tooth curve, the cycloidal, which happens to have the circle for a roller, can be intelligently handled by it, and the natural result is, that that curve has received the bulk of the attention.

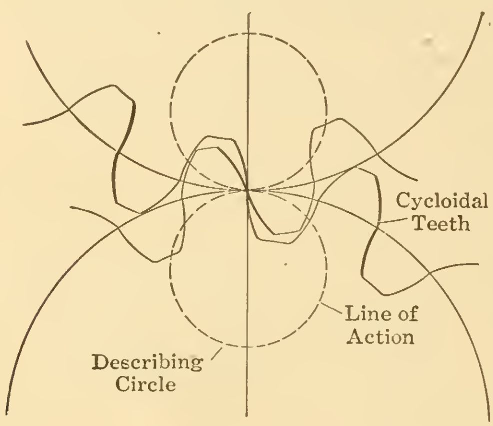


FIG. 34. RELATION OF THE LINE OF ACTION TO THE DESCRIBING CIRCLE.

For example, the simplest and best of all the odontoids (pure form of tooth curve), the involute, is entirely beyond its reach, because its roller is the logarithmic spiral, a transcendental curve that can be reached only by the higher mathematics.

No tooth curve, which, like the involute, crosses the pitch line at any angle but a right angle, can be traced by a point in a simple curve. The tracing point must be the pole of a spiral, and therefore a mechanical impossibility. A practicable rolled odontoid must cross the pitch line at right angles.

To use the rolled curve theory as a base of operations will confine the discussion to the cycloidal tooth, for the involute can only be reached by abandoning its true logarithmic roller, and taking advantage of one of its peculiar properties, and the segmental, sinusoidal, parabolic, and pin tooth, as well as most other available odontoids, cannot be discussed at all."

THE LAW OF TOOTH CONTACT

To transmit uniform motion, any form of tooth curve is subject to this general law: "The common normal to the tooth must pass through the pitch point." That is, a line drawn from the pitch point P through the contact point of any pair of teeth, as at b, must be at right angles or normal to the engaging portions of both teeth. See Fig. 35.

In the involute system the line of action always passes through the pitch point P, and the engaging teeth take their base from the points f and y, where the line of action intersects the base circles. Conversely, a line drawn from the instantaneous radii of any two teeth engaged will pass through their

point of contact if the teeth are correctly formed. For example: In Fig. 35, the point of contact between the teeth *C* and *D* is at *b*, on the line of action, the

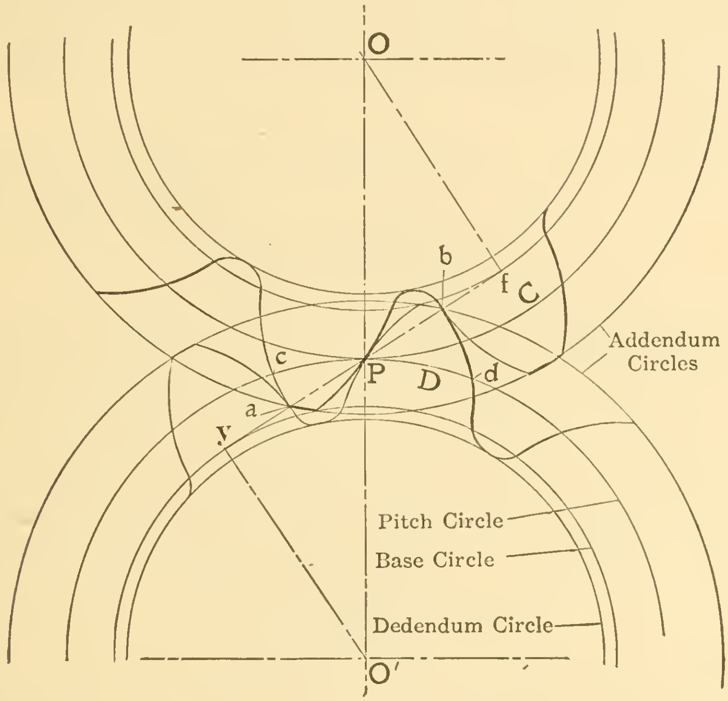


FIG. 35. THE ARC OF ACTION.

radius of the engaged portion of the tooth *C* is at *f*, and the radius of the tooth *D* is at *y*, fulfilling the required conditions.

THE ARC OF ACTION

The tooth action between two gears is between the points *a* and *b*, at which points the line of action intersects the addendum circles of the two gears. The actual length of contact is along the pitch lines occupied by the teeth whose addendum circles intersect the line of action, or between the points *c* and *d*. See Fig. 35.

The distance *P*—*d* passed over while the point of contact approaches the pitch point is the arc of approach, the distance *P*—*c* passed over as the point of contact leaves the pitch point is the arc of recess.

By increasing the addendum of the driving gear the arc of approach is reduced and the arc of recess is increased. The friction of the arc of approach is greater than in the arc of recess.

THE BUTTRESSED TOOTH

The buttressed tooth shown in Fig. 36 is described by Professor Robert Willis in a paper published in the Transactions of the Institute of Civil Engineers, London, 1838. It is apparent that the object is to obtain a strong tooth

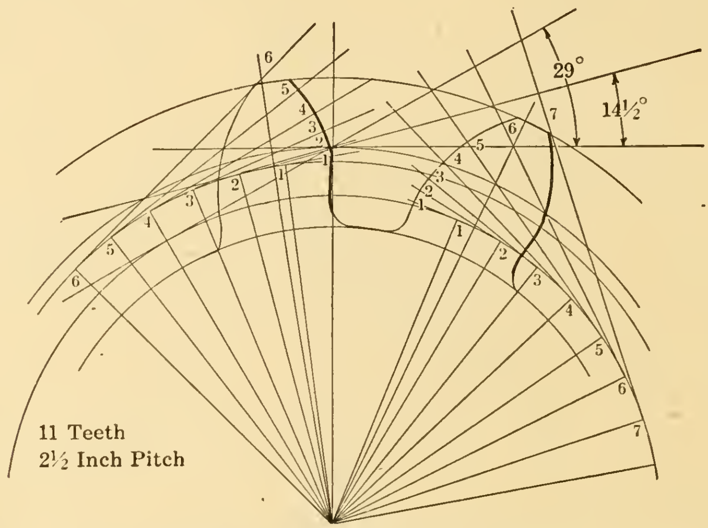


FIG. 36. THE BUTTRESSED TOOTH.

for a pair of gears operating continuously in one direction. This is accomplished by increasing the angle of obliquity of the back of the tooth, the face of the tooth being any angle desired. If the back of the tooth is correctly formed the

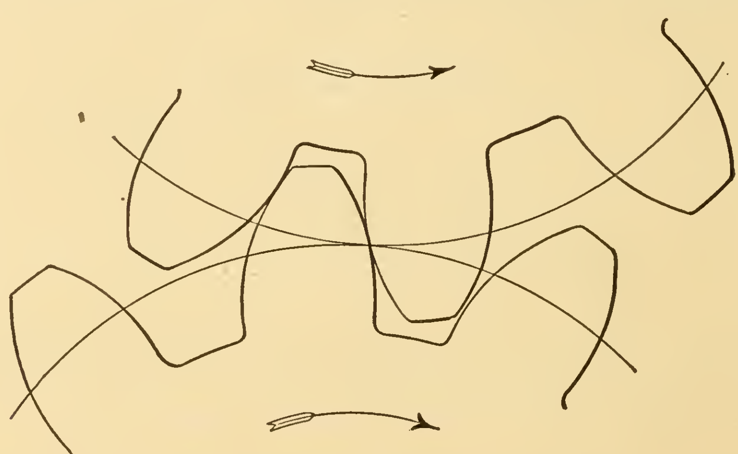


FIG. 37. BUTTRESSED TEETH IN CONTACT

gears will operate satisfactorily in either direction although with an increased pressure on their bearings when using the back face of the teeth owing to the

increased obliquity of action. For many purposes there is no objection to this, and it is a great wonder that this tooth is not more extensively used.

Of course, there must be a limit to the angle of the back of the tooth. For practical purposes the curve at the top of tooth at the back should not extend further than the center line of the tooth; for an addendum of $\frac{1}{p}$ or $0.6866p_1$, this will occur at an angle of about 32 degrees. A greater angle than this will subject the tooth to breakage at the point. In Fig. 37 is shown a pair of buttressed tooth gears in contact.

STEPPED GEARS

A stepped spur gear consists of two or more gears keyed to the same shaft, the teeth on each gear being slightly advanced. If mated with a similar gear the tooth contact will be increased, which increases the smoothness of action. A common form of this type of gear is that of two gears cast in one piece with a separating shroud. For a cut gear there must be a groove turned between the faces of sufficient width to allow the planing tool or cutter to clear. A tooth is placed opposite a space, when the gear is made in two sections.

HUNTING TOOTH

It has been customary to make a pair of cast tooth gears with a hunting tooth, in order that each tooth would engage all of the teeth in the mating gear, the idea being that they would eventually be worn into some indefinite but true shape. Some designers have even gone so far as to specify a pair of "hunting-tooth miter gears." That is, one "miter" gear would have, say, 24 teeth and its mate 25 teeth.

There never was any call for the introduction of the hunting tooth even in cast gears, but in properly cut gears any excuse for its use has certainly ceased to exist.

TEMPLET MAKING

In making the templets for gear teeth there are several points of importance to be kept in mind, namely:

Templets should be made of light sheet steel instead of zinc which is often employed; the surface of steel should be coppered by the application of blue vitriol.

For spiral or worm gears, templets should always be made for the normal pitch.

For spacing and tooth thickness, always use chordal measurements. Check the chordal distance over the end teeth of templet. This is of the utmost importance.

Put enough teeth in the templet to show the entire tooth action, and try the templets on centers before making up the cutters or formers.

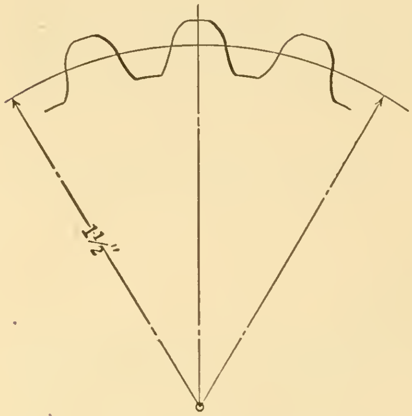


FIG. 38. TOOTH OUTLINE AS PHOTOGRAPHED FROM LARGE SCALE DRAWING.

It is a good idea to make a standard templet of each pitch as they are required, to try out other templets that must be made later on.

When a templet is required for a fine pitch gear it is good practice to lay out the teeth on white paper 10 or even 20 times the actual size and reduce by photography. On this drawing the center should be plainly marked and inclosed in a heavy circle, also a short section of the pitch line should be made heavy with a connecting radial line indicating the radius of pitch circle.

If the pitch radius required is $1\frac{1}{2}$ inches, it should be made, say, 15 inches on the drawing. The drawing is then photographed, the camera being set until the radial line, which was drawn 15 inches, measures $1\frac{1}{2}$ inches on the ground glass. See Fig. 38.

DEFINITION OF PITCHES

Diametral pitch is the number of teeth to each inch of the pitch diameter.

Circular pitch is the distance from the edge of one tooth to the corresponding edge of the next tooth measured along the pitch circle.

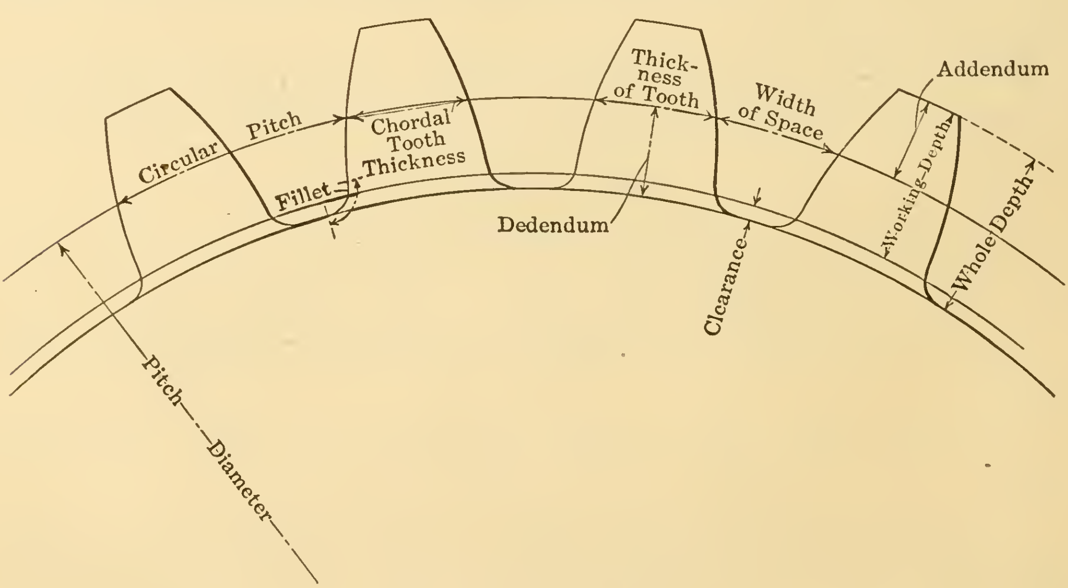


FIG. 39. TOOTH PARTS.

DIAMETRAL PITCH	CIRCULAR PITCH	THICKNESS OF TOOTH OF PITCH LINE	WHOLE DEPTH	DEDENDUM	ADDENDUM
½	6.2832″	3.1416″	4.3142″	2.3142″	2.0000″
¾	4.1888	2.0944	2.8761	1.5728	1.3333
1	3.1416	1.5708	2.1571	1.1571	1.0000
1¼	2.5133	1.2566	1.7257	0.9257	0.8000
1½	2.0944	1.0472	1.4381	0.7714	0.6666
1¾	1.7952	0.8976	1.2326	0.6612	0.5714
2	1.5708	0.7854	1.0785	0.5785	0.5000
2¼	1.3963	0.6981	0.9587	0.5143	0.4444
2½	1.2566	0.6283	0.8628	0.4628	0.4000
2¾	1.1424	0.5712	0.7844	0.4208	0.3636
3	1.0472	0.5236	0.7190	0.3857	0.3333
3½	0.8976	0.4488	0.6163	0.3306	0.2857
4	0.7854	0.3927	0.5393	0.2893	0.2500
5	0.6283	0.3142	0.4314	0.2314	0.2000
6	0.5236	0.2618	0.3595	0.1928	0.1666
7	0.4488	0.2244	0.3081	0.1653	0.1429
8	0.3927	0.1963	0.2696	0.1446	0.1250
9	0.3491	0.1745	0.2397	0.1286	0.1111
10	0.3142	0.1571	0.2157	0.1157	0.1000
11	0.2856	0.1428	0.1961	0.1052	0.0909
12	0.2618	0.1309	0.1798	0.0964	0.0833
13	0.2417	0.1208	0.1659	0.0890	0.0769
14	0.2244	0.1122	0.1541	0.0826	0.0714
15	0.2094	0.1047	0.1438	0.0771	0.0666
16	0.1963	0.0982	0.1348	0.0723	0.0625
17	0.1848	0.0924	0.1269	0.0681	0.0588
18	0.1754	0.0873	0.1198	0.0643	0.0555
19	0.1653	0.0827	0.1135	0.0609	0.0526
20	0.1571	0.0785	0.1079	0.0579	0.0500
22	0.1428	0.0714	0.0980	0.0526	0.0455
24	0.1309	0.0654	0.0898	0.0482	0.0417
26	0.1208	0.0604	0.0829	0.0445	0.0385
28	0.1122	0.0561	0.0770	0.0413	0.0357
30	0.1047	0.0524	0.0719	0.0386	0.0333
32	0.0982	0.0491	0.0674	0.0362	0.0312
34	0.0924	0.0462	0.0634	0.0340	0.0294
36	0.0873	0.0436	0.0599	0.0321	0.0278
38	0.0827	0.0413	0.0568	0.0304	0.0263
40	0.0785	0.0393	0.0539	0.0289	0.0250
42	0.0748	0.0374	0.0514	0.0275	0.0238
44	0.0714	0.0357	0.0490	0.0263	0.0227
46	0.0683	0.0341	0.0469	0.0252	0.0217
48	0.0654	0.0327	0.0449	0.0241	0.0208
50	0.0628	0.0314	0.0431	0.0231	0.0200
56	0.0561	0.0280	0.9385	0.0207	0.0178
60	0.0524	0.0262	0.0360	0.0193	0.0166

TABLE 1—DIAMETRAL PITCH

Relation between Diametral Pitch and Circular Pitch, with corresponding Tooth Dimensions

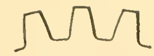
CIRCULAR PITCH	DIAMETRAL PITCH	THICKNESS OF TOOTH OF PITCH LINE	WHOLE DEPTH	DEDENDUM	ADDENDUM
6″	0.5236	3.0000″	4.1196″	2.2098″	1.9098″
5″	0.6283	2.5000	3.4330	1.8415	1.5915
4″	0.7854	2.0000	2.7464	1.4732	1.2732
3 1/2″	0.8976	1.7500	2.4031	1.2890	1.1140
3″	1.0472	1.5000	2.0598	1.1049	0.9550
2 3/4″	1.1424	1.3750	1.8882	1.0028	0.8754
2 1/2″	1.2566	1.2500	1.7165	0.9207	0.7958
2 1/4″	1.3963	1.1250	1.5449	0.8287	0.7162
2″	1.5708	1.0000	1.3732	0.7366	0.6366
1 7/8″	1.6755	0.9375	1.2874	0.6906	0.5968
1 3/4″	1.7952	0.8750	1.2016	0.6445	0.5570
1 5/8″	1.9333	0.8125	1.1158	0.5985	0.5173
1 1/2″	2.0944	0.7500	1.0299	0.5525	0.4775
1 7/16″	2.1855	0.7187	0.9870	0.5294	0.4576
1 3/8″	2.2848	0.6875	0.9441	0.5064	0.4377
1 5/16″	2.3936	0.6562	0.9012	0.4837	0.4178
1 1/4″	2.5133	0.6250	0.8583	0.4604	0.3979
1 3/16″	2.6465	0.5937	0.8156	0.4374	0.3780
1 1/8″	2.7925	0.5625	0.7724	0.4143	0.3581
1 1/16″	2.9568	0.5312	0.7295	0.3913	0.3382
1″	3.1416	0.5000	0.6866	0.3683	0.3183
15/16″	3.3510	0.4687	0.6437	0.3453	0.2984
7/8″	3.5904	0.4375	0.6007	0.3223	0.2785
13/16″	3.8666	0.4062	0.5579	0.2993	0.2586
3/4″	4.1888	0.3750	0.5150	0.2762	0.2387
11/16″	4.5696	0.3437	0.4720	0.2532	0.2189
5/8″	5.0265	0.3125	0.4291	0.2301	0.1989
9/16″	5.5851	0.2812	0.3862	0.2071	0.1790
1/2″	6.2832	0.2500	0.3433	0.1842	0.1592
7/16″	7.1808	0.2187	0.3003	0.1611	0.1393
2/5″	7.8540	0.2000	0.2746	0.1473	0.1273
3/8″	8.3776	0.1875	0.2575	0.1381	0.1194
1/3″	9.4248	0.1666	0.2287	0.1228	0.1061
5/16″	10.0531	0.1562	0.2146	0.1151	0.0995
2/7″	10.9956	0.1429	0.1962	0.1052	0.0909
1/4″	12.5664	0.1250	0.1716	0.0921	0.0796
2/9″	14.1372	0.1111	0.1526	0.0818	0.0707
1/5″	15.7080	0.1000	0.1373	0.0737	0.0637
3/16″	16.7552	0.0937	0.1287	0.0690	0.0592
1/6″	18.8496	0.0833	0.1144	0.0614	0.0531
1/7″	21.9911	0.0714	0.0981	0.0526	0.0455
1/8″	25.1327	0.0625	0.0858	0.0460	0.0398
1/9″	28.2743	0.0555	0.0763	0.0409	0.0354
1/10″	31.4159	0.0500	0.0687	0.0368	0.0318
1/16″	50.2655	0.0312	0.0429	0.0230	0.0199

TABLE 2—CIRCULAR PITCH

Relation between Circular Pitch and Diametral Pitch, with corresponding Tooth Dimensions

20 D. P.
0.1571 Inch C. P.

18 D. P.
0.1745 Inch C. P.



16 D. P.
0.1963 Inch C. P.

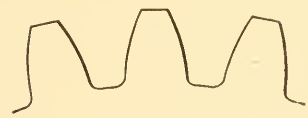
14 D. P.
0.2244 Inch C. P.

12 D. P.
0.2618 Inch C. P.

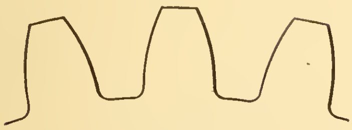
10 D. P.
0.3142 Inch C. P.

9 D. P.
0.3491 Inch C. P.

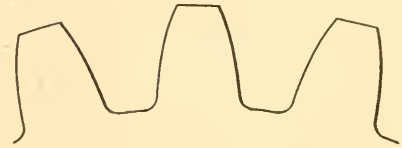
8 D. P.
0.3927 Inch C. P.



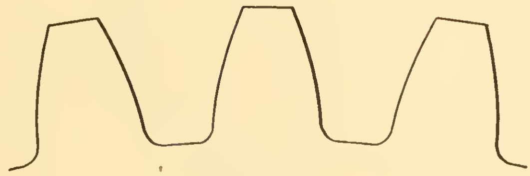
7 D. P.
0.4488 Inch C. P.



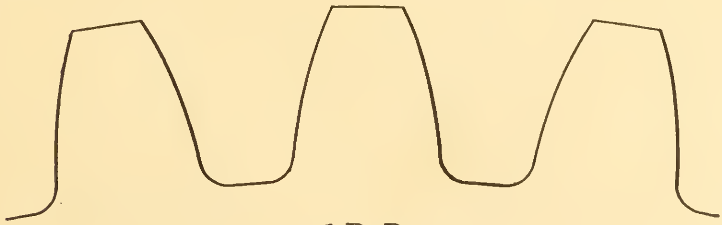
6 D. P.
0.5236 Inch C. P.



5 D. P.
0.6283 Inch C. P.

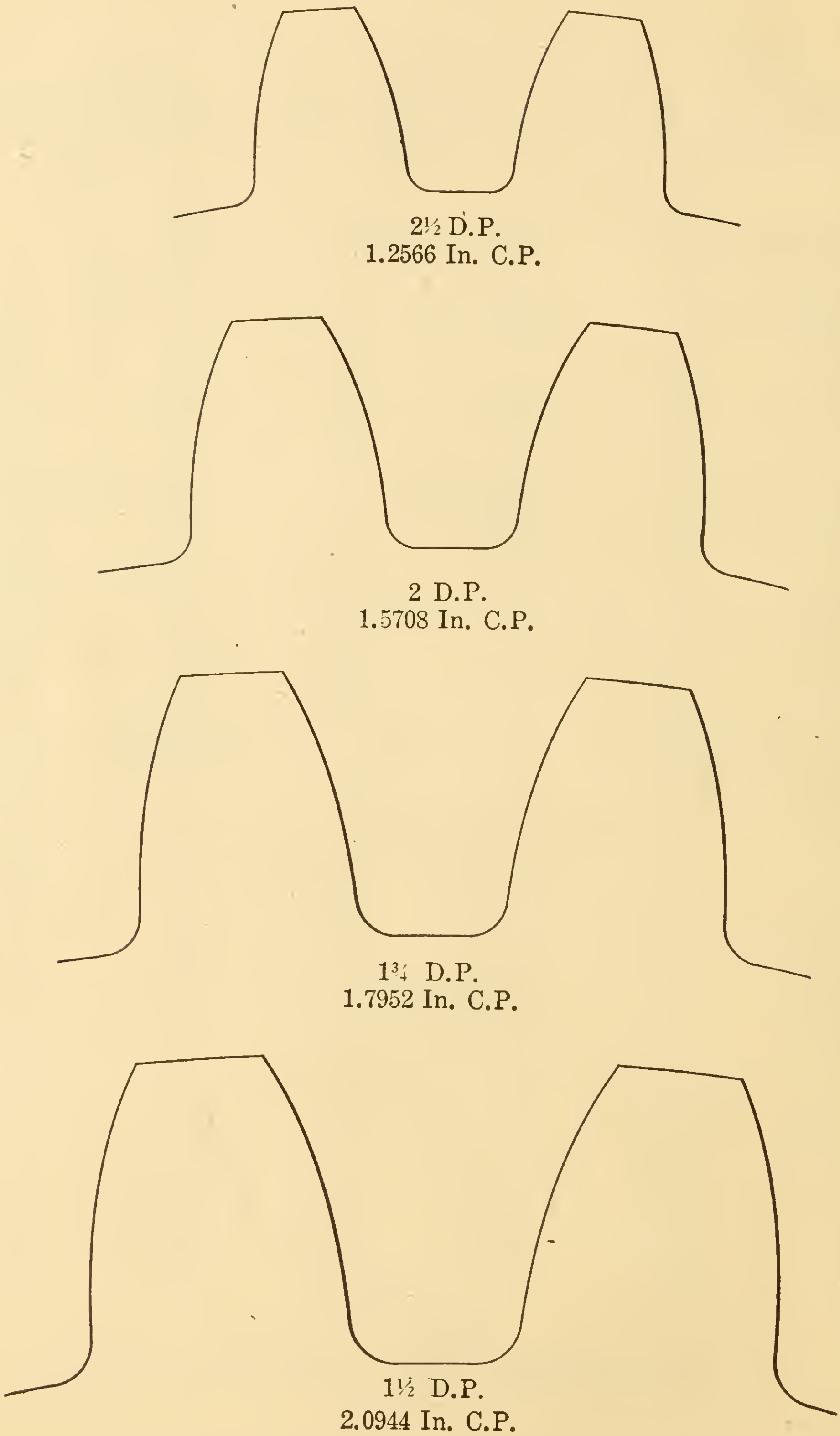


4 D. P.
0.7854 Inch C. P.

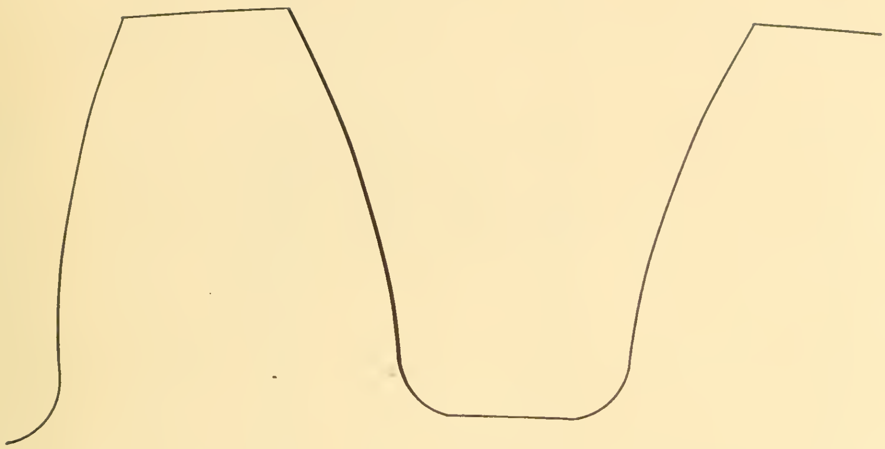


3 D. P.
1.0472 Inch C. P.

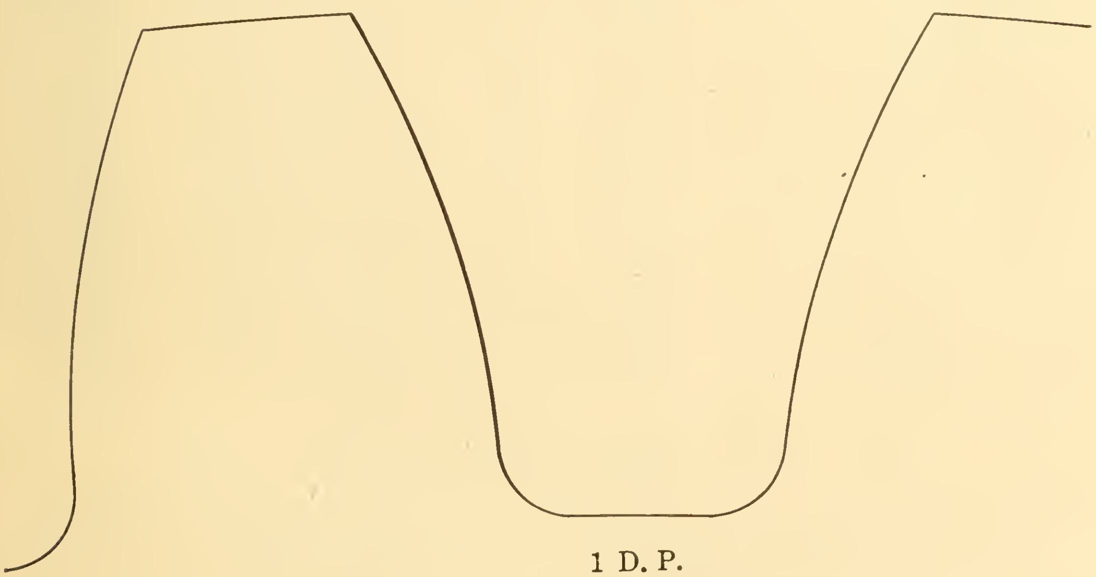
COMPARATIVE SIZES OF GEAR TEETH—INVOLUTE FORM.



COMPARATIVE SIZES OF GEAR TEETH—INVOLUTE FORMS.



1¼ D. P.
2.5133 Inch C. P.



1 D. P.
3.1416 Inch C. P.

COMPARATIVE SIZES OF GEAR TEETH—INVOLUTE FORM.

NO. TEETH	PITCH DIAMETER	NO. TEETH	PITCH DIAMETER	NO. TEETH	PITCH DIAMETER	NO. TEETH	PITCH DIAMETER
8	2.550	43	13.687	78	24.828	113	35.968
9	2.870	44	14.006	79	25.146	114	36.286
10	3.183	45	14.324	80	25.465	115	36.605
11	3.501	46	14.642	81	25.783	116	36.923
12	3.820	47	14.961	82	26.101	117	37.241
13	4.138	48	15.279	83	26.420	118	37.560
14	4.456	49	15.597	84	26.738	119	37.878
15	4.775	50	15.915	85	27.056	120	38.196
16	5.093	51	16.234	86	27.375	121	38.514
17	5.411	52	16.552	87	27.693	122	38.833
18	5.730	53	16.870	88	28.011	123	39.151
19	6.048	54	17.189	89	28.330	124	39.469
20	6.366	55	17.507	90	28.648	125	39.788
21	6.684	56	17.825	91	28.966	126	40.106
22	7.003	57	18.144	92	29.284	127	40.424
23	7.321	58	18.462	93	29.603	128	40.743
24	7.639	59	18.780	94	29.921	129	41.061
25	7.958	60	19.099	95	30.239	130	41.379
26	8.276	61	19.417	96	30.558	131	41.697
27	8.594	62	19.735	97	30.876	132	42.016
28	8.913	63	20.053	98	31.194	133	42.334
29	9.231	64	20.372	99	31.513	134	42.652
30	9.549	65	20.690	100	31.831	135	42.971
31	9.868	66	21.008	101	32.148	136	43.289
32	10.186	67	21.327	102	32.468	137	43.607
33	10.504	68	21.645	103	32.785	138	43.926
34	10.822	69	21.963	104	33.103	139	44.243
35	11.141	70	22.282	105	33.421	140	44.562
36	11.459	71	22.600	106	33.740	141	44.881
37	11.777	72	22.918	107	34.058	142	45.199
38	12.096	73	23.237	108	34.376	143	45.517
39	12.414	74	23.555	109	34.695	144	45.835
40	12.732	75	23.873	110	35.013	145	46.154
41	13.051	76	24.192	111	35.331	146	46.472
42	13.369	77	24.510	112	35.650	147	46.790

TABLE 3—PITCH DIAMETERS FOR ONE-INCH CIRCULAR PITCH

Teeth from 8 to 147

FOR ANY OTHER PITCH—MULTIPLY BY THAT PITCH

METRIC PITCH

The module is the addendum, or the pitch diameter in millimeters divided by the number of teeth in the gear.

The pitch diameter in millimeters is the module multiplied by the number of teeth in the gear. All calculations are in millimeters.

M = module (addendum)
D' = pitch diameter
D = outside diameter
N = number of teeth
W = working depth of tooth
W' = whole depth of tooth
t = thickness of tooth at pitch line
f = clearness
r = root

$$M = \frac{D'}{N} \text{ or } \frac{D}{N+2}, \qquad D' = NM, \qquad D = (N+2)\,M$$

$$N = \frac{D'}{M} \text{ or } \frac{D}{M} - 2, \qquad W = 2M \qquad W' = W + f$$

$$t = M\,1.5708, \qquad f = \frac{M\,1.5708}{10}, \text{ or } M\,0.157, \qquad r = M + f$$

MODULE	ENGLISH DIAMETRAL PITCH	MODULE	ENGLISH DIAMETRAL PITCH	MODULE	ENGLISH DIAMETRAL PITCH
0.5	50.800			7	3.628
1	25.400	3	8.466	8	3.175
1.25	20.320	3.5	7.257	9	2.822
1.5	16.933	4	6.350	10	2.540
1.75	14.514	4.5	5.644	11	2.309
2	12.700	5	5.080	12	2.117
2.25	11.288	5.5	4.618	14	1.814
2.5	10.160	6	4.233	16	1.587
2.75	9.236				

Module in Millimeters

TABLE 4—PITCHES COMMONLY USED

CHORDAL PITCH

The chordal pitch is the shortest distance between two teeth measured on the pitch line, in other words, the distance to which the dividers would be set to space around the gear on the pitch line. This pitch is not used except for laying out large gears and segments that cannot be cut on a gear cutter. For such cases, also for laying out templets, it is absolutely necessary to use the chordal pitch, as the chordal pitch of the pinion is different from the chordal pitch of the gear, the circular pitch of each being equal.

N = number of teeth,
C' = chordal pitch,

D' = pitch diameter,
e = sine of one half of angle subtended by side at center.

$$e = \text{sine}\ \frac{180^\circ}{N}.$$

$$D' = \frac{C'}{e}.$$

$$C' = D'\ e, \text{ or } D'\ sine\ \frac{180^\circ}{N}.$$

Table 5, diameters for chordal pitch, will be found useful for sprocket gears.

NO. TEETH	PITCH DIAMETER	NO. TEETH	PITCH DIAMETER	NO. TEETH	PITCH DIAMETER	NO. TEETH	PITCH DIAMETER
4	1.414	39	12.427	74	23.562	109	34.701
5	1.701	40	12.746	75	23.880	110	35.019
6	2.000	41	13.064	76	24.198	111	35.337
7	2.305	42	13.382	77	24.517	112	35.655
8	2.613	43	13.699	78	24.835	113	35.974
9	2.924	44	14.018	79	25.153	114	36.292
10	3.236	45	14.335	80	25.471	115	36.610
11	3.549	46	14.653	81	25.790	116	36.929
12	3.864	47	14.972	82	26.108	117	37.247
13	4.179	48	15.291	83	26.426	118	37.565
14	4.494	49	15.608	84	26.744	119	37.883
15	4.810	50	15.927	85	27.063	120	38.202
16	5.126	51	16.244	86	27.381	121	38.520
17	5.442	52	16.562	87	27.699	122	38.838
18	5.759	53	16.880	88	28.017	123	39.156
19	6.076	54	17.200	89	28.335	124	39.475
20	6.392	55	17.516	90	28.654	125	39.793
21	6.710	56	17.835	91	28.972	126	40.111
22	7.027	57	18.152	92	29.290	127	40.429
23	7.344	58	18.471	93	29.608	128	40.748
24	7.661	59	18.789	94	29.927	129	41.066
25	7.979	60	19.107	95	30.245	130	41.384
26	8.297	61	19.425	96	30.563	131	41.703
27	8.614	62	19.744	97	30.881	132	42.021
28	8.931	63	20.062	98	31.200	133	42.339
29	9.249	64	20.380	99	31.518	134	42.657
30	9.567	65	20.698	100	31.836	135	42.976
31	9.884	66	21.016	101	32.154	136	43.294
32	10.202	67	21.335	102	32.473	137	43.612
33	10.520	68	21.653	103	32.791	138	43.931
34	10.838	69	21.971	104	33.109	139	44.249
35	11.156	70	22.289	105	33.428	140	44.567
36	11.474	71	22.607	106	33.740	141	44.890
37	11.791	72	22.926	107	34.058	142	45.204
38	12.110	73	23.244	108	34.376	143	45.522

TABLE 5—PITCH DIAMETERS FOR ONE-INCH CHORDAL PITCH

Teeth from 4 to 143

FOR ANY OTHER PITCH MULTIPLY BY THAT PITCH

CHORDAL THICKNESS OF TEETH

In order to correctly measure the teeth, the chordal thickness must be used, as illustrated by Fig. 40. Also as the location of the pitch line on the sides of

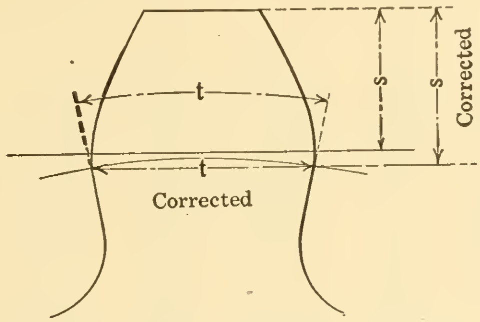


FIG. 40. CHORDAL TOOTH THICKNESS.

the teeth falls below the pitch line at the center of tooth. The measurement for the addendum must also be corrected, if any degree of accuracy is expected. Table 6, gives these corrected dimensions for various standard pitches.

Number of Teeth.	1 D. P.		1½ D. P.		2 D. P.		2½ D. P.		Number of Teeth.
	Thickness.	Addendum.	Thickness.	Addendum.	Thickness.	Addendum.	Thickness.	Addendum.	
8	1.5607	1.0769	1.0405	0.7179	0.7804	0.5385	0.6243	0.4308	8
9	1.5628	1.0684	1.0419	0.7123	0.7814	0.5342	0.6251	0.4273	9
10	1.5643	1.0616	1.0429	0.7077	0.7821	0.5308	0.6257	0.4246	10
11	1.5654	1.0559	1.0436	0.7039	0.7827	0.5279	0.6261	0.4224	11
12	1.5663	1.0514	1.0442	0.7009	0.7831	0.5257	0.6265	0.4206	12
14	1.5675	1.0440	1.0450	0.6960	0.7837	0.5220	0.6270	0.4176	14
17	1.5686	1.0362	1.0457	0.6908	0.7843	0.5181	0.6274	0.4145	17
21	1.5694	1.0294	1.0463	0.6863	0.7847	0.5147	0.6277	0.4118	21
26	1.5698	1.0237	1.0465	0.6825	0.7849	0.5118	0.6279	0.4095	26
35	1.5702	1.0176	1.0468	0.6784	0.7851	0.5088	0.6281	0.4070	35
55	1.5706	1.0112	1.0471	0.6741	0.7853	0.5056	0.6282	0.4045	55
135	1.5707	1.0047	1.0471	0.6698	0.7853	0.5023	0.6283	0.4019	135

Number of Teeth.	3 D. P.		3½ D. P.		4 D. P.		5 D. P.		Number of Teeth.
	Thickness.	Addendum.	Thickness.	Addendum.	Thickness.	Addendum.	Thickness.	Addendum.	
8	0.5202	0.3589	0.4459	0.3077	0.3902	0.2692	0.3121	0.2154	8
9	0.5209	0.3561	0.4465	0.3052	0.3907	0.2671	0.3126	0.2137	9
10	0.5214	0.3538	0.4469	0.3033	0.3911	0.2654	0.3129	0.2123	10
11	0.5218	0.3519	0.4473	0.3017	0.3913	0.2640	0.3131	0.2112	11
12	0.5221	0.3505	0.4475	0.3004	0.3916	0.2628	0.3133	0.2103	12
14	0.5225	0.3480	0.4479	0.2983	0.3919	0.2610	0.3135	0.2088	14
17	0.5228	0.3454	0.4482	0.2961	0.3921	0.2590	0.3137	0.2072	17
21	0.5231	0.3431	0.4485	0.2941	0.3923	0.2573	0.3139	0.2059	21
26	0.5233	0.3412	0.4485	0.2925	0.3925	0.2559	0.3140	0.2047	26
35	0.5234	0.3392	0.4486	0.2907	0.3926	0.2544	0.3140	0.2035	35
55	0.5235	0.3371	0.4487	0.2889	0.3927	0.2528	0.3141	0.2022	55
135	0.5236	0.3349	0.4488	0.2871	0.3927	0.2512	0.3141	0.2009	135

TABLE 6—CHORDAL THICKNESSES AND ADDENDA OF GEAR TEETH OF DIAMETRAL PITCH

Boston Gear Works

Number of Teeth.	6 D. P. Thickness.	6 D. P. Addendum.	7 D. P. Thickness.	7 D. P. Addendum.	8 D. P. Thickness.	8 D. P. Addendum.	9 D. P. Thickness.	9 D. P. Addendum.	Number of Teeth.
8	0.2601	0.1795	0.2230	0.1538	0.1951	0.1346	0.1734	0.1197	8
9	0.2605	0.1781	0.2233	0.1526	0.1954	0.1336	0.1736	0.1187	9
10	0.2607	0.1769	0.2235	0.1517	0.1955	0.1327	0.1738	0.1180	10
11	0.2609	0.1760	0.2236	0.1508	0.1957	0.1320	0.1739	0.1173	11
12	0.2610	0.1752	0.2238	0.1502	0.1958	0.1314	0.1740	0.1168	12
14	0.2612	0.1740	0.2239	0.1491	0.1959	0.1305	0.1742	0.1160	14
17	0.2614	0.1727	0.2241	0.1480	0.1961	0.1295	0.1743	0.1151	17
21	0.2616	0.1716	0.2242	0.1471	0.1962	0.1287	0.1744	0.1144	21
26	0.2616	0.1706	0.2243	0.1462	0.1962	0.1280	0.1744	0.1137	26
35	0.2617	0.1696	0.2243	0.1454	0.1963	0.1272	0.1745	0.1131	35
55	0.2618	0.1685	0.2244	0.1445	0.1963	0.1264	0.1745	0.1124	55
135	0.2618	0.1675	0.2244	0.1435	0.1963	0.1256	0.1745	0.1116	135

Number of Teeth.	10 D. P. Thickness.	10 D. P. Addendum.	11 D. P. Thickness.	11 D. P. Addendum.	12 D. P. Thickness.	12 D. P. Addendum.	13 D. P. Thickness.	13 D. P. Addendum.	Number of Teeth.
8	0.1561	0.1077	0.1419	0.0979	0.1301	0.0897	0.1201	0.0828	8
9	0.1563	0.1068	0.1421	0.0971	0.1302	0.0890	0.1202	0.0822	9
10	0.1564	0.1061	0.1422	0.0965	0.1304	0.0885	0.1203	0.0816	10
11	0.1565	0.1056	0.1423	0.0960	0.1305	0.0880	0.1204	0.0812	11
12	0.1566	0.1051	0.1424	0.0956	0.1305	0.0876	0.1205	0.0809	12
14	0.1567	0.1044	0.1425	0.0949	0.1306	0.0870	0.1206	0.0803	14
17	0.1569	0.1036	0.1426	0.0942	0.1307	0.0863	0.1207	0.0797	17
21	0.1569	0.1029	0.1427	0.0936	0.1308	0.0858	0.1207	0.0792	21
26	0.1570	0.1024	0.1427	0.0931	0.1308	0.0853	0.1207	0.0787	26
35	0.1570	0.1018	0.1427	0.0925	0.1309	0.0848	0.1208	0.0782	35
55	0.1571	0.1011	0.1428	0.0919	0.1309	0.0843	0.1208	0.0777	55
135	0.1571	0.1005	0.1428	0.0913	0.1309	0.0837	0.1208	0.0772	135

Number of Teeth.	14 D. P. Thickness.	14 D. P. Addendum.	15 D. P. Thickness.	15 D. P. Addendum.	16 D. P. Thickness.	16 D. P. Addendum.	17 D. P. Thickness.	17 D. P. Addendum.	Number of Teeth.
8	0.1115	0.0769	0.1040	0.0718	0.0975	0.0673	0.0918	0.0633	8
9	0.1116	0.0763	0.1042	0.0712	0.0977	0.0669	0.0919	0.0628	9
10	0.1117	0.0758	0.1043	0.0709	0.0978	0.0664	0.0920	0.0624	10
11	0.1118	0.0754	0.1044	0.0704	0.0978	0.0659	0.0921	0.0621	11
12	0.1119	0.0751	0.1044	0.0701	0.0979	0.0657	0.0921	0.0618	12
14	0.1119	0.0746	0.1045	0.0696	0.0980	0.0652	0.0922	0.0614	14
17	0.1120	0.0740	0.1046	0.0691	0.0980	0.0648	0.0923	0.0609	17
21	0.1121	0.0735	0.1046	0·0686	0.0981	0.0643	0.0923	0.0605	21
26	0.1121	0.0731	0.1046	0.0682	0.0981	0.0640	0.0923	0.0602	26
35	0.1122	0.0727	0.1047	0.0678	0.0981	0.0636	0.0924	0.0598	35
55	0.1122	0.0722	0.1047	0.0674	0.0981	0.0632	0.0924	0.0595	55
135	0.1122	0.0718	0.1047	0.0670	0.0981	0.0628	0.0924	0.0591	135

Chordal Thicknesses and Addenda of Gear Teeth of Diametral Pitch—*Continued*

Number of Teeth.	18 D. P. Thickness.	18 D. P. Addendum.	19 D. P. Thickness.	19 D. P. Addendum.	20 D. P. Thickness.	20 D. P. Addendum.	24 D. P. Thickness.	24 D. P. Addendum.	Number of Teeth.
8	0.0867	0.0598	0.0821	0.0567	0.0780	0.0538	0.0650	0.0448	8
9	0.0868	0.0593	0.0822	0.0562	0.0781	0.0534	0.0651	0.0445	9
10	0.0869	0.0589	0.0823	0.0558	0.0782	0.0530	0.0651	0.0443	10
11	0.0869	0.0586	0.0824	0.0555	0.0783	0.0528	0.0652	0.9439	11
12	0.0870	0.0584	0.0824	0.0553	0.0784	0.0525	0.0653	0.0437	12
14	0.0871	0.0580	0.0825	0.0549	0.0784	0.0522	0.0653	0.0435	14
17	0.0871	0.0575	0.0826	0.0545	0.0784	0.0518	0.0653	0.0432	17
21	0.0872	0.0572	0.0826	0.0542	0.0785	0.0514	0.0654	0.0429	21
26	0.0872	0.0568	0.0826	0.0538	0.0785	0.0511	0.0654	0.0426	26
35	0.0872	0.0565	0.0826	0.0535	0.0785	0.0508	0.0654	0.0424	35
55	0.0873	0.0562	0.0827	0.0532	0.0785	0.0505	0.0654	0.0421	55
135	0.0873	0.0558	0.0827	0.0528	0.0785	0.0502	0.0654	0.0419	135

Chordal Thicknesses and Addenda of Gear Teeth of Diametral Pitch—*Continued*

Number of Teeth.	⅝″ C. P. Thickness.	⅝″ C. P. Addendum.	¾″ C. P. Thickness.	¾″ C. P. Addendum.	⅞″ C. P. Thickness.	⅞″ C. P. Addendum.	1″ C. P. Thickness.	1″ C. P. Addendum.	Number of Teeth.
8	0.3105	0.2142	0.3725	0.2570	0.4347	0.2997	0.4968	0.3426	8
9	0.3109	0.2125	0.3730	0.2550	0.4353	0.2976	0.4974	0.3400	9
10	0.3112	0.2112	0.3734	.02534	0.4357	0.2957	0.4978	0.3378	10
11	0.3114	0.2100	0.3737	0.2520	0.4360	0.2941	0.4982	0.3360	11
12	0.3116	0.2091	0.3739	0.2510	0.4363	0.2938	0.4986	0.3346	12
14	0.3118	0.2077	0.3741	0.2492	0.4366	0.2908	0.4988	0.3322	14
17	0.3120	0.2061	0.3744	0.2473	0.4369	0.2886	0.4992	0.3298	17
21	0.3122	0.2048	0.3746	0.2457	0.4371	0.2868	0.4994	0.3276	21
26	0.3123	0.2036	0.3748	0.2443	0.4372	0.2851	0.4997	0.3258	26
35	0.3124	0.2024	0.3748	0.2429	0.4373	0.2833	0.4999	0.3238	35
55	0.3124	0.2011	0.3748	0.2414	0.4374	0.2816	0.4999	0.3218	55
135	0.3124	0.1999	0.3748	0.2398	0.4374	0.2798	0.4999	0.3198	135

Number of Teeth.	1¼″ C. P. Thickness.	1¼″ C. P. Addendum.	1½″ C. P. Thickness.	1½″ C. P. Addendum.	1¾″ C. P. Thickness.	1¾″ C. P. Addendum.	2″ C. P. Thickness.	2″ C. P. Addendum.	Number of Teeth.
8	0.6210	0.4284	0.7450	0.5140	0.8694	0.5994	0.9936	0.6852	8
9	0.6218	0.4250	0.7460	0.5100	0.8706	0.5952	0.9948	0.6800	9
10	0.6224	0.4224	0.7468	0.5068	0.8714	0.5914	0.9956	0.6756	10
11	0.6228	0.4200	0.7474	0.5040	0.8720	0.5882	0.9964	0.6720	11
12	0.6232	0.4182	0.7478	0.5020	0.8726	0.5876	0.9972	0.6692	12
14	0.6236	0.4154	0.7482	0.4984	0.8732	0.5816	0.9976	0.6644	14
17	0.6240	0.4122	0.7488	0.4946	0.8738	0.5772	0.9984	0.6596	17
21	0.6244	0.4096	0.7492	0.4914	0.8742	0.5736	0.9988	0.6552	21
26	0.6246	0.4072	0.7496	0.4886	0.8744	0.5702	0.9994	0.6516	26
35	0.6248	0.4048	0.7498	0.4858	0.8746	0.5666	0.9998	0.6476	35
55	0.6250	0.4022	0.7499	0.4828	0.8748	0.5632	0.9999	0.6436	55
135	0.6250	0.3998	0.7499	0.4796	0.8748	0.5596	0.9999	0.6396	135

Table 7—Chordal Thicknesses and Addenda of Gear Teeth of Circular Pitch
Boston Gear Works

INVOLUTE CUTTERS

Until quite recently involute cutters were made in sets of eight, as follows:

Number of Cutter	
1	for 135 teeth to rack
2	for 55 to 134 teeth
3	for 35 to 54 teeth
4	for 26 to 34 teeth
5	for 21 to 26 teeth
6	for 17 to 20 teeth
7	for 14 to 16 teeth
8	for 12 to 13 teeth

Modern conditions, however, require a more accurate tooth than can be produced by this number of cutters. A set of fifteen, utilizing the half numbers is now in common use.

Number of Cutter	
1	for 135 teeth to a rack
1½	for 80 to 134 teeth
2	for 55 to 79 teeth
2½	for 42 to 54 teeth
3	for 35 to 41 teeth
3½	for 30 to 34 teeth
4	for 26 to 29 teeth
4½	for 23 to 25 teeth
5	for 21 to 22 teeth
5½	for 19 to 20 teeth
6	for 17 to 18 teeth
6½	for 15 to 16 teeth
7	for 14 teeth
7½	for 13 teeth
8	for 12 teeth

To produce accurate gears, templets for tooth thickness, made according to Tables 6 and 7, should be used instead of using one templet for each pitch and depending upon the workman's judgment as to how much shake to allow for different numbers of teeth. These templets, made up according to Tables 6 and 7, which are based on the use of eight cutters for each pitch, should be sufficiently accurate for all practical purposes.

SECTION II

Spur Gear Calculations

To find the pitch diameters of two gears, the number of teeth in each and the distance between centers being given: Divide twice the distance between centers by the sum of the number of teeth: Find the pitch diameter of each gear separately by multiplying this quotient by its number of teeth.

Example: Find the pitch diameters of a pair of spur gears, 21 and 60 teeth, for 25-inch centers.

$$\frac{2 \times 25}{21 + 60} = 6.17284,$$

$6.17284 \times 21 = 12.96296$ inches, or the pitch diameter of the pinion
$6.17284 \times 60 = 37.03704$ inches, or the pitch diameter of the gear

The distance between the centers is one-half the sum of the pitch diameters. In the above example the center distance would prove to be:

$$\frac{12.96296 + 37.03704}{2} = 25 \text{ inches}$$

NOTATIONS FOR FORMULAS

p = diametral pitch

$$\left.\begin{array}{l}\left.\begin{array}{l} D' = \text{pitch diameter} \\ D = \text{outside diameter} \\ V = \text{velocity} \end{array}\right\} \text{gear} \\ \\ \left.\begin{array}{l} d' = \text{pitch diameter} \\ d = \text{outside diameter} \\ v = \text{velocity} \end{array}\right\} \text{pinion} \end{array}\right\} \text{These gears run together}$$

a = distance between the centers
b = number of teeth in both

TO FIND	HAVING	RULE	FORMULA	EXAMPLE
b	a and p	The continued product of center distance pitch and 2	$a\,p\,2$	$15 \times 3 \times 2 = 90$
a	D' and d'	One-half the sum of the pitch diameters	$\frac{D' + d'}{2}$	$\frac{20 + 10}{2} = 15''$
a	b and p	Divide the total number of teeth by twice the pitch	$\frac{b}{2p}$	$\frac{90}{2 \times 3} = 15$
N	$n\,v$ and V	Divide the product of the number of teeth and velocity of pinion by the velocity of gear	$\frac{n\,v}{V}$	$\frac{30 \times 2}{1} = 60$
N	$b\,v$ and V	Divide the product of the total number of teeth and velocity of pinion by the sum of the velocities	$\frac{b\,v}{v + V}$	$\frac{90 \times 2}{2 + 1} = 60$
n	$b\,v$ and V	Divide the product of the total number of teeth and the velocity of gear by the sum of the velocities	$\frac{b\,V}{v + V}$	$\frac{90 \times 1}{2 + 1} = 30$
n	$N\,v$ and V	Divide the product of the number of teeth in gear and its velocity by the velocity of pinion	$\frac{N\,V}{v}$	$\frac{60 \times 1}{2} = 30$
n	$p\,D'\,V$ and v	Divide the continued product of the pitch, pitch diameter and velocity of the gear by the velocity of pinion.	$\frac{p\,D'\,V}{v}$	$\frac{3 \times 20 \times 1}{2} = 30$
v	$N\,V$ and n	Divide the product of the number of teeth and velocity of gear by the number of teeth in pinion	$\frac{N\,V}{n}$	$\frac{60 \times 1}{30} = 2$
v	$p\,D'\,V$ and n	Divide the continued product of the pitch, pitch diameter and velocity of gear by the number of teeth in pinion	$\frac{p\,D'\,V}{n}$	$\frac{3 \times 20 \times 1}{30} = 2$
V	$n\,v$ and N	Divide the product of the number of teeth in pinion and its velocity by the number of teeth in gear	$\frac{n\,v}{N}$	$\frac{30 \times 2}{60} = 1$
D'	$a\,v$ and V	Divide the continued product of the center distance, velocity of pinion and 2, by the sum of the velocities.	$\frac{2\,a\,v}{v + V}$	$\frac{2 \times 15 \times 2}{2 + 1} = 20$
d'	$a\,v$ and V	Divide the continued product of the center distance, velocity of gear and 2, by the sum of the velocities	$\frac{2\,a\,V}{v + V}$	$\frac{2 \times 15 \times 1}{2 + 1} = 10$

TABLE 8—FORMULAS FOR A PAIR OF MATING SPUR GEARS

TO FIND	HAVING	RULE	FORMULA
The Diametral Pitch.	The Circular Pitch..	Divide 3.1416 by the Circular Pitch......................	$p = \frac{3.1416}{p'}$
The Diametral Pitch.	The Pitch Diameter and the Number of Teeth............	Divide Number of Teeth by Pitch Diameter............	$p = \frac{N}{D'}$
The Diametral Pitch.	The Outside Diameter and the Number of Teeth.........	Divide Number of Teeth plus 2 by Outside Diameter........	$p = \frac{N+2}{D}$
Pitch Diameter.	The Number of Teeth and the Diametral Pitch............	Divide Number of Teeth by the Diametral Pitch.............	$D' = \frac{N}{p}$
Pitch Diameter.	The Number of Teeth and Outside Diameter..............	Divide the product of Outside Diameter and Number of Teeth by Number of Teeth plus 2...	$D' = \frac{D\,N}{N+2}$
Pitch Diameter.	The Outside Diameter and the Diametral Pitch.........	Subtract from the Outside Diameter the quotient of 2 divided by the Diametral Pitch.........	$D' = D - \frac{2}{p}$
Pitch Diameter.	Addendum and the Number of Teeth..	Multiply Addendum by the Number of Teeth............	$D' = s\,N$
Outside Diameter.	The Number of Teeth and the Diametral Pitch............	Divide Number of Teeth plus 2 by the Diametral Pitch.......	$D = \frac{N+2}{p}$
Outside Diameter.	The Pitch Diameter and the Diametral Pitch............	Add to the Pitch Diameter the quotient of 2 divided by the Diametral Pitch.............	$D = D' + \frac{2}{p}$
Outside Diameter.	The Pitch Diameter and the Number of Teeth............	Divide the product of the Pitch Diameter and Number of Teeth plus 2 by the Number of Teeth	$D = \frac{(N+2)\,D'}{N}$
Outside Diameter.	The Number of Teeth and Addendum....	Multiply the Number of Teeth plus 2 by Addendum.........	$D = (N+2)\,s$
Number of Teeth.	The Pitch Diameter and the Diametral Pitch............	Multiply Pitch Diameter by the Diametral Pitch............	$N = D'\,p$
Number of Teeth.	The Outside Diameter and the Diametral Pitch.........	Multiply Outside Diameter by the Diametral Pitch and subtract 2.....................	$N = D\,p - 2$
Thickness of Tooth.	The Diametral Pitch.	Divide 1.5708 by the Diametral Pitch.......................	$t = \frac{1.5708}{p}$
Addendum.	The Diametral Pitch.	Divide 1 by the Diametral Pitch, or $s = \frac{D'}{N}$................	$s = \frac{1}{p}$
Dedendum.	The Diametral Pitch.	Divide 1.157 by the Diametral Pitch......................	$s + f = \frac{1.157}{p}$
Working Depth.	The Diametral Pitch.	Divide 2 by the Diametral Pitch.	$W' = \frac{2}{p}$
Whole Depth.	The Diametral Pitch.	Divide 2.157 by the Diametral Pitch......................	$W' + f = \frac{2.157}{p}$
Clearance.	The Diametral Pitch.	Divide 0.157 by the Diametral Pitch......................	$f = \frac{0.157}{p}$
Clearance.	Thickness of Tooth..	Divide Thickness of Tooth at pitch line by 10.............	$f = \frac{t}{10}$

TABLE 9—SPUR GEAR CALCULATIONS FOR DIAMETRAL PITCH

14½ *Degree Standard*

TO FIND	HAVING	RULE	FORMULA
The Circular Pitch.	The Diametral Pitch.	Divide 3.1416 by the Diametral Pitch....................	$p' = \frac{3.1416}{p}$
The Circular Pitch.	The Pitch Diameter and the Number of Teeth............	Divide Pitch Diameter by the product of 0.3183 and Number of Teeth..................	$p' = \frac{D'}{0.3183\ N}$
The Circular Pitch.	The Outside Diameter and the Number of Teeth..........	Divide Outside Diameter by the product of 0.3183 and Number of Teeth plus 2..............	$p' = \frac{D}{0.3183\ N+2}$
Pitch Diameter.	The Number of Teeth and the Circular Pitch............	The continued product of the Number of Teeth, the Circular Pitch and 0.3183............	$D' = N\ p'\ 0.3183$
Pitch Diameter.	The Number of Teeth and the Outside Diameter...........	Divide the product of Number of Teeth and Outside Diameter by Number of Teeth plus 2...	$D' = \frac{N\ D}{N+2}$
Pitch Diameter.	The Outside Diameter and the Circular Pitch............	Subtract from the Outside Diameter the product of the Circular Pitch and 0.6366............	$D' = D - (p'\ 0.6366)$
Pitch Diameter.	Addendum and the Number of Teeth..	Multiply the Number of Teeth by the Addendum..............	$D' = N\ s$
Outside Diameter.	The Number of Teeth and the Circular Pitch............	The continued product of the Number of Teeth plus 2 the Circular Pitch and 0.3183....	$D = (N+2)\ p'\ 0.3183$
Outside Diameter.	The Pitch Diameter and the Circular Pitch............	Add to the Pitch Diameter the product of the Circular Pitch and 0.6366.................	$D = D + (p'\ 0.6366)$
Outside Diameter.	The Number of Teeth and the Addendum	Multiply Addendum by Number of Teeth plus 2	$D = s\ (N+2)$
Number of Teeth.	The Pitch Diameter and the Circular Pitch............	Divide the product of Pitch Diameter and 3.1416 by the Circular Pitch................	$N = \frac{D'\ 3.1416}{p'}$
Thickness of Tooth.	The Circular Pitch...	One-half the Circular Pitch.....	$t = \frac{p'}{2}$
Addendum.	The Circular Pitch...	Multiply the Circular Pitch by 0.3183, or $s = \frac{D'}{N}$.........	$s = p'\ 0.3183$
Dedendum.	The Circular Pitch...	Multiply the Circular Pitch by 0.3683......................	$s + f = p'\ 0.3683$
Working Depth.	The Circular Pitch...	Multiply the Circular Pitch by 0.6366.....................	$W = p'\ 0.6366$
Whole Depth.	The Circular Pitch...	Multiply the Circular Pitch by 0.6866.....................	$W' = p'\ 0.6866$
Clearance.	The Circular Pitch...	Multiply the Circular Pitch by 0.05......................	$f = p\ 0.05$
Clearance.	Thickness of Tooth..	One-tenth the Thickness of Tooth at Pitch Line..............	$f = \frac{t}{10}$

TABLE 10—SPUR GEAR CALCULATIONS FOR CIRCULAR PITCH

14½ *Degree Standard*

SECTION III

Speeds and Powers

TRANSMISSION OF POWER BY GEARING WITH PARTICULAR REFERENCE TO SPUR AND BEVEL GEARS

SPEED RATIO

The problem of finding the proper diameter or speed of a gear or pulley is simple enough when once thoroughly understood.

The gear may be represented by its number of teeth, pitch diameter, pitch radius, or speed ratio, as the case may be. In the explanation to follow the number of teeth is used. The speed is in revolutions per minute.

Rule: Divide the product of the speed and number of teeth of one gear by the speed *or* number of teeth of its mate to secure the lacking dimension.

That is, if both the speed and number of teeth are known for one gear, multiply the speed by the number of teeth, and divide this product by the known quantity of the mating gear to secure *its* number of teeth or speed, as the case may be.

Or the same result may be obtained by proportion, the values being placed as follows:

$$n : N :: R : r \qquad (1)$$

n = number of teeth in pinion
r = revolutions per minute of pinion
N = number of teeth in gear
R = revolutions per minute of gear

Example: A gear having 60 teeth makes 300 revolutions per minute, what will be the speed of an engaging pinion having 15 teeth?

$$n : N : R : r$$

$$15 : 60 : 300 : x$$

Therefore, $x = \dfrac{60 \times 300}{15}$, or 1200 revolutions per minute for pinion n

To compute these values for a train of gears, use the continued product of the pinions and the continued product of the gears as a single gear and pinion and proceed as above.

Example: In Fig. 41, the gear N has 100 teeth, N', 70 teeth, N'', 60 teeth, n, 15 teeth; n', 18 teeth; and n'', 24 teeth. The gear N makes 10 revolutions per minute. What will be the speed of the pinion n''?

$$N, N' \text{ and } N'' = 100 \times 70 \times 60 = 420{,}000$$
$$n, n' \text{ and } n'' = 15 \times 18 \times 24 = 6{,}480.$$
$$n : N : : R : r$$
$$6{,}480 : 420{,}000 : : 10 : x$$

Therefore, $x = \dfrac{420{,}000 \times 10}{6{,}480}$, or 648 revolutions per minute for pinion n''.

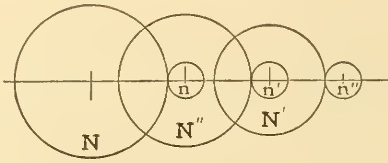


FIG. 41. GEAR TRAIN.

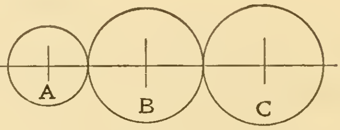


FIG. 42. INTERMEDIATE GEAR DOES NOT AFFECT THE SPEED RATIO.

The velocities of a train of gears may also be found as follows: N, N', N'' and n, n', n'', etc., representing the number of teeth in the gears and pinions.

$$r = \frac{R\, N\, N'\, N''}{n\, n'\, n''}. \qquad (2)$$

$$R = \frac{r\, n\, n'\, n''}{N\, N'\, N''}. \qquad (3)$$

The intermediate gear B, as shown in Fig. 42, while it changes the direction of the rotation of the gears, A and C does not alter their speed ratio, the circumferential velocities of all three gears being equal.

ARRANGEMENT OF GEAR TRAINS

For compound reduction there must be four gears, as per Fig. 43, the gears B and C being keyed to an intermediate shaft, the power being transferred to the machine by the shaft-carrying gear D.

When a great reduction is required, say 64 to 1, there may be two intermediate shafts, as in Fig. 44.

This reduction might be accomplished by using a drive, as in Fig. 43, dividing the total reduction between two sets of gears, but a triple reduction is used by way of illustration. The best results are always obtained by dividing the reduction as evenly as possible among the different pairs of gears. For instance: for a double reduction, as in Fig. 43, the ratio of each pair should be made as

near the square root of the total reduction as possible. In case of the triple reduction, Fig. 44, the ratio of each pair should be the cube root of the total reduction, or $\sqrt[3]{64} = 4$. That is, there are three sets of gears, each having a speed ratio of 4 to 1. If double reduction had been used the reduction of each gear would have been $\sqrt{64} = 8$, or two sets of gears each having a speed ratio of 8 to 1.

Gear trains proportioned in this way give the highest possible efficiency. For instance: an unsuccessful single gear reduction of 16 to 1 might be

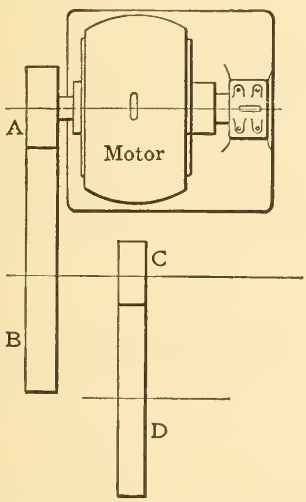


FIG. 43. DOUBLE GEAR REDUCTION.

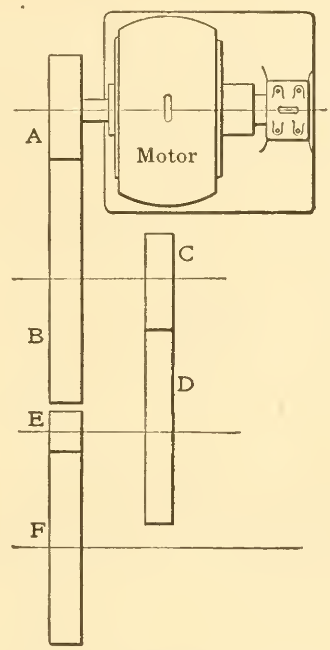


FIG. 44. TRIPLE GEAR REDUCTION.

made efficient by substituting two pairs of gears, each having a ratio of 4 to 1. Making the compound gears 8 to 1 and 2 to 1 would help, but would not be as efficient as the equal reduction. This will be especially noticeable in long leads in the lathe or milling machines.

POWER RATIO

The relative powers of a train of gears are inversely proportional to their circumferential velocities. The circumferential velocity of each pair of gears in a train being equal, the driving pinion, as shown in Figs. 45 and 46, is ignored in the calculations for a single pair, the circumferential velocity and the load on the teeth being the same as for the mating gear. The problem is to determine the power ratio between the drum r and the gear R.

Ignoring friction, the values of this drive may be found by proportion, arranged as follows:

$$W : R :: F : r \qquad (4)$$

Enough must be added to the load W or taken from the effective lifting force F to overcome the frictional resistance of the teeth and bearings. This loss must be estimated and the percentage of loss added to the load W, the ratio of R and r being determined according to this new ratio.

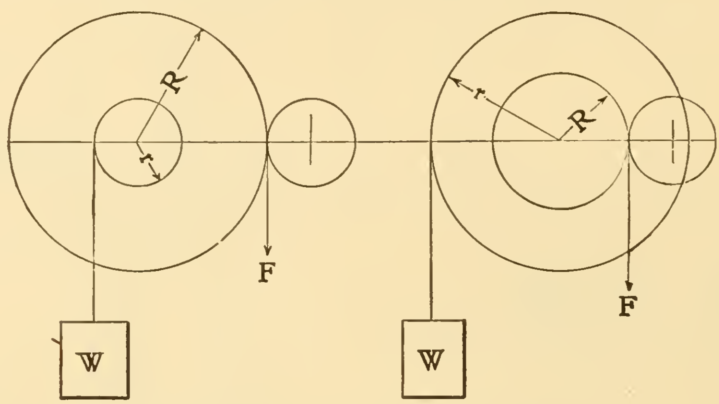


FIG. 45. FIG. 46.
POWER RATIO DIAGRAMS.

Example: Referring to Fig. 45: if the radius of the gear R is 18 inches, the radius of the drum r three inches, what power will be required at F to raise 300 pounds at W?

$$W : R :: F : r$$
$$300 : 18 :: x : 3$$

Therefore, $x = \dfrac{300 \times 3}{18}$, = 50 pounds required at F.

Suppose the loss in efficiency to be 10 per cent and the radius of the gear R 18 inches. What must be the radius of the drum r to raise 300 pounds at W?

$$300 + 10 \text{ per cent} = 330 \text{ pounds.}$$

$$W : R :: F : T$$
$$300 : 18 :: 50 : x$$

Therefore, $x = \dfrac{18 \times 50}{330}$, = 2.7 inches for the radius of drum r.

For a train of gears, the continued products of the driving and driven gears may be considered as single gears. Or the power ratio may be considered between each pair inversely proportional to their velocity ratios.

Example: Referring to Fig. 47; what force is required at F to raise 2500 pounds at W, the loss in efficiency being 30 per cent?

$$R\,R'\,R'' = 20 \times 18 \times 10 = 3600$$
$$r\,r'\,r'' = 6 \times 8 \times 5 = 240$$
$$W = 2500 + 30 \text{ per cent} = 3250.$$

$$W : R : : F : r$$
$$3250 : 3600 : : x : 240,\ x = \frac{3250 \times 240}{3600}, \text{ or 217 pounds at } F.$$

$$\text{Also } F = \frac{W\,r\,r'\,r''}{R\,R'\,R''}, = \frac{3250 \times 6 \times 8 \times 5}{20 \times 18 \times 10}, = 217 \text{ pounds.} \qquad (5)$$

$$\text{And } W = \frac{F\,R\,R'\,R''}{r\,r'\,r''}, = \frac{217 \times 20 \times 18 \times 10}{6 \times 8 \times 5}, = 3250 \text{ pounds.} \qquad (6)$$

AN EXAMPLE IN HOIST GEARING

Example: What gears will be required to lift a load of 2400 pounds at a uniform rate of speed, employing a 10 horse-power motor running 1120 revolutions per minute, driving with a rawhide pinion 4 inches pitch diameter? See Fig. 48.

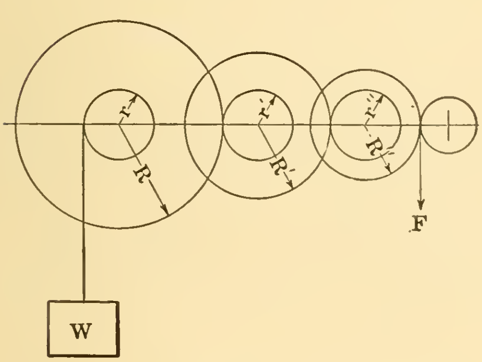


FIG. 47. POWER RATIO OF GEAR TRAINS.

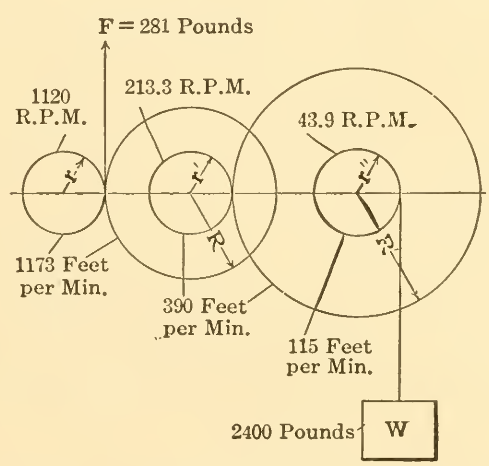


FIG. 48. EXAMPLE OF GEAR DRIVE FOR HOIST.

$$\text{Velocity of pinion in feet per minute, } V = d'\ 0.2618\ R.\,P.\,M. \qquad (7)$$

$$\text{The safe load, } W + \frac{HP \times 33{,}000}{V} \qquad (8)$$

Therefore, $V = 4 \times 0.2618 \times 1120 = 1173$ feet per minute.

And $W = \dfrac{10 \times 33000}{1173}$, or 281 pounds, which is the load to be carried by the pinion.

Assuming that 20 per cent is lost by the friction of the gear teeth, bearings, etc., the real load to be raised by the force of 280 pounds at the pitch line of the driving pinion is:

$$2400 + 20 \text{ per cent.} = 2880 \text{ pounds.}$$

The necessary velocity ratio of the gears to equal this ratio of power is, therefore:

$$\frac{2880}{281}, = \frac{10.25}{1}.$$

This reduction must be made between R' and r'', and R and r', the pinion r not being considered as its velocity is the same as that of the gear R, therefore the load on the teeth will be the same.

Since it is always best to make the reduction in even steps, and double reduction is desirable for a ratio of 10.25 to 1 take the square root of the total reduction, 10.25, which is approximately 3.2 to 1 for each reduction. Practically, however, a reduction of $\frac{3.4}{1}$ and $\frac{3}{1}$ will answer.

The ratio between R' and r'' is made $\frac{3.4}{1}$. Assuming the diameter of the drum r'' to be 10 inches, the pitch diameter of the gear R' will be $3.4 \times 10 = 34$ inches. The ratio between R and r' is $\frac{3}{1}$, assuming the pitch diameter of the pinion r' to be 7 inches, the pitch diameter of the gear R will be $7 \times 3 = 21$ inches.

The power or circumferential force of the gear R is, of course, that of the driving pinion r, 281 pounds. Therefore, the power of the pinion r', and consequently that of the gear R', is $281 \times 3 = 843$ pounds.

The problem is now reduced to two simple ones, that is—to design a pair of gears r and R to transmit a force of 281 pounds at a speed of 1173 feet per minute, and a second pair r' and R' to transmit a force of 843 pounds at a speed of 390 feet per minute.

It is necessary to assume a pitch judged to be suitable for the different drives and to try its value for carrying the required load by the Lewis formula, obtaining the safe load per inch of face, and make the face sufficiently wide to transmit the power.

For the first pair of gears, r and R, assume 4 diametral pitch—0.7854-inch circular pitch—allowing 5000 pounds per square inch as a safe stress for rawhide. Number of teeth in pinion $r = 4 \times 4 = 16$.

Safe load per inch of face $= spy \dfrac{600}{600 + V}$. (See formula 24.)

Or $5000 \times 0.785 \times 0.077 \dfrac{600}{600 + 1173} = 100$ pounds per inch of face.

Making the face of the gears r and R 3 inches it will safely carry $3 \times 100 = 300$ pounds, which is sufficient.

For the second pair, r' and R', try 3 diametral pitch—1.0472-inch circular pitch—both gears of cast iron. Figure the strength of the pinion, as it is the weaker of the two. Allow 8000 pounds per square inch as a safe stress.

For a pinion 7 inches pitch diameter, 3 pitch, the number of teeth equals $7 \times 3 = 21$ teeth. Factor y for 21 teeth equals 0.092. $W = 8000 \times 1.047 \times 0.092 \dfrac{600}{600 + 390}$, or 472 pounds per inch of face.

Making the face 3 inches, the gears will carry a load of $3 \times 472 = 1416$ pounds. These gears will therefore be heavier than necessary, but owing to the nature of the service this should be the case, especially as they are made of cast iron.

From the ratio of this train of gears it will be found that the load will be raised at $\dfrac{1173}{3 \times 3.4} = 115$ feet per minute, using the full speed of the motor. If the load must be raised at a greater speed than 115 feet, a more powerful motor would be required, and if at a lower speed there must be a greater gear reduction. For instance, if the hoisting speed had been 80 feet per minute the speed ratio would be $\dfrac{1173}{80} = \dfrac{14.7}{1}$, instead of $\dfrac{10.2}{1}$ as in the example.

The above problem is generally put before the designer in a different manner—that is the load and speed at which the load is to be raised are given, the size of motor and ratio of gearing, etc., to be determined.

Example: A load of 2400 pounds is to be raised at the uniform rate of 115 feet per minute; what size motor and what gears will be required?

Assuming as before a loss of 20 per cent in efficiency in the driven gears, bearing, etc., this load will require:

$\dfrac{2880 \times 115}{33{,}000}$, = 10 horse power (2400 pounds + 20 per cent = 2880 pounds).

Using a rawhide pinion four-inch pitch diameter on the motor, we consider the problem in the same manner as in previous examples making the ratio of the gears; $\dfrac{2880}{281} = \dfrac{10.25}{1}$

The problem of determining the proper gears is the same.

RAILWAY GEARS

Speed in feet per minute at rim of car wheel $V' = 88 \times$ speed of car in miles per hour. (9)

Speed in feet per minute at pitch line of gear $V' = 88 \times$ miles per hour $\times R$. (10)

Ratio of gear to wheel $R = \dfrac{\text{pitch diameter of gear}}{\text{diameter of wheel}}$. (11)

Force at pitch line of gear $F = \dfrac{HP \times 33{,}000}{V}$, or $\dfrac{Kw \times 44{,}102}{V}$. (12)

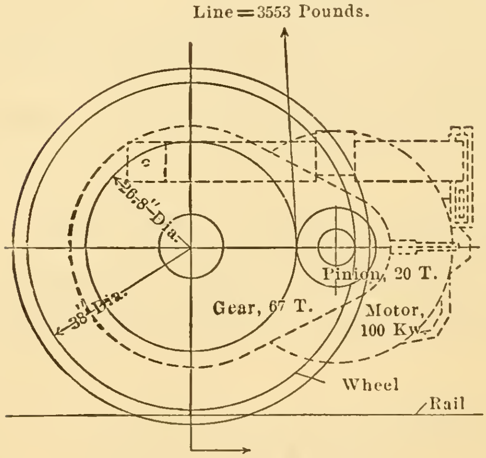
Fiber stress in tooth $S = \dfrac{F}{p' f y \dfrac{600}{600 + V}}$, (See formula 18.) (13)

Traction effort at wheel $T = \dfrac{F}{R}$. (14)

Horse power $HP = \dfrac{T M}{0.375}$. (15)

Speed of car in miles per hour $M = \dfrac{\text{Dia. of wheel} \times \text{teeth in pinion} \times \text{revolution per minute of pinion.}}{\text{teeth in gear} \times 336}$ (16)

Traction effort $T = \dfrac{\text{teeth in gear} \times 24 \times \text{gear efficiency} \times \text{torque of motor}}{M \times \text{diameter of wheel}}$. (17)



From Formula 12, $F = \dfrac{5040 \times 26.8}{38} = 3550$ Pounds

FIG. 49. RAILWAY GEARS.

Example: A car weighing 60 tons driven by four motors accelerating at the rate of 1½ miles per hour, per second, reaches the peak of its starting torque when at a speed of 20 miles per hour. The gears are 20 and 67 teeth 2½ diametral pitch (1.26 inches circular pitch) 5¼ inch face. The diameter of the car wheels is 38 inches. It is required to know the maximum fiber stress in pinion tooth. The power exerted by motors at its peak is 400 kilowatts (800 amperes at 500 volts). See Fig. 49.

Kilowatts per motor $Kw = \frac{400}{4} = 100\ Kw$

Pitch diameter of gear $D' = \frac{67}{2\frac{1}{2}} = 26.8''$

Ratio of gear to wheel $R = \frac{26.8}{38} = 0.705$

Speed of gear in feet per minute

$V = 88 \times 20 \times 0.705 = 1241$ feet per minute.

Force at pitch line $F = \frac{100 \times 44{,}102}{1241} = 3553$ pounds.

Fiber stress in pinion tooth $S = \dfrac{3553}{1.26 \times 5.25 \times 0.09 \times \dfrac{600}{600+1241}} = 18{,}400$ pounds per square inch.

STRENGTH OF GEAR TEETH

Lewis

W = load transmitted in pounds (same as value F),
p' = circular pitch,
f = face,
y = factor for different numbers and forms of teeth (Table 11),
S = safe working stress of material,
V = velocity in feet per minute,

$$W = S\,p'\,f\,y\,\frac{600}{600 + V}. \qquad (18)$$

NUMBER OF TEETH	VALUE OF FACTOR y			NUMBER OF TEETH	VALUE OF FACTOR y		
	INVOLUTE 20°	INVOLUTE 15° CYCLOIDAL	RADIAL FLANKS		INVOLUTE 20°	INVOLUTE 15° CYCLOIDAL	RADIAL FLANKS
12	0.078	0.067	0.052	27	0.111	0.100	0.064
13	0.083	0.070	0.053	30	0.114	0.102	0.065
14	0.088	0.072	0.054	34	0.118	0.104	0.066
15	0.092	0.075	0.055	38	0.122	0.107	0.067
16	0.094	0.077	0.056	43	0.126	0.110	0.068
17	0.096	0.080	0.057	50	0.130	0.112	0.069
18	0.098	0.083	0.058	60	0.134	0.114	0.070
19	0.100	0.087	0.059	75	0.138	0.116	0.071
20	0.102	0.090	0.060	100	0.142	0.118	0.072
21	0.104	0.092	0.061	150	0.146	0.120	0.073
23	0.106	0.094	0.062	300	0.150	0.122	0.074
25	0.108	0.097	0.063	Rack	0.154	0.124	0.075

TABLE 11—VALUES OF FACTOR y FOR LEWIS FORMULA

Safe working stress S for 0.30 carbon steel	=	15000
Safe working stress S for 0.50 carbon steel	=	25000
Safe working stress S for cast iron	=	8000
Safe working stress S for rawhide	=	5000

AVERAGE VALUES FOR S

Mr. Lewis' formula for the strength of gears originally read: $W = S\,p'\,f\,y$, a table being given in which the allowable stress of the material S was reduced as the speed of the gear was increased as follows:

SPEED OF TEETH IN FEET PER MINUTE	100 OR LESS	200	300	600	900	1200	1800	2400
Cast Iron........	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Steel............	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300

SAFE WORKING STRESSES S IN POUNDS PER SQUARE INCH FOR DIFFERENT SPEEDS

Later Carl G. Barth introduced an equation, $\frac{600}{600 + V}$, which gives practically the same result as the table when added to the formula, the value S being the safe working stress per square inch of the material used, or

$$W = S\,p'\,f\,y\;\frac{600}{600 + V}.$$

Mr. Barth's equation is the one commonly accepted. It is evident, however, that this value will vary for different conditions, the design and workmanship being important factors in its proper determination.

The load is reduced as the speed increases on account of impact. It is evident that an accurately spaced and generated gear should have a much higher value than one cut by ordinary methods.

It is also evident that helical and herringbone gears, owing to the nature of their tooth contact, should have a much higher value, as they operate under entirely different conditions, therefore are capable of heavier loads at higher speeds for the same area of tooth contact. Rawhide gears should also have a higher factor, as rawhide will absorb shocks that would affect harder materials.

In the absence of all vibration, and with an indeflectable material, this equation could be eliminated from the formula for strength and wear. These are conditions that can never be attained, but it is evident that this value will stand extended investigation.

VALUES OF P

produce fiber stress of 1000 lbs. per square inch.

Number of Teeth in Gear	CIRCULAR PITCH																	Multiplier for	
	3.1416	2.52	2.1	1.8	1.57	1.4	1.25	1.14	1.05	0.98	0.89	0.84	0.79	0.7	0.63	0.57	0.52	20° involute	Radial Flank
	DIAMETRAL PITCH																		
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	5	5½	6		
12	210	168	140	120	105	93	84	76	70	65	60	56	52	47	42	38	35	1.16	.775
13	220	176	147	126	110	98	88	80	73	68	63	59	55	49	44	40	37	1.18	.756
14	226	180	151	129	113	100	90	82	75	69	65	60	56	50	45	41	38	1.22	.750
15	236	189	157	135	118	105	94	86	79	73	67	63	59	52	47	43	39	1.22	.734
16	242	194	161	138	121	107	97	88	81	74	69	64	60	54	48	44	40	1.22	.728
17	251	200	167	143	125	112	100	91	84	77	72	67	63	56	50	46	42	1.20	.712
18	261	208	174	149	130	116	104	95	87	80	75	70	65	58	52	47	43	1.18	.700
19	273	218	182	156	136	121	109	100	91	84	78	73	68	61	54	50	45	1.15	.678
20	283	227	189	162	142	126	113	103	94	87	81	75	71	63	57	51	47	1.14	.667
21	289	232	193	165	144	128	116	105	96	89	83	77	72	64	58	52	48	1.13	.663
23	295	236	197	169	147	131	118	107	98	91	84	79	74	65	59	53	49	1.12	.659
25	305	245	203	174	152	136	122	111	102	94	87	81	76	68	61	55	51	1.11	.649
27	314	250	210	180	157	140	126	114	104	97	90	84	78	70	63	57	52	1.11	.640
30	320	256	213	183	160	142	128	116	106	99	91	85	80	71	64	58	53	1.12	.637
34	327	262	218	188	164	146	131	119	109	101	94	88	82	73	66	59	54	1.14	.634
38	336	269	224	192	168	149	134	122	112	103	96	90	84	75	67	61	56	1.14	.626
43	346	277	230	198	173	154	138	126	115	106	99	92	86	77	69	63	57	1.15	.619
50	352	282	235	200	176	156	140	128	117	108	100	94	88	78	70	64	58	1.16	.615
60	358	286	238	204	179	159	143	130	119	110	102	95	89	80	71	65	59	1.18	.613
75	364	292	243	208	182	162	146	132	121	112	104	97	91	81	73	66	61	1.19	.612
100	371	297	247	212	185	164	148	135	124	114	106	99	93	82	74	67	62	1.20	.610
150	377	302	251	215	188	167	151	137	126	116	108	100	94	84	75	68	63	1.22	.608
300	384	308	256	219	192	171	154	140	128	118	110	102	96	85	77	70	64	1.23	.607
Rack	390	312	260	223	195	173	156	142	130	120	112	104	97	87	78	71	65	1.24	.605

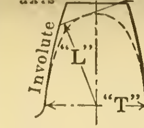
Fiber Stresses

Material	Average of Tests Made by W. E. & Mfg. Co. Ultimate	Elastic Limit	※ Safe Fiber Stress Lbs. per Sq. In.
Cast Iron	22,000		8,000
Cast Steel	60,000	28,000	20,000
Forged Steel	65,000	31,000	25,000

※ The safe fiber stresses given here are for static loads acting at point of one tooth.

For consideration of other conditions and methods of figuring applied to motor gears see special sheet.

METHOD OF FIGURING.



P = total load in pounds acting on one tooth.
T = tooth thickness at line of weakness.
L = moment arm, normal to P.

Bending moment M = P L

Fiber Stress $S = \frac{My}{I} = \frac{Ply}{I}$ in which

y = inches from neutral axis to outside fiber.
I = moment of inerta at line of weakness $= \frac{f T^3}{12}$ in which
f = face of gear in inches.

From $S = \frac{PLy}{I}$, $p = \frac{SI}{Ly} = \frac{S\left(\frac{f T^3}{12}\right)}{L \frac{T}{2}} = \frac{S f T^2}{6 L}$

The values of "L" and "T" used in compiling accompanying table were obtained by measurement from a series of tooth layouts.

EXAMPLE: Given a cast iron spur gear of 60 teeth, 3 diametral pitch, 4" face, running at pitch line speed of 700 ft. per min.

(1) What is safe working load P?
(2) What horsepower will gear transmit?

SOLUTION: From table of values of P, under 3 diametral pitch for 60 teeth, read 119. From speed coefficient curve for 700 ft. per Min. read .47. From fiber stress table for cast iron read 8000.

Then (1) P = 119 x 4" x .47 x 8 = 1790 lbs.

(2) Horsepower $= \frac{1790 \times 700}{33000} = 38$ H.P.

SPEED COEFFICIENT CURVE.

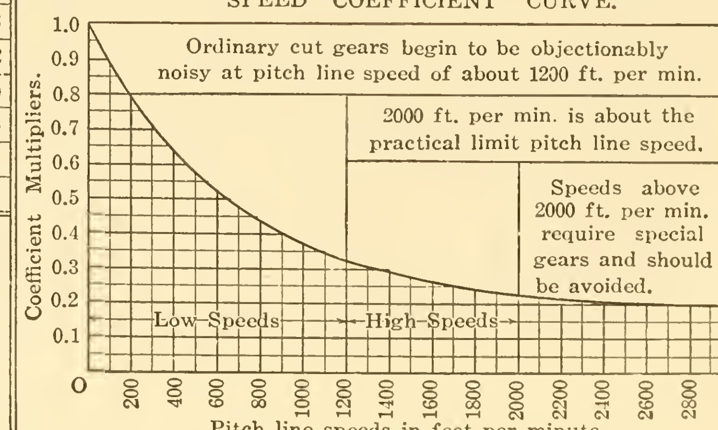


TABLE 12—STRENGTH OF SPUR GEAR TEETH.—GEORGE H. FOLLOWS

Table 13 was prepared by Henry Hess and published in AMERICAN MACHINIST to allow the pitch and the face to be found with very little arithmetic. It is based on Lewis' method of calculating the strength of gear teeth.

As the bulk of gears in use are either 15 degrees involute or cycloid the table has been made up of these forms.

The arrangement of the table is such that, given the number of teeth in the gear, and the quotient obtained from dividing the working load per tooth by the greatest fiber stress, the pitch and face width can be directly picked out. The face width is given in inches and also as a ratio to the circular pitch; it is usual to make the face from two or three times the circular pitch—2.5 is a fair average. For overhung gears 2 to 2.5 is proper; for gears supported on both sides 2.5 to 3 is good practice. For convenience sake these ratios are repeated at the tops of the vertical columns, while the face widths to the nearest sixteenth higher corresponding to each ratio for each pitch are given at the foot of the various columns.

As diametral pitches are generally used for light work the table is arranged for these from 8 to 3.5, and for heavier work for circular pitches from 1 to 4 inches. The equivalent circular and diametral pitches are marked in lighter faced type.

Directions and examples are given on page 61.

The formula from which the tabular values were determined is

$$k = \frac{W}{S} = p'^2 r \left(0.124 - \frac{0.684}{n}\right),$$

which is but another way of writing the Lewis formula, which, with the notation changed to agree with formula above, is

$$W = S p' f \left(0.124 - \frac{0.684}{n}\right),$$

the quantity in the parenthesis being the Lewis variable for cycloidal and 15 degrees involute teeth. The change in form is made by introducing for f its value—$p' r$—as per notation below:

W = working load in pounds.
S = greatest fiber stress in pounds per square inch.
p' = circular pitch in inches.
f = face width in inches.
n = number of teeth in pinion.
r = ratio of face width to circular pitch $= \frac{f}{p'}$.
p = diametral pitch.

To find the circular pitch p' or diametral pitch p and the face width f:

Divide the known load W by the permissible greatest fiber stress S; find the resulting value k in the body of the table in line with the number of teeth in the pinion. Use the pitch given at the top and the face width given at the foot of the column.

Example: A gear of eighteen teeth is loaded with 495 pounds per tooth; the permissible fiber stress is 3,000 pounds per square inch. Then $\frac{W}{S} = \frac{495}{3{,}000} =$ 0.165. Opposite eighteen teeth find $k = 0.167$ under a diametral pitch 3.5 and over a face width of 2.25″. Or find $k = 0.166$ under a circular pitch of 1 inch and over a face width of 2 inches. Either solution will do.

To find the greatest fiber stress S:

In the column headed with the pitch used and marked with the face width used at the foot, find opposite the tooth number a constant. Divide this into the working load imposed on the tooth to get the greatest fiber stress.

Example: The working load on a tooth of 4-inch circular pitch and 10-inch face in a 100-tooth gear is 28,000 pounds. Opposite 100 teeth, under $p' = 4$ inches and over $f = 10$ inches, find 4.684. Dividing this into the load gives

$\frac{28{,}000}{4.684} = 5{,}970$ pounds per square inch greatest fiber stress.

Table 14 for the working strength of gear teeth has been furnished by the New Britain Machine Company. It is based on the Lewis formula, and, unlike other tables, gives the strength of the teeth when their size is indicated by diametral pitch. But one width of face is given for each pitch, this face being, as near as may be, that used by the makers of standard gears for the market.

The figures in the body of the table give the working load in pounds for a speed of 100 feet per minute for cast-iron gears of the pitch and face found at the top of the corresponding column and of the number of teeth given at the left of the corresponding line. The left-hand figure at the top of each column gives the diametral pitch and the right-hand figure the face in inches.

For higher speeds the loads are to be reduced by the equation $\frac{600}{600 + V}$ used as a multiplier, and for steel gears the loads may be increased in proportion to the safe load for that material.

NO. OF TEETH	$p' =$ 0.393″			0.524″			0.628″			0.785″			0.898″			1″			1.25″			1.5″		
	$p =$ 8			6			5			4			3.5			3.142			2.513			2.094		
	$r =$ 2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3
12	.021	.026	.031	.035	.046	.055	.053	.066	.079	.083	.103	.124	.108	.135	.162	.134	.168	.201	.209	.262	.314	.302	.378	.452
13	.022	.027	.033	.037	.048	.058	.055	.069	.083	.086	.108	.129	.113	.141	.169	.140	.175	.210	.218	.273	.328	.315	.395	.472
14	.022	.028	.033	.039	.049	.059	.057	.071	.086	.089	.111	.133	.116	.145	.174	.144	.180	.216	.225	.282	.337	.324	.407	.486
15	.023	.029	.035	.041	.050	.062	.059	.074	.089	.092	.116	.139	.121	.151	.182	.150	.188	.225	.235	.293	.352	.338	.423	.506
16	.024	.030	.036	.042	.053	.063	.061	.076	.091	.095	.118	.142	.124	.155	.186	.154	.193	.231	.241	.301	.360	.346	.434	.519
17	.025	.031	.037	.044	.055	.066	.063	.079	.095	.098	.123	.148	.130	.161	.193	.160	.200	.240	.250	.313	.375	.360	.451	.540
18	.025	.032	.038	.046	.057	.068	.065	.082	.098	.102	.128	.153	.134	.167	.200	.166	.207	.249	.259	.324	.389	.374	.469	.560
19	.027	.034	.040	.048	.060	.072	.069	.085	.107	.107	.134	.160	.140	.175	.210	.174	.217	.262	.272	.339	.408	.392	.491	.587
20	.028	.035	.042	.049	.062	.075	.070	.089	.108	.111	.139	.165	.146	.182	.217	.180	.225	.270	.282	.352	.421	.405	.508	.606
21	.028	.036	.043	.050	.063	.076	.073	.091	.109	.113	.142	.169	.148	.185	.223	.184	.230	.277	.288	.359	.432	.415	.521	.623
23	.029	.036	.044	.051	.064	.078	.074	.093	.111	.116	.145	.174	.151	.189	.228	.188	.235	.282	.294	.367	.440	.423	.532	.634
25	.030	.038	.045	.053	.067	.080	.076	.096	.114	.119	.149	.179	.156	.195	.235	.194	.243	.292	.303	.380	.454	.437	.550	.654
27	.031	.039	.046	.055	.069	.083	.079	.099	.118	.123	.154	.185	.161	.201	.242	.200	.250	.390	.313	.390	.469	.450	.566	.675
30	.032	.040	.047	.056	.070	.084	.080	.101	.121	.126	.157	.188	.164	.205	.246	.204	.255	.306	.319	.397	.478	.459	.578	.688
34	.032	.040	.048	.057	.071	.086	.082	.103	.123	.129	.161	.193	.168	.209	.252	.208	.261	.312	.326	.407	.488	.468	.593	.703
38	.033	.041	.050	.059	.073	.088	.084	.104	.127	.132	.165	.197	.171	.216	.258	.214	.267	.321	.333	.417	.500	.482	.606	.723
43	.034	.043	.051	.061	.075	.091	.087	.108	.130	.135	.169	.203	.177	.222	.266	.220	.275	.331	.343	.429	.515	.495	.624	.743
50	.035	.043	.052	.062	.077	.092	.089	.110	.133	.138	.172	.206	.181	.225	.272	.224	.280	.336	.349	.436	.524	.503	.635	.755
60	.035	.044	.053	.063	.078	.094	.090	.112	.135	.141	.176	.211	.184	.229	.276	.228	.284	.342	.355	.444	.535	.512	.645	.769
75	.036	.045	.054	.064	.079	.096	.092	.114	.137	.143	.179	.214	.186	.233	.280	.232	.289	.348	.361	.452	.543	.521	.656	.783
100	.037	.046	.055	.065	.081	.097	.093	.116	.140	.145	.182	.218	.191	.237	.285	.236	.293	.354	.368	.460	.554	.530	.666	.797
150	.037	.046	.055	.066	.082	.099	.095	.118	.142	.148	.185	.222	.193	.241	.288	.240	.299	.360	.374	.468	.565	.539	.677	.810
300	.038	.047	.056	.067	.084	.100	.096	.120	.144	.151	.189	.226	.197	.245	.293	.244	.304	.366	.381	.476	.573	.548	.689	.824
Rack	.038	.048	.058	.068	.085	.102	.098	.122	.147	.153	.191	.228	.200	.249	.299	.248	.310	.372	.387	.483	.580	.557	.700	.836
Face	11/16″	7/8″	1 1/16″	1 1/16″	1 5/16″	1 9/16″	1 1/4″	1 9/16″	1 7/8″	1 9/16″	2″	2 3/8″	1 13/16″	2 1/4″	2 11/16″	2″	2 1/2″	3″	2 1/2″	3 1/8″	3 3/4″	3″	3 3/4″	4 1/2″

FORMULA: $k = \frac{W}{S} = p'^2 r y$

k = constants of table. W = working load, pounds. S = maximum fiber stress, pounds per square inch.

p' = circular pitch, inches. p = diametral pitch. r = ratio $\frac{\text{face}}{p'}$. y = Lewis' variable for number of teeth, Table 11.

NO. OF TEETH	$p' =$ 1.75″			2″			2.25″			2.5″			3″			3.5″			4″		
	$p =$ 1.795			1.571			1.396			1.256			1.04			0.897			0.785		
	$r =$ 2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3	2	2.5	3
12	.410	.496	.616	.536	.670	.804	.679	.846	1.018	.838	1.048	1.258	1.208	1.509	1.811	1.642	2.052	2.462	2.146	2.683	3.217
13	.429	.535	.643	.560	.700	.840	.706	.885	1.062	.875	1.094	1.310	1.260	1.572	1.890	1.715	2.142	2.573	2.240	2.800	3.360
14	.442	.552	.662	.576	.720	.864	.726	.911	1.092	.898	1.125	1.350	1.296	1.620	1.944	1.762	2.205	2.646	2.304	2.880	3.456
15	.460	.575	.687	.600	.752	.900	.758	.951	1.135	.938	1.175	1.406	1.350	1.689	2.025	1.838	2.300	2.755	2.400	3.004	3.600
16	.471	.590	.706	.616	.771	.923	.777	.975	1.170	.965	1.205	1.443	1.386	1.734	2.076	1.883	2.363	2.825	2.464	3.088	3.692
17	.491	.613	.736	.640	.800	.960	.810	1.013	1.214	1.000	1.250	1.500	1.440	1.800	2.160	1.960	2.450	2.940	2.560	3.200	3.849
18	.510	.636	.762	.664	.830	.995	.840	1.050	1.280	1.038	1.298	1.555	1.494	1.869	2.238	2.034	2.545	3.052	2.652	3.440	3.980
19	.532	.667	.799	.696	.870	1.045	.880	1.100	1.320	1.088	1.358	1.633	1.566	1.956	2.350	2.128	2.665	3.197	2.776	3.480	4.176
20	.552	.690	.826	.720	.900	1.080	.911	1.139	1.362	1.125	1.408	1.685	1.620	2.028	2.430	2.205	2.758	3.304	2.880	3.600	4.311
21	.563	.703	.846	.736	.922	1.106	.930	1.162	1.402	1.150	1.440	1.726	1.653	2.070	2.488	2.254	2.825	3.385	2.944	3.684	4.420
23	.575	.722	.863	.752	.941	1.128	.952	1.188	1.425	1.175	1.470	1.763	1.689	2.118	2.538	2.303	2.881	3.455	3.008	3.760	4.508
25	.593	.742	.890	.776	.972	1.166	.980	1.227	1.470	1.213	1.515	1.818	1.743	2.184	2.619	2.377	2.975	3.566	3.104	3.884	4.776
27	.613	.767	.902	.800	1.000	1.200	1.012	1.264	1.518	1.250	1.563	1.875	1.800	2.250	2.700	2.450	3.063	3.675	3.200	4.000	4.800
30	.626	.781	.938	.816	1.020	1.224	1.035	1.290	1.545	1.278	1.592	1.913	1.836	2.298	2.751	2.506	3.126	3.745	3.264	4.080	4.892
34	.640	.795	.955	.836	1.040	1.248	1.057	1.321	1.583	1.305	1.626	1.950	1.881	2.360	2.808	2.562	3.185	3.822	3.340	4.160	4.960
38	.656	.815	.982	.856	1.068	1.286	1.085	1.355	1.628	1.340	1.670	2.008	1.929	2.406	2.889	2.625	3.276	3.937	3.432	4.280	5.136
43	.674	.843	1.012	.880	1.100	1.302	1.113	1.390	1.670	1.375	1.720	2.065	1.980	2.475	2.970	2.695	3.367	4.043	3.520	4.400	5.280
50	.689	.855	1.030	.896	1.120	1.344	1.135	1.416	1.698	1.400	1.750	2.097	2.016	2.517	3.015	2.744	3.427	4.105	3.584	4.480	5.364
60	.700	.871	1.045	.912	1.136	1.368	1.151	1.440	1.725	1.425	1.778	2.138	2.052	2.559	3.069	2.793	3.479	4.182	3.648	4.530	5.460
75	.711	.888	1.065	.930	1.157	1.392	1.175	1.463	1.762	1.450	1.813	2.175	2.088	2.604	3.126	2.842	3.543	4.259	3.712	4.624	5.556
100	.725	.904	1.085	.944	1.177	1.416	1.195	1.486	1.792	1.475	1.844	2.213	2.124	2.649	3.168	2.891	3.605	4.333	3.776	4.720	5.652
150	.735	.918	1.105	.960	1.197	1.440	1.215	1.515	1.821	1.500	1.875	2.250	2.160	2.694	3.234	2.940	3.668	4.410	3.840	4.800	5.748
300	.749	.934	1.121	.976	1.318	1.464	1.238	1.540	1.850	1.525	1.905	2.284	2.196	2.739	3.291	2.989	3.731	4.486	3.904	4.872	5.844
Rack	.761	.950	1.140	1.192	1.340	1.488	1.260	1.565	1.880	1.550	1.938	2.325	2.238	2.787	3.345	3.052	3.797	4.563	3.968	4.960	5.940
Face	3½″	4⅜″	5¼″	4″	5″	6″	4½″	5⅝″	6¾″	5″	6¼″	7½″	6″	7½″	9″	7″	8¾″	10½″	8″	10″	12″

TABLE 13—VALUES OF FACTOR K: CYCLOID AND 15 DEGREE INVOLUTE; ADDENDUM = 0.31831 p'.

NO. OF TEETH	y	DIAMETRAL PITCH AND FACE OF STANDARD GEARS												
		4 × 2	5 × 1¾	6 × 1½	7 × 1⅜	8 × 1¼	9 × 1⅛	10 × 1	11 × ⅞	12 × ¾	14 × ⅝	16 × ½	20 × ⅜	24 × ¼
12	.067	842	589	421	331	263	210	168	134	105	75	53	31	17
13	.070	880	616	440	346	275	220	176	140	110	78	55	33	18
14	.072	904	633	452	356	282	226	181	144	113	81	56	34	19
15	.075	942	660	472	370	294	235	188	150	118	84	59	35	20
16	.077	967	677	483	380	302	242	193	154	121	86	60	36	20
17	.080	1005	704	503	395	314	251	201	160	125	89	63	38	21
18	.083	1043	730	522	410	326	261	208	166	130	93	65	39	22
19	.087	1093	765	548	430	342	273	218	174	136	97	68	41	23
20	.090	1130	791	566	445	353	283	226	180	141	101	70	42	24
21	.092	1157	809	578	454	361	289	231	184	144	103	72	43	24
23	.094	1180	827	592	464	369	295	236	188	148	105	74	44	25
25	.097	1218	853	610	479	381	305	244	194	152	108	76	46	25
27	.100	1257	880	628	494	393	314	251	200	157	112	78	47	26
30	.102	1280	897	642	504	400	320	256	204	160	114	80	48	27
34	.104	1310	915	654	514	408	327	261	208	163	116	82	49	27
38	.107	1344	942	673	528	420	336	269	214	168	120	84	50	28
43	.110	1382	967	692	543	432	346	276	220	173	123	86	52	29
50	.112	1410	985	705	553	440	352	281	224	176	125	88	53	29
60	.114	1433	1003	717	563	447	358	286	228	179	127	89	54	30
75	.116	1458	1021	730	573	455	365	291	232	182	130	91	55	30
100	.118	1483	1038	743	583	463	371	296	236	185	132	93	56	31
150	.120	1508	1055	755	592	472	377	302	240	188	134	94	56	31
300	.122	1532	1073	767	602	479	383	307	244	192	137	96	57	32
Rack	.124	1560	1092	780	612	487	389	312	248	195	139	97	58	32

TABLE 14—SAFE WORKING LOADS ON THE TEETH OF STANDARD CUT GEARS OF CAST IRON RUNNING AT A SPEED OF 100 FEET PER MINUTE

FACTOR OF SAFETY FOR GEARS

The load on the teeth of gears made from forgings may be such as to strain the material close to its elastic limit (based upon the worn thickness of tooth), if it is free from flaws. For castings this is not a safe rule, as there are always hidden defects to a greater or less extent. As long as the strain is kept below this point, excessive wear will not take place, but if this point is exceeded but slightly, rapid wear, or fracture of the teeth, is sure to result. For reasonable service, a factor of safety of 1.5 should be used if the load is uniform. Thus, for a forged steel gear having an elastic limit of 20,000 pounds per square inch, the safe load would be $\frac{20,000}{1.5} = 13,330$ pounds per square inch. For cast steel, free from apparent defects, a factor of 2 is recommended; thus, for this same strength in steel in a casting the safe load would be $\frac{20,000}{2} = 10,000$ pounds per square inch.

The elastic limit meant in this connection is the real elastic limit of the material as taken by an accurate extensometer and not by the drop of the beam, or by caliper measurement, as has been commercial practice. This instrument detects the first indications of permanent set in the test piece, showing that the safe load for that material has been exceeded; the drop of the beam is not apparent for some time after this. For untreated mild steels this point is sometimes at one-half the drop of beam; for the higher grades, however, the two points are closer together.

Such an instrument is described by T. O. Lynch in a paper on "The Use of the Extensometer for Commercial Work" read before the American Society for Testing Materials; Philadelphia, published in the Proceedings for 1908, volume 8.

It must be pointed out that gear steels should have an ample reduction of area to guard against sudden fracture. Test pieces should be cut with the center of tooth a little below the bottom line, say 0.07 of the circular pitch, as illustrated by Fig. 50, as it is through this point that the tooth generally breaks out.

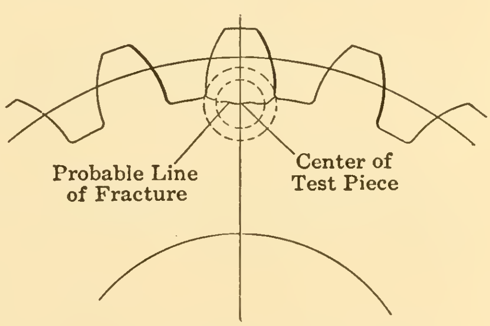


FIG. 50. LOCATION OF TEST PIECE.

The strength of the material in gears will be found to vary as much as 30 per cent. in different parts of the tooth and rim, therefore a settled point for cutting out test pieces is necessary if uniform, safe, or accurate results are to be expected. It does not greatly matter if the threaded portion of the test piece projects into the tooth space, as it will on all gears 2½ diametral pitch and finer, so long as the 0.505-inch portion of the piece is clear. When a 0.505-inch test piece (⅕ of a square inch in area) cannot be obtained in this manner, make one 0.2525 inch in diameter (1/20 of a square inch in area), leaving the threaded portion ⅝ inch instead of ¾ inch, which is standard.

Note that elastic limits given in table for wear of gear teeth is by drop of beam.

STRENGTH OF BEVEL GEARS

In general apply the Lewis formula for spur gears, figuring the safe load from the average pitch diameter, or, stated a little differently, the velocity in feet per minute and the pitch is to be taken at the average pitch diameter, otherwise the gear is to be treated as a spur.

Let N = number of teeth.
p' = circular pitch.
D' = pitch diameter.
b = face width.
p'_a = average circular pitch.
D_a = average pitch diameter.
a = apex distance.
E = center angle.
S = safe working stress.
V_a = velocity (average) in feet per minute.

In order to get the average pitch we must first determine the apex distance a. Now

$$a = \frac{D'}{2 \sin. E}.$$

The average pitch is the pitch at the center of the gear face f. Above this section the tooth strength increases and below this point it decreases. The mean strength of the tooth is, therefore, located at this point. Thus it is the proper dimension to use in determining the strength of the tooth. This average pitch is found from the following equation:

$$p'_a = \frac{p'\left(a - \frac{f}{2}\right)}{a}.$$

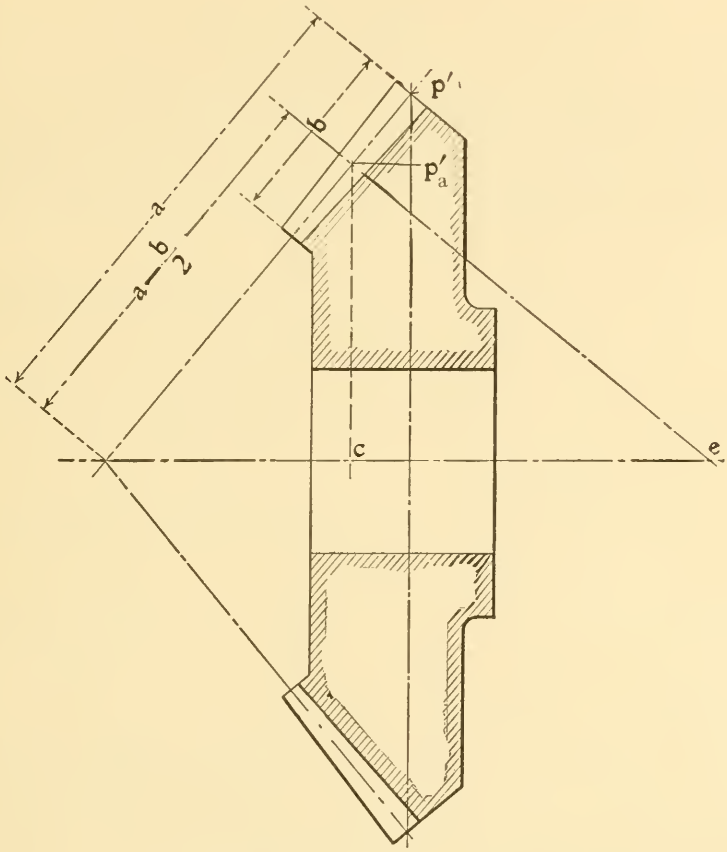


FIG. 51. DIAGRAM FOR STRENGTH OF BEVEL GEARS.

This formula is derived as follows: By referring to Fig. 51, it is evident that the pitch of the tooth at p' is to the apex distance a as the pitch at p'_a is to the mean apex distance $\left(a - \frac{b}{2}\right)$.

The average pitch diameter D_a is found from formula (3) by substituting the average pitch. The average velocity V_a is found from formula (4) by using the average pitch diameter. Then

$$a = \frac{D}{2 \sin. E}. \tag{1}$$

$$p'_a = \frac{p'\left(a - \frac{f}{2}\right)}{a}. \tag{2}$$

$$D_a = N\ p'_a\ 0.3183. \tag{3}$$

$$V_a = 0.2618\ D_a\ (\text{r.p.m.}). \tag{4}$$

$$\textit{Safe load} = S\ p'_a\ f\ y\left(\frac{600}{600 + V_a}\right). \tag{5}$$

$$Horse\text{-}power = \frac{Safe\ load \times V_a}{33{,}000}. \tag{6}$$

The values for the factor y are from the Lewis formula.

An illustrative example is as follows: What power may be transmitted by a pair of miter gears of the following dimensions: 30 teeth, 2-inch pitch, 5-inch face, 19.107-inch pitch diameter, at 50 revolutions per minute? The material is cast iron.

$a = \dfrac{19.107}{2 \times 0.707} = 13.51$ inches, the apex distance;

$p'_a = \dfrac{2 \times (13.51 - \frac{5}{2})}{13.51} = 1.63$ inches, the average pitch;

$D_a = 30 \times 1.63 \times 0.3183 = 15.5$ inches, the average pitch diameter;

$V_a = 0.2618 \times 15.5 \times 50 = 203$, the average velocity in feet per minute;

The safe load $= 8000 \times 1.63 \times 5 \times 0.102 \times \left(\dfrac{600}{600 + 203}\right) = 4900$ pounds.

The horse-power $= \dfrac{4900 \times 203}{33{,}000} = 30.1.$

The teeth in bevel gears are more strongly shaped than the teeth of spur gears of the same pitch and number. This increase is represented by the radius $e - p'_a$ in Fig. 51, compared with the radius at the point p'. The corresponding number of teeth for this larger radius is found by the expression:

$$\frac{N}{cos.\ E}.$$

When selecting the constant y, however, it is well to disregard this increase, as it will tend to compensate for the loss in efficiency due to the use of bevel gears.

THE STRENGTH OF SHROUDED GEAR TEETH

In regard to strength of shrouded gear teeth, Wilfred Lewis submits the following, originally published in American Machinist of Jan. 30, 1902.

"I do not know of any careful analyses of or experiments on the strength of shrouded gear teeth, but I have some recollection of a tradition in vogue about twenty-five years ago that from one-fourth to one-half might be added to the strength by shrouding.

"There are, however, a number of cases to be considered; the shrouding may extend to the pitch line only or to the ends of the teeth, and it may be single or double. Formerly the practice of shrouding pinions was more common than it is to-day, because the advent of steel as a cheap construction material makes it possible to obtain unshrouded pinions of greater strength than the cast-iron

gears with which they engage, and now steel pinions have generally supplanted the old cast-iron shrouded ones which were naturally more roughly shaped, because harder to fit, and more difficult to assemble by reason of the shrouding. In my investigation of the strength of gear teeth I therefore assumed that the time for shrouded gears has passed, at least as far as machine tools were concerned, but they are possibly used as freely as ever on roll trains and some other classes of machinery, so that the problem may still be worthy of consideration from a practical standpoint. Rankine, in his 'Applied Mechanics,' rather summarily disposes of the strength of gear teeth by assuming the load that may be carried on one corner to be all that any tooth is good for. When so loaded, it is shown that the corner will break off at an angle of 45 degrees, and the strength of a tooth of any width is then no greater than that of a tooth whose width is twice its hight. So, if the hight of a tooth is 0.65 pitch, the strength, according to Rankine, should be taken for a width of only 1.3 pitch. Faces wider than this would be no stronger, and shrouding at one end would make no difference.

"But his assumption is untenable, because no maker of machinery who values his reputation will put gears into service bearing only at one end, and should they be so started, an even distribution of pressure is sooner or later effected by the natural process of wear.

"A comparison of strength between shrouded and unshrouded gears should therefore be made on the assumption of uniform distribution of pressure across the faces of their teeth, and for this purpose it will be expedient to neglect the influence of tooth forms, which would complicate and prolong the investigation, and treat all teeth simply as rectangular prisms, which may or may not be supported by shrouding. Rankine and Unwin have both been contented to estimate the actual strength of teeth as though they were rectangular prisms, and, although this is far from the truth, it is certainly more admissible as a basis of comparison for another variable than as an approximation for a direct result. The effect of this assumption will be to exaggerate the value of shrouding, and for the present it will be sufficient to indicate roughly the maximum benefit to be anticipated.

"In Fig. 52 a gear tooth is shrouded at one end, and the problem is to determine its strength as compared with the same tooth not shrouded. For convenience, the thickness, or half the pitch may be taken as unity, and the hight as 1.3. The load W is assumed to be applied uniformly along the end of the tooth over the face b, making the full load $b\ W$. If there were no shrouding, the strength of this tooth would be measured by the transverse resistance at its root t, and if broken at the root as shown in Fig. 53, we may consider how much strength could be given to it by shrouding alone.

"A tooth thus broken would have some strength as a cantilever imbedded in the shrouding, but more as a shaft subjected to torsion, and for the shape here assumed the torsional strength alone will probably exceed the combined strength due to torsion, and flexure for any actual shape.

"The rectangular tooth whose sides are h and t cannot be treated as an ordinary shaft because its neutral axis is at one side instead of, as usual, at the center of gravity. It must therefore be treated as one half of a shaft whose

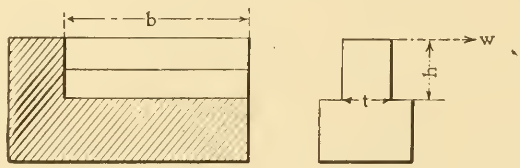


FIG. 52.

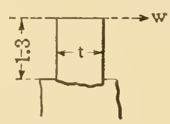


FIG. 53.

DIAGRAMS ILLUSTRATING THE STRENGTH OF SHROUDED GEAR TEETH.

sides are $t \times 2\,h$ or 1×2.6, for which the moment of resistance is about $.6\,S$, where S is the shearing stress at the end of a tooth. For the unit load W we have $.6\,S = 1.3\,W$, or $S = 2.17\,W$, and for the width b we have $S_1 = b\,S = 2.17\,b\,W$.

"Thus the maximum intensity of shearing S_1 is found to be a little more than twice the full load $b\,W$.

"On the other hand, for an unshrouded tooth the transverse stress f at the root of a tooth depends only upon W and the relation is expressed by the equation $f = 7.8\,W$. In these terms,

$$W = \frac{f}{7.8},$$

$$= \frac{S_1}{2.17\,b},$$

whence

$$f = \frac{3.5 S_1}{b},$$

and assuming that the shearing stress S_1 may be $0.8 f$, we have $b = 2.8$. This means that a tooth 2.8 wide has as much strength as may possibly be added by shrouding at one end, but the question remains to be considered, under what conditions and to what extent can this possible strength be made effective? The development of stress is always accompanied by strain, and in the case of a shrouded tooth the unit load W must be divided between torsion and flexure. Obviously, if the tooth is very long, its stiffness under torsion will be so little as compared with its stiffness under flexure, that the benefit from shrouding will be inappreciable, and on the other hand if very short, the torsional stiffness will be preponderate. When a uniform distribution of load has been attained, as it must be by the action of wear, that part of a tooth farthest from the shroud-

ing will sustain the greatest transverse stress and the load W will be divided at all points along the face of the tooth between torsion and bending directly as the stiffness encountered in these two directions or inversely as the relative strains. Each strain is relieved by the other, but the limit of strength is reached when either attains its maximum.

"Considering a cantilever loaded at the end, we have for the deflection y, under the stress f,

$$y = \frac{f h^2}{1.5 E},$$

where $h = 1.3$ and E is the modulus of elasticity for flexure. Substituting this value of h we have

$$y = \frac{1.1 f}{E}. \qquad (1)$$

"Considering the tooth as a rectangular shaft in torsion, it will be seen that the shearing stress for a distributed load decreases from the maximum S_1 at the shrouding to nothing at the other end of the tooth. For the unit load W the shearing stress is S, for an element $d\,x$ the stress is $S\,d\,x$, and for this stress at the distance x from the shrouding, the torsional deflection

$$d\,z = \frac{S\,x\,d\,x}{G},$$

where G is the modulus for shearing. The total deflection for the load distributed over a length x therefore is

$$z = \frac{S\,x^2}{2\,G},$$

or for the face b we have

$$z = \frac{S\,b^2}{2\,G}.$$

"But since $G = 0.4\,E$, we may write

$$z = \frac{S\,b^2}{0.8\,E}. \qquad (2)$$

"The value of S has been found to be $S = 2.17\,W$, and we have also found $f = 7.8\,W$, therefore

$$S = \frac{2.17}{7.8} f,$$
$$= 0.28 f.$$

and

$$z = \frac{S\,b^2}{8\,E}.$$

may be written

$$z = \frac{0.35 f\,b^2}{E}. \qquad (3)$$

"The deflection of an unshrouded tooth under the load W has been shown by equation (1), and, dividing this into equation (3), we have for the relation between y and z

$$\frac{z}{y} = 0.3b^2.$$

For a very narrow tooth, letting $b = 1$, we have $z = 0.3\,y$; but since y and z are necessarily equal, when the tooth under consideration is attached at its root and also to the shrouding, the load W will be supported at both points, and it will necessarily be divided between them in the proportion of y to z, or as 1 to 3. The shrouding will carry $\frac{W}{1.3}$, or $0.77\,W$, and the root of the tooth $0.23\,W$.

"Similarly making, $b = 2$, or one pitch, we have $z = 1.2\,y$, and at this point the shrouding will carry $0.45\,W$ and the root of the tooth $0.55\,W$. Also, when $b = 3$, we have $z = 2.7\,y$, reducing the load on shrouding to $0.27\,y$ and increasing the load at the root to $0.73\,W$. At $b = 4$, or 2 pitch, $z = 4.8\,y$, the load on shrouding drops to $0.17\,W$ and the load at the root rises to $0.83\,W$.

"The average width of gear faces is probably about 2.5 pitch, and for $b = 5$ we have about $0.12\,W$ carried by the shrouding and $0.88\,W$ carried by the tooth acting as a cantilever. We may therefore conclude that the strength of an ordinary pinion shrouded at one end only is not increased more than 12–88 or about 13 per cent., by the shrouding. Indeed, this is only the result of a first approximation, and for the successive proportions of W thus credited to the shrouding new values of z might be estimated to be used as the basis of a second approximation. But we will not continue the process—it is sufficient to know that our valuation of the effect of shrouding is high. A double-shrouded pinion running with a gear whose face is $2.5\,p'$ will be about 3 pitch between shroudings and its strength will be about the same as that just found for $b = 3$. An ordinary pinion will not therefore be increased in strength by double shrouding more than 37 per cent., and it is probably safe to say that a more elaborate investigation will reduce the additional strength to 10 per cent. for single shrouding and 30 per cent. for double shrouding.

"When the shrouding extends to the pitch line only, the shearing strength of its attachment to a tooth is reduced, but the elastic relations upon which the strength at the root depends remain practically the same. In this case the shearing strength instead of the transverse strength limits the strength of a tooth, and the strength is apparently less than for full shrouding.

"The effect of shrouding is clearly to prevent the adjacent part of a tooth from exercising its strength as a cantilever. The shrouding carries what the tooth itself might carry almost as well. A heavy link in a light chain adds nothing to the strength of the chain, and teeth which are not strong all over need not be strengthened in spots. A little more face covering the space occupied by

shrouding is more to the purpose for durability as well as for strength, and when this fact is appreciated I believe the practice of shrouding will disappear in rolling mills, as it has done in machine shops.

"In regard to the working stress allowable for cast iron and steel, I may say that 8000 pounds was given as safe for cast-iron teeth, either cut or cast, and that 20,000 pounds was intended for ordinary steel whether cast or forged. These were the unit stresses recommended for static loads, and as the speed increased they were reduced by an arbitrary factor, depending upon the speed.

"The iron should be of good quality capable of sustaining about a ton on a test bar 1 inch square between supports 12 inches apart, and of course the steel should be solid and of good quality. The value given for steel was intended to include the lower grades, but when the quality is known to be high, correspondingly higher values may be assigned.

"In conclusion I may say that the crude investigation here given seems to justify the traditions referred to that from ¼ to ½ may be added to the strength of teeth by shrouding. If the teeth are very narrow, ½ may be added, but generally, I believe, ¼ is enough and since writing the above I find that D. K. Clark almost splits the difference by adding ⅓ for double shrouding. But the development of the full strength of gear teeth depends nearly as much upon the strength and stiffness of the gear journals as upon the teeth themselves, and no rules can be given for indiscriminate use."

WEAR OF GEAR TEETH

The Lewis formula is the only accurate method of figuring the power of gears so far as the strength of the teeth is concerned, but takes no account whatever of wear, and the value of the tooth surfaces to resist crushing of the material. Trouble from this source is a common experience, although not properly understood, as it is sometimes difficult to account for the "mysterious" failure of gears that were apparently of ample strength. It is noticed that gears generally fail through wear and not by fracture of the teeth, also that the teeth often break at a load far below that which is considered safe. More attention, therefore, should be given to this point.

A certain combination of diameters will carry but a certain load per unit of face, irrespective of the pitch of the gears, so that there is no gain in an increased pitch above that just sufficient to resist fracture. This pitch may be found as usual by the Lewis formula, but the actual strength of the material should be used. The material in a 1-inch pitch tooth is stronger proportion-

ately than the same material in a 2-inch pitch tooth. This is caused by the fact that nearer the exterior the material is stronger and of a closer grain, due to rapid cooling. A tooth is stronger at the top than at its root. It would seem as if tests of material should be made for flexure and not for tensile strength, as the tooth breaks through bending. The average ultimate strength of cast iron for flexure is 38,000 pounds per square inch, while the tensile strength is 24,000 pounds per square inch.

Aside from this feature the surface hardness should also be considered irrespective of the strength of material. For instance, a pressure of 5000 pounds per unit of contact would be allowed for a case-hardened steel surface, while but 1500 would be allowed for the same material in its unhardened condition.

The relative hardness of material, in conjunction with the co-efficient of friction for different grades and hardness of material engaging will supply the safe load A per unit of area.

It is true that the arc of rolling contact in gears is very small; the balance is sliding contact, which increases proportionately over the rolling contact as the pitch points separate, or as the tooth disengages, and decreases as the tooth enters contact until the pitch points again engage where it is rolling.

The wearing qualities of the teeth depend greatly upon their condition when put into service. If a little care is used to obtain a smooth surface at the start and allow the teeth to find their proper bearing, the gears will wear indefinitely longer than if put under full load when new, no matter how accurately the teeth are cut. Also a gear once started to cut can often be saved by the timely application of a fine file, finally smoothing the teeth with an oil stone.

A series of experiments to determine the proper load per unit of contact (A) for different grades and hardness of material used would certainly lead to a fuller knowledge of the capacity of gears for the transmission of power and leave less to supposition on the part of the designer. On account of the peculiar nature of the tooth contact it is quite likely that the best manner to reach accurate results would be with gears made from the materials under consideration. The values given in Chart 2 are the best obtainable at the present writing.

The idea of limiting the load to the proportion of the gear diameters irrespective of the pitch (for a unit of face) may at first appear startling, but when we consider that the radii from which the tooth is drawn are always proportional to the *pitch diameter* of the gear and not to the *pitch*, and that the teeth in contact are actually two cylinders rolling and slipping upon each other, it appears more reasonable. See Fig. 54. It should be understood, however,

that the diameter and position of these rollers change constantly throughout the contact, and that a gear made in strict accordance with Fig. 54 would not give a uniform movement. It illustrates the principle however.

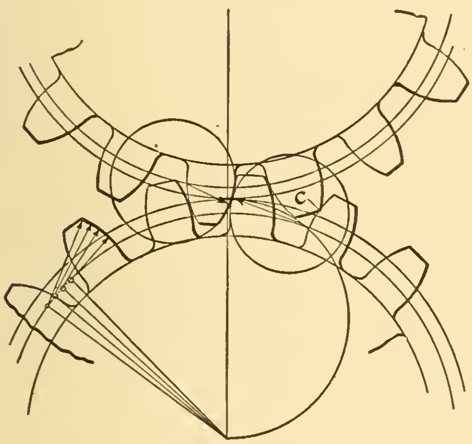


FIG. 54. GEAR TOOTH ACTION.

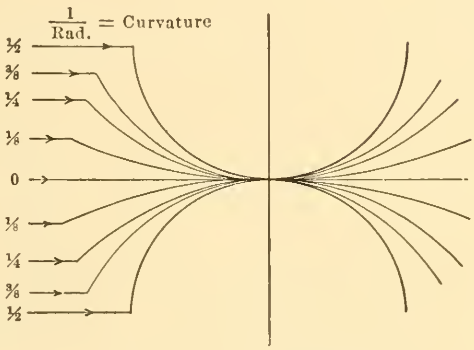


FIG. 55. CURVATURES.

To secure safe results the flank or shortest radius is used in these formulas.

The following is the gist of an article by Harvey D. Williams, in the American Machinist, with its application to gear transmissions:

"The curvature of a plane curve is defined by mathematicians as the change of direction per unit of length, and is equal to the reciprocal of the radius of curvature at the point considered. Thus in going once around a circle of radius R the distance traversed is the circumference $2 \pi R$, while the change of direction is in circular measure or radius 2π.

"The change in direction per unit of length is therefore

$$\frac{2 \pi}{2 \pi R} = \frac{1}{R}.$$

"Accordingly the curvature of a 2-inch circle is 1, that of a 1-inch circle is 2, that of a ½-inch circle is 4, and that of a straight line is $\frac{1}{\infty} = 0$, etc. . . . .

"The curvature of a straight line being zero, that of an arc may be said to be its curvature in relation to the straight line, or its *relative curvature* to the straight line. Similarly, in comparing the curvature of two arcs, it will be convenient to use the term 'relative curvature' instead of the difference of curvature, meaning thereby the algebraic difference of the curvature as dimensioned in Fig. 55.

"It will be seen that when two plane curves are tangent to each other externally the relative curvature is to be found by adding the respective curvatures, and when they are tangent internally the relative curvature is to be found by subtracting. . . .

"The amount of contact between plane curved profiles is measured by the reciprocal of the relative curvature." See formulas 21, 22, and 23.

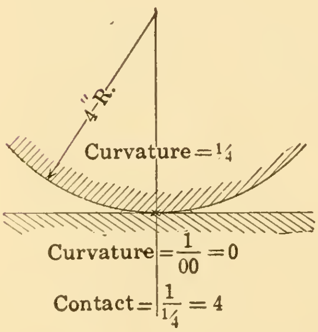


FIG. 56.

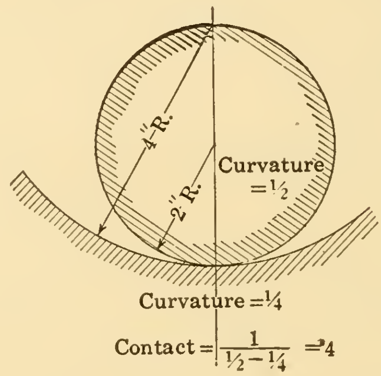


FIG. 57.

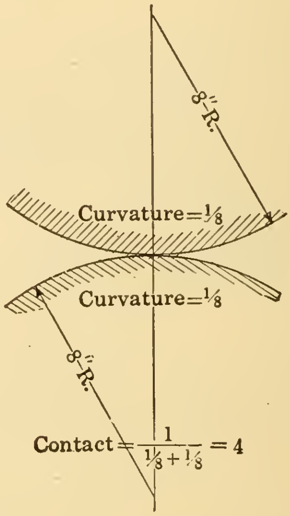


FIG. 58.

CONTACTS OF THE SAME CURVATURE.

In each of the cases shown by Figs. 56, 57, and 58 the contact is 4, and if these profiles were made of the same material and the same width of face, they would be equally efficient as regards their ability to withstand pressure.

DEVELOPMENT OF FORMULAS

Let

C = contact.
R^1 = flank radius of the gear.
r^1 = flank radius of the pinion.
f = face width.
V = velocity in feet per minute.
A = safe crushing load per unit of contact.
D^1 = pitch diameter of the gear.
d^1 = pitch diameter of the pinion.
a = angle of obliquity.
W^c = safe load on the tooth to resist crushing and wear.
W = safe load on the tooth to resist fracture.

Then

$$R^1 = \frac{D^1}{2} \sin a, \qquad (19)$$

and

$$r^1 = \frac{d^1}{2} \sin a. \qquad (20)$$

For spur gears

$$C = \frac{1}{\frac{1}{r^1} + \frac{1}{R^1}}. \qquad (21)$$

For internal gears

$$C = \frac{1}{\frac{1}{r^1} - \frac{1}{R^1}}. \qquad (22)$$

For racks

$$C = \frac{1}{\frac{1}{r^1} \pm 0} = \frac{r^1}{1} = r^1. \qquad (23)$$

$$W^e = C\, f\, A \left(\frac{600}{600 + V} \right). \qquad (24)$$

It is desirable that the gear and pinion should wear equally to avoid the necessity of engaging a new pinion with a partly worn gear, thereby decreasing the life of both. It is assumed that wear is proportional to the hardness of the material; obviously the pinion should be harder than the gear in proportion to the ratio of the drive. Therefore, to secure equal wear in a pair of gears having, say, a ratio of 4 to 1, the pinion should be made four times as hard as the gear.

I have thought that a hard pinion would tend to preserve a softer gear, but as no data are found to sustain this theory, and as the value calculated for Chart 1 tends toward a much softer gear than was originally thought proper to make the best wearing combination for certain ratios, this has not been taken into account. It is assumed, therefore, that a hard pinion will neither preserve nor influence the wear of a softer gear. Therefore, it may be assumed safely that a hard gear will not influence the wear of a softer pinion. Chart 1 is made on this basis. The wear of gear and pinion are determined independently if the proper combinations of hardness are not used.

The wear is based entirely on the pinion hardness, the gear performing the same amount of work, but having less wear on account of the greater number of teeth in use in proportion to the ratio of the gears. For instance, in a gear drive having a ratio of 4 to 1, the gear may be 75 per cent. softer than the pinion for equal wear.

Thus the wear of the gear is found according to the pinion hardness that is proper for the ratio of the gears irrespective of the material actually used for the pinion.

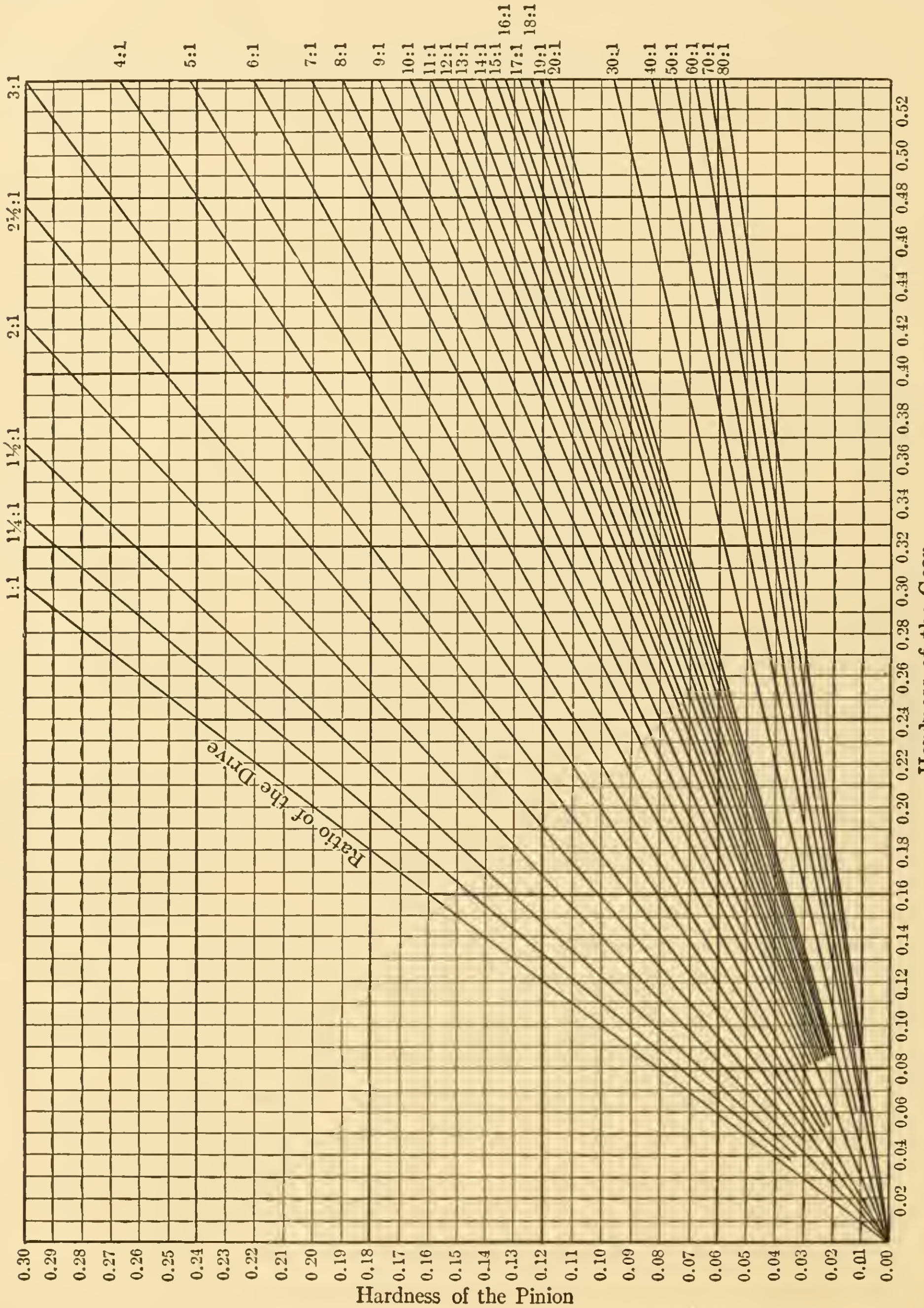


CHART I. RELATIONS BETWEEN GEAR AND PINION HARDNESS FOR VARIOUS RATIOS OF DRIVES.

For example: If a pinion of a hardness represented by 0.15 (see Chart 1) engages a gear of 0.35 hardness, the ratio being 4 to 1, the wear of the gear will be in accordance with the pinion hardness found opposite the line of ratio and

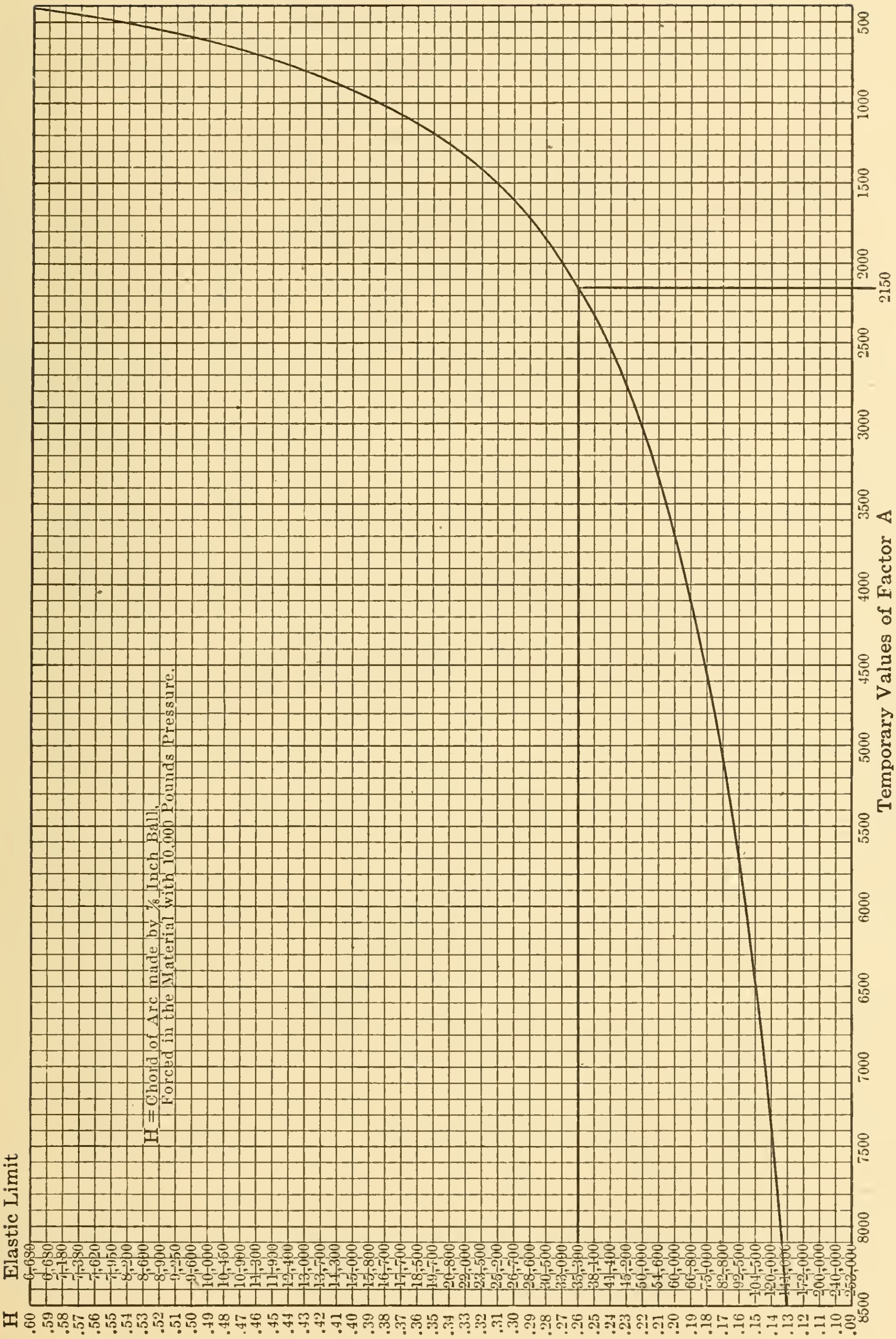


CHART 2. RELATIONS BETWEEN THE ELASTIC LIMIT AND VALUES OF THE FACTOR A.

over the gear hardness (0.35), which in this case is 0.17½. In this event the gear will wear out first. On the other hand, if the pinion had been of 0.20 hardness, the pinion would wear out first. The value for the wear of the gear would remain 0.17½.

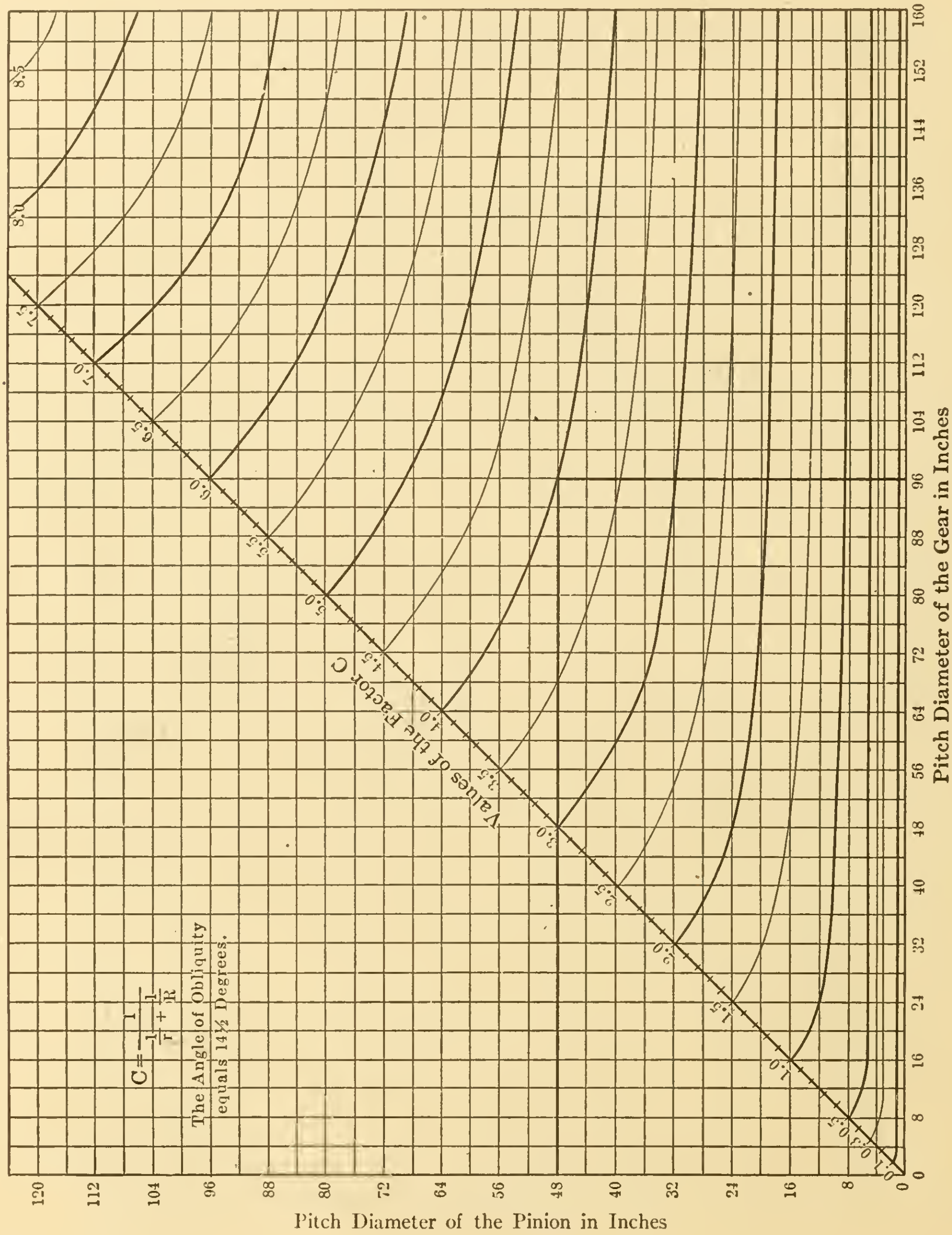


CHART 3. RELATIONS BETWEEN THE PITCH DIAMETERS OF MATING SPUR GEARS AND THE FACTOR C.

It is thought that the elastic limit of a material follows the hardness, therefore the wear may be determined from the elastic limit. The points of hardness, however, are used in the accompanying charts for convenience; Chart 2

gives the corresponding values. The hardness values given in these tables were obtained by pressing a ⅞-inch hardened steel ball into the surface of the material with a pressure of 10,000 pounds. The dimensions given are diameters of the indentations thus made. See Fig. 59. The comparisons in hardness were

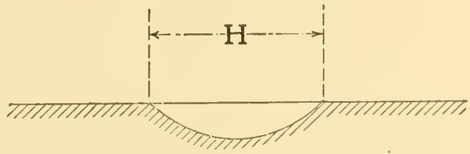


FIG. 59. CHORD MEASUREMENT OF HARDNESS TEST.

made inversely to the square of these diameters. The elastic limit was determined for one of these values; the comparison for others may be found by the same inverse proportion.

Thus, the square of 0.20 = 0.04 and the square of 0.30 = 0.09. Therefore, 0.20 would be $\frac{0.09}{0.04} = 2\frac{1}{4}$ times harder than 0.30. This is a proper combination for a gear ratio of $2\frac{1}{4}$ to 1. The elastic limit for 0.20 is 60,000 pounds per square inch. The elastic limit of 0.30 $= \frac{60,000}{2\frac{1}{4}} =$ 26,700 pounds per square inch.

For comparison with the Brinell scale; hardness value 0.22 inch in Chart 2 measures 5.6 millimeters; the hardness numeral for this impression being 168, the impression is made with a 10-millimeter ball at a pressure of 3000 kilograms.

As the elastic limit of cast iron is very close to its ultimate tensile strength the ultimate strength may be used to determine the hardness. This was at first very confusing before it was found that the hardness followed the elastic limit, as cast iron under the ball test referred to would show a hardness equal to machine steel of twice its ultimate tensile strength.

The values of A according to the hardness or elastic limit of the material (Chart 2) have been assumed as correct for gears operating 10 hours per day for a period of two years. If found in error their multiplier given in Table 15 for the time and conditions of service may be shifted without changing the original values of Charts 1 and 2. This table may be elaborated to any desired extent to cover various conditions. It is evident that a pair of gears will not last as long fastened to the ceiling or to insecure timbers as if mounted upon a proper concrete foundation. There are all manner of machine constructions to be considered as well as unknown overloads, the influence of fly-wheels and other things that are usually neglected. All this, however, is simple indeed when the Stygian darkness in which we are now wandering is considered.

TIME OF SERVICE	UNIFORM LOAD		
	CONTINUOUS	10 HOURS DAILY	5 HOURS DAILY
For 3 months	3.00	8.00	18.00
For 6 months	1.50	4.00	9.00
For 9 months	1.00	2.67	6.00
For 1 year	0.75	2.00	4.50
For 2 years	0.38	1.00	2.25
For 3 years	0.25	0.67	1.50
For 4 years	0.19	0.50	1.13
For 5 years	0.15	0.40	0.90
For 6 years	0.13	0.33	0.75
For 7 years	0.11	0.29	0.65
For 8 years	0.10	0.25	0.56
For 9 years	0.09	0.22	0.50
For 10 years	0.08	0.20	0.45

TABLE 15—MULTIPLIERS FOR FACTOR 'A'

According to the Conditions of Service and Desired Life of Gears

For gears subjected to 25 per cent. overload, multiply result by 0.80; for gears subjected to 50 per cent. overload, multiply result by 0.70; for gears subjected to 75 per cent. overload, multiply result by 0.60; for gears subjected to 100 per cent. overload, multiply result by 0.50; for gears operating in dust-proof oil case, multiply result by 1.50.

EXAMPLES

What is the safe load for a pair of spur gears properly mounted on concrete foundations to operate continuously for a period of five years before replacing? The gears are to run in oil in a dust-proof case driving an electric generator making 300 revolutions per minute from a turbine revolving at 1200 revolutions per minute. The overload at no time will exceed 25 per cent. The gear has 84 teeth, 3 diametral pitch, 12-inch face and 28-inch pitch diameter. The pinion has 21 teeth, 3 diametral pitch, 12-inch face and 7-inch pitch diameter. The ratio is 4 to 1. The circumferential speed at the pitch line is 2200 feet per minute. The pinion should be four times as hard as the gear for equal wear and, according to Chart 1, if a cast-steel gear of 30,500 pounds' elastic limit which is assumed, to have a hardness value of 0.28, the pinion hardness should be represented by 0.14, which represents an elastic limit of 120,000 pounds per square inch. This may be obtained by hardening, chrome nickel or other high-grade steel. High-carbon steel should be avoided for this purpose on account of its tendency to crystallize.

Referring to Chart 2 it is found that the temporary value of A is 7500 pounds. The multiplier for time of service is 0.15; the multiplier for overload is 0.80; the multiplier for the oil case is 1.50. Thus the final value of $A = 7500 \times 0.15 \times 0.80 \times 1.50 = 1350$ pounds.

In the formula

$$W^c = C f A \frac{600}{600 + V},$$

where

W^c = the safe load in pounds (to be determined),
C = 0.70 (from Chart 3),
f = the face, 12 inches,
$A = 1350$, and $\frac{600}{600 + 2200}$ (from Chart 4) = 0.21,

then $W^c = 0.70 \times 12 \times 1350 \times 0.21 = 2380$ pounds,

or $$\frac{2380 \times 2200}{33{,}000} = 158 \text{ horse-power.}$$

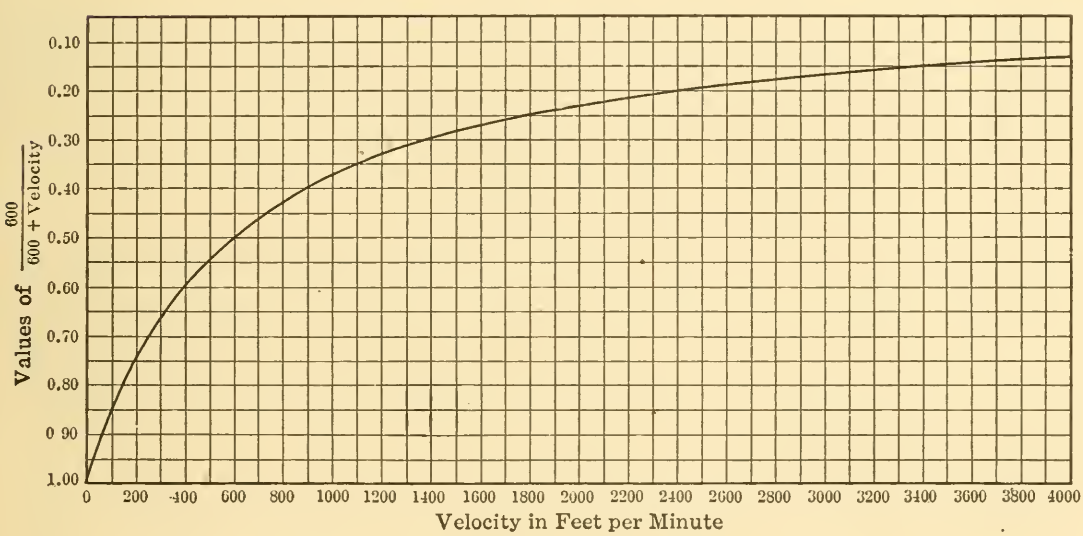


CHART 4. RELATIONS BETWEEN THE QUANTITY $\frac{600}{600 + velocity}$ AND THE VELOCITY.

The strength of the teeth must now be checked by the Lewis formula to guard against fracture at this load. We find by this method a safe working load of 317 horse-power.

This illustrates that the teeth are capable of carrying 317 horse-power, but as shown by the value W^c they would wear out in about one-half the specified time if such a load were applied. If the example had read "ten hours per day"

instead of "continuous," all other conditions remaining the same, we would have: $A = 7500 \times 0.40 \times 0.80 \times 1.50 = 6642$ pounds,

$$\frac{6642 \times 2200}{33,000} = 440 \text{ horse-power.}$$

For this load, however, the teeth would be liable to fracture.

RELATIVE IMPORTANCE OF STRENGTH AND HARDNESS

It would appear that the actual *strength* of the tooth is to be a secondary consideration, figured only as a preventive against fracture. The real points to be considered are: First, the proper proportion of the gear diameters; second, the hardness of the material and the best combination of hardness for wear. George B. Grant was very near the truth in saying, "It does not proportionately increase the strength of a tooth to double its pitch." With herringbone gears it would seem that the strength need hardly be considered, as it is practically impossible to break out a single tooth of sufficient angle, an entire section must be removed. They must be *worn* out.

Aside from the hardness, the value of the material to avoid crystallization must be considered, as gears in which the teeth are apparently extremely tough will often become brittle and drop off after a comparatively short service from this cause. For this reason high-carbon steel should be avoided and hardness obtained by case-hardening, or by the addition of manganese, nickel, chromium, vanadium, or some other hardening ingredient.

It is hardly necessary to add that proper lubrication adds greatly to the efficiency of a gear drive, except, of course, where they are exposed to brick or cement dust, where it is often advisable to run them dry, as the oil will hold particles of grit and cause the teeth to cut. A jet of air applied at the point of contact is also found beneficial in such cases, as it will remove particles of grit from the teeth before they enter contact.

The constructions of housings upon which the gears are mounted is of the utmost importance, as the absence of vibration is essential to high efficiency. Where the housings are insecure it is often found that a rawhide pinion will sometimes give better service than one made of iron, as the rawhide will give and absorb vibration that would destroy the harder material.

Another point that naturally suggests itself is the proper value of a suitable lubricant. It is evident that the efficiency of 95 per cent. running dry and 98 per cent. when immersed in oil does represent the total saving. Lubrication means in many cases the difference between a successful drive and a failure which is not apparent from any superficial tests made for power efficiency only.

The question is often asked, "What is the life of a gear?" It is evident that

continual service will wear out *any* material no matter how hard. We will say for example that a load of 900 pounds per unit of area will wear out a pair of cast-iron gears in one year's continual service. What pressure must be applied, therefore, to wear out these gears in six months or to allow them to run for five years? It would appear that this could be taken care of nicely by factor A, and as in Table 15, provided, of course, that other conditions are correct and that we have the proper analysis of our materials. We should determine not only what grades of material will wear best together, but also how long they will wear.

A gear may be said to be worn out when the teeth have been reduced to not less than one-half their original thickness—this will subject them to the limit of ultimate stress of the material if allowed their full load according to the Lewis formula. If this is exceeded the teeth would be liable to fracture. See Fig. 61.

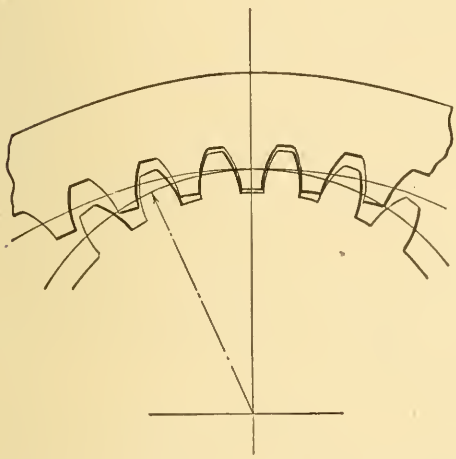


FIG. 60. INTERNAL GEAR MESHING WITH SPUR.

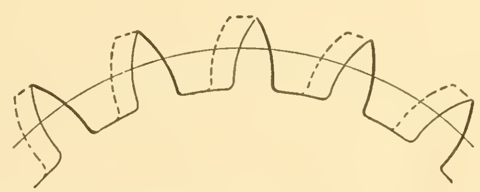


FIG. 61. LIMITING WEAR OF GEAR TEETH.

It is evident that during the latter part of the life of a gear the teeth will wear more rapidly, as the backlash will allow the teeth to hammer with variations in the load or in reversing.

It is well known that all materials are subject to what is known as fatigue, that is, a piece of steel that will stand an intermittent strain of 2000 pounds successfully is liable to fail if this load is permanently applied. This should enter into our problem, as it is often required to design a pair of gears to transmit a certain load continuously, with the guarantee that they will render successful service for, say, two years before renewing. The correct solution of this problem would require a thorough knowledge on all the points brought up in this paper, also a proper determination of their efficiency, unless, of course, the gears were made amply large, or there had been some precedent upon which to base the calculations.

Another point to be considered is what difference (if any) should be made in

the comparative hardness in favor of the pinion, so that it may have wear proportional to the ratio of the pair.

IMPORTANCE OF PROPER DESIGN

Aside from all these conditions, to obtain anything like accurate results the gears must be mounted in such a manner as to obviate practically all vibration. The teeth must be accurately formed and spaced to insure that the impulse received by the driven gear is uniform and without variation, and the thickness of the teeth must be such as to avoid practically all backlash, except just enough to secure free operation, as teeth are often broken by crowding on close centers. The gears must also be properly designed to withstand any strains to which the teeth are subjected. The proper distribution of the material will add greatly to the wear and strength. It is well to have the gear as rigid as possible. It should be remembered that accuracy in cutting the teeth will be of little avail if they are not correctly mounted upon their shafts. If the shaft is a little under size, the key will cause it to run out of true. This will also apply when the shaft is a neat fit and the taper key is driven too tightly.

In the absence of practically all experimental data I have not attempted to put forward anything more than a general outline of the situation and to bring up essential points for consideration. It is trusted that they will be received as such. In view of the growing importance of gears for the transmission of power the points referred to are certainly worthy of attention.

SPEED OF SPUR GEARS

When the question is asked, "What is the greatest circumferential speed at which a spur gear may operate?" we are told, "When 1200 feet per minute is exceeded either rawhide or herringbone gears must be used, and even when properly mounted 3000 feet is the limit of speed for any gear."

To illustrate the fallacy of this statement consider a pair of spur pinions, each of 12 teeth, 6 pitch, 2-inch pitch diameter, running at a speed of 1200 feet per minute. A moment's consideration will show that the noise generated by such a drive would be excessive, as this would mean 2280 revolutions per minute. These gears would make their presence known at a speed of 400 feet per minute, which represents 761 revolutions per minute.

At first thought it appears that the number of teeth in contact would represent the comparative speed value, but, as it is sometimes possible to obtain as many teeth in contact in a small pair of gears as in a large pair, owing to the difference in pitch, the proportion of gear diameter must also be taken into account. As the proportionate value of the gear diameter is represented by the

number of teeth in contact, the pitch remaining constant; this value may be gaged according to the circular pitch. Fig. 62 will illustrate this.

The relative speed value is found in the product of the number of teeth in contact and the circular pitch, or $n\ p'$.

n = number of teeth in contact,
p' = circular pitch.

The next step is the determination of a factor from actual practice, ρ, which multiplied by the product of the pitch, p', and the number of teeth in contact, n, will give the safe speed. The formula now becomes:

$$Safe\ speed = p'\ n\ \rho. \qquad (26)$$

The factor ρ may be made to cover almost any type of gear or condition of service. If the safe speed is known for certain machine construction, the value of the factor ρ may be determined as below for similar drives.

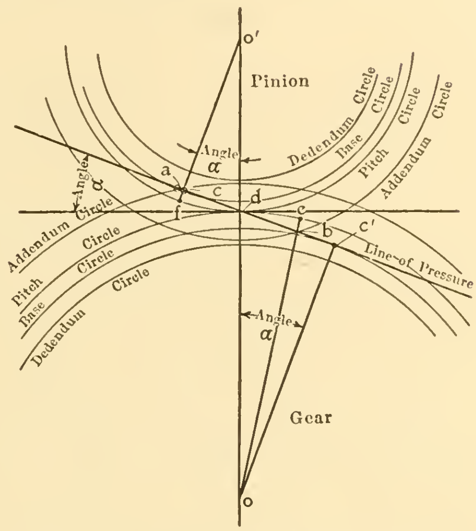


FIG. 62. DIAGRAM ILLUSTRATING METHOD OF DETERMINING NUMBER OF TEETH IN CONTACT.

$$Value\ of\ \rho = \frac{Safe\ speed}{p'\ n}. \qquad (27)$$

For general use the values of ρ have been estimated as in Table 16.

STYLE OF GEAR	COMMERCIAL CUT GEARS	GENERATED TEETH
Spur gear, pattern molded	0 to 300	....
Spur gear, machine molded	110 to 450	....
Spur gear, commercial cut	600	....
Spur gears, cut with exact cutters, accurately spaced	700	800
Spur gears, cut stepped teeth	820	....
Spur gears, fiber	900	1000
Spur gears, rawhide	1000	....
Herringbone gears, angle of spiral, 10 degrees	700	1100
Herringbone gears, angle of spiral, 20 degrees	800	1400
Herringbone gears, angle of spiral, 30 degrees	1100	1900
Herringbone gears, angle of spiral, 45 degrees	2400	4000

TABLE 16—VALUES OF FACTOR ρ

NUMBER OF TEETH IN CONTACT

As the necessary formula to determine the number of teeth in contact is considered too cumbersome for practical use, a graphical solution is given, as follows:

Referring to Fig. 62, the length of contact is measured between the intersection points of the line of pressure and the addendum circles of the gear and pinion, or between a and b. In case this intersection falls outside the intersection of a line drawn at right angles with the pressure line to the center of the gear as at the point c, the contact is measured from the point c, as this indicates that the distance $a\ c$ must be deducted for interference. The gear tooth is rounded from this point out, or the flank of the pinion tooth is undercut to accomplish the same purpose.

If, on the other hand, this point falls outside the intersection of the pressure line and the addendum circle as at c' this extra length must be deducted, as there is no contact until the point of the mating tooth has passed the addendum circle. In order that the number of teeth in contact can be stepped off, the lines $o'\ c$ and $o\ b$ are extended to the pitch or to points e and f. The length of contact is then measured from e to d on the gear, and from d to f on the pinion along this pitch circumference. This distance divided by the circular pitch equals the number of teeth in contact.

LIMITING SPEEDS

Estimate the maximum speed *to avoid danger of fracture* for the best type of gear and condition of service as 500 feet per minute per 1000 pounds safe working stress of the material of which the gear is constructed. Thus the maximum speeds would be as follows:

4,000 feet per minute for cast iron of 8,000 pounds per square inch.
8,000 feet per minute for cast steel of 16,000 pounds per square inch.
10,000 feet per minute for machinery steel of 25,000 pounds per square inch.

To attain anything like these speeds, however, the gears must be exceptionally accurate and well balanced, also the housings must be sufficiently heavy to obviate practically all vibration. The restriction placed by the above limits, however, will avoid the possibility of allowing the higher speeds. According to a series of experiments made by Prof. Charles H. Benjamin, American Machinist, December 28, 1901, page 1421, the bursting speed of a solid, cast-iron gear blank is found to be 24,000 feet per minute. The centrifugal tension at this speed is 15,600 pounds per square inch. The same wheel, split between the arms, burst at an average speed of 11,500 feet per minute.

CONSTRUCTION OF THE GEAR

Approximate multipliers should be used for various designs in reference to limiting speeds.

Properly proportioned solid gear	1.00
Gear split through the arms	0.75
Gear split between the arms	0.50
Link flywheel construction	0.60

These values correspond closely to those given by the Fidelity and Casualty Company, which sets the limit of speed for a solid flywheel (cast iron) at 6000 feet per minute. No value is given for the wheel split through the arms.

The principal cause of failure (as pointed out by Professor Benjamin in the article above referred to) in gears split between the arms is the necessity of placing bolting lugs on the inside of the rim. These lugs naturally tend to increase the stress at their point of location and fracture the rim in this locality at correspondingly low speeds.

LOCATION OF THE GEAR

The influence of the location on the speed cannot be well determined. In general, however, for gears mounted upon insecure foundations or secured to the wall or ceiling of light buildings more or less allowance must be made. One case in mind is a 10-horse-power motor direct-connected to a line shaft making 150 revolutions per minute. The motor is securely bolted to the joists: the gears have 106 and 22 teeth, respectively, 4 diametral pitch, 5-inch face, cast iron and rawhide; the speed is 1050 feet per minute. These gears were exceptionally noisy and were replaced by herringbone gears of the same normal pitch, with the angle of spirality 20 degrees. The gear in this instance was cast iron and the pinion machinery steel. These gears were no better than the first pair and were replaced by gears with a spiral angle of 45 degrees. These proved to be but little better, and as a last resort a pair was installed with an angle of 76 degrees. These gears were fairly quiet when operating under full load, but very distressing when running light. This condition is sometimes found in spur gears with excessive backlash. It was noticed that the teeth gave evidence of rapid wear due to the reduced face contact. Rubber cushions placed under the motor and shaft hangers and filling the space between the hub and rim of gear with wood made no perceptible difference in the noise of this drive, which was finally abolished.

In this connection I have knowledge of a pair of herringbone gears, cast iron and machine steel, 80 and 24 teeth, 6 normal pitch, 20- and 6-inch pitch

diameters, angle of spiral 45 degrees that are practically noiseless at a speed of 3150 feet per minute. At this speed, however, the load is comparatively light. These gears are driven by a 10-horse-power motor and are entirely satisfactory.

RELATION OF PRESSURE TO SPEED

The question naturally arises, "In what way does the tooth pressure influence the speed of gears?" From a power standpoint this is ordinarily taken care of by Mr. Barth's expression $\frac{600}{600 + V}$. But this does not fix the speed at which noise may be avoided, which is the subject of this paper.

It has been assumed that the tooth pressure allowable at the speeds given by the formula (26) are within the limits placed by the foregoing formula (24) for *wear.*

As previously pointed out "the *strength* of teeth in herringbone gears need hardly be considered, as it is impossible to break out a single tooth provided the angle be great enough to engage another tooth before the first lets go." However, the actual contact, which depends upon the angle, should be used instead of the actual face when determining the load for wear, as increasing the angle decreases the noise, but increases the wear.

ILLUSTRATIVE EXAMPLE

Required the safe speed of a pair of solid steel spur gears of the following dimensions:

Gear, 80 teeth, 2-inch pitch, 6-inch face, 50.93-inch pitch diameter, pinion 48 teeth, 2-inch pitch, 6-inch face, 30.558-inch pitch diameter.

Cutters for these gears to be made for the exact number of teeth.

The computations for speed involve the number of teeth in contact = 2.5, value of ρ according to Table 16 = 800, safe speed = $p' \, n \, \rho = 2 \times 2\frac{1}{2} \times 800 = 4000$ feet per minute. According to factors given for limiting speeds, these gears would be amply safe to resist fracture, but would be at the extreme limit for cast iron of a safe stress of 8000 pounds per square inch. The computations for wear involve, assuming that the pinion is made from steel of an elastic limit of 50,000 pounds per square inch, the limiting tooth pressure at this speed would be equal to the value W^c as follows:

$$W^c = C f A \frac{600}{600 + V} = 2.4 \times 6 \times 3000 \times 0.13 = 5600 \textit{ pounds},$$

in which

$$C = 2.4,$$
$$f = 6,$$
$$A = 3000,$$

and

$$\frac{600}{600 + V} = 0.13.$$

The computations for strength involve the use of the Lewis formula, based on carrying the entire load on one tooth,

$$W = S\,p'\,f\,y\,\frac{600}{600 + V} = 25{,}000 \times 2 \times 6 \times 0.111 \times 0.13 = 4600 \textit{ pounds},$$

in which

S = 25,000 pounds per square inch (one-half the elastic limit),
p' = 2 inches,
f = 6 inches,

and

y = 0.111 for 48 teeth.

As the load should not exceed the strength of the tooth, this pressure is the limit to be used. The corresponding horse-power transmitted would equal

$$\frac{W\,V}{33{,}000} = \frac{4600 \times 4000}{33{,}000} = 557 \textit{ horse-power.}$$

These gears if properly mounted should be satisfactory at a speed of 4000 feet per minute, the value W^c indicating that they could be used for 10 hours per day service for two and one-quarter years at a uniform pressure of 4600 pounds. If longer life or service per day is required the load on the teeth must be reduced accordingly.

HIGH SPEED GEARING *

For the transmission of power it frequently becomes necessary to use toothed gearing, subjected to high peripheral speed conjointly with high pressure per unit of tooth contact, and the object of these remarks is to record what has been successfully done in recent years, as much higher speeds are now successfully attained than formerly. Considered in a static sense, the gear tooth satisfies the condition of stress if it is proportioned to endure forces acting transversely on it, and the pressure per unit of contract is not of such intensity as permanently to deform the curved bearing surface of the teeth. When in motion, the curved surfaces slide upon each other as they enter and leave contact, and when this sliding action is accompanied with high pressure, the limit of endurance is soon reached, and in the case of the inferior materials this occurs at comparatively low speeds and pressures. In addition to this, more or less impact usually occurs, especially when the resistance is of a fluctuating character or the loads are suddenly applied. The effects of this hammering action

* A paper read before the Engineers' Club of Philadelphia, by James Christie.

are discernible by a flattening of the curved faces of the teeth, after which the proper engagement of the teeth ceases and the gear is speedily destroyed.

To prevent this, it is desirable to cut the teeth so accurately that no side clearance or "backlash" exists, and this is now usually done on first-class gearing of even the largest dimensions. Owing to the low elastic limit of cast iron and the bronzes we cannot expect these metals to endure so high a pressure as steel, and steel appears to be the most trustworthy material to endure the highest pressures and speeds. This assertion, however, does not apply to all grades of steel. Soft steel surfaces abrade or cut very readily despite all methods of lubrication, and surfaces of this material should never be allowed to engage in sliding contact. Gearing of soft steel is usually destroyed by abrasion at quite moderate speeds. Rolling-mill pinions of steel, containing 0.3 per cent. carbon, have been destroyed in a few months, whereas the same pattern in steel of 0.6 per cent. carbon has done similar work for several years without distress. Of course it is necessary to shape the teeth to a proper curve to insure proper engagement and uniform angular velocity.

Some years ago there was required suitable gearing to connect the engines to a rolling mill in this vicinity. The diameters of the wheels were 37.6 and 56.4 inches respectively. They were intended to revolve at speeds of 150 and 100 revolutions per minute and expected to transmit about 2500 horse-power. The character of the service was such that renewal was a serious matter and long endurance very desirable. A high grade of steel was selected especially in the pinion, in which the greatest wear would occur, and which, owing to the location, was the most difficult to replace. The pinion was forged from fluid compressed steel of the following composition:

	Per cent.
Carbon	0.86
Manganese	0.51
Silicon	0.27
Phosphorus and sulphur, both below	0.03

The spur wheel was an annealed steel casting:

	Per cent.
Carbon	0.47
Manganese	0.66
Phosphorus and sulphur, both	0.05

The tooth dimensions were: Pitch, 4.92 inches; face, 24 inches. See Fig. 63.

These were accurately cut with involute curves generated by a rolling tangent of 16 degrees obliquity. No side clearance was allowed. After

starting the mill, it was found that a higher speed was practicable than was originally contemplated. Higher pressures on the teeth were also applied,

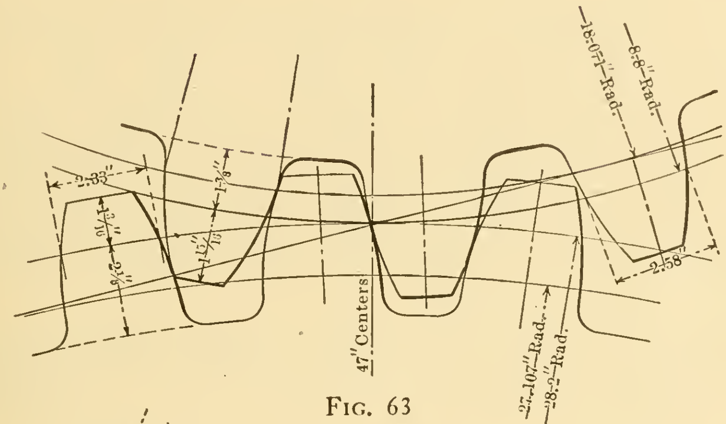


FIG. 63

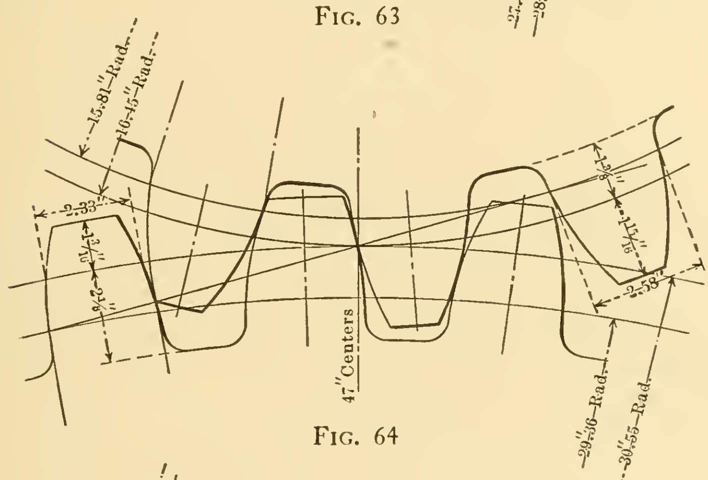


FIG. 64

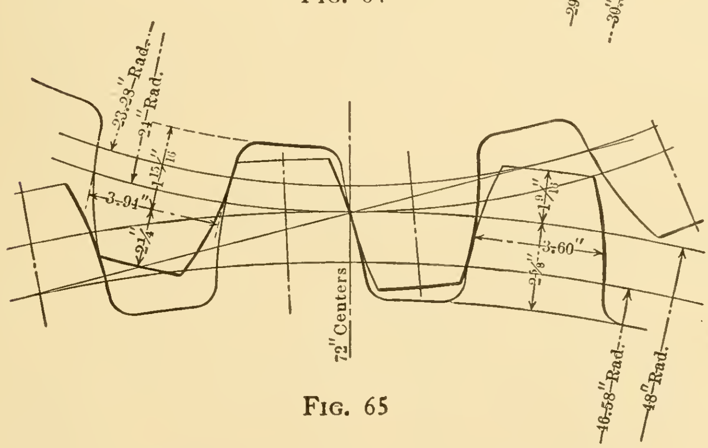


FIG. 65

EXAMPLES OF HIGH SPEED TOOTH GEARING.

so that ultimately about 3300 horse-power was transmitted through the gearing, corresponding to a pressure of nearly 2100 pounds per inch of face. The speed was variable, but occasionally attained a velocity of 260 revolutions per

minute for the pinion, corresponding to a peripheral velocity of 2500 feet per minute. This gearing has been in constant operation for several years and behaves satisfactorily.

The highest recorded speed for gearing that I can recall is that described by Mr. Geyelin in the Club "Proceedings" of June, 1894. The mortise bevels had a peripheral velocity of 3900 feet per minute, but the pressure per inch of face was only about 680 pounds, the diameter and speed being made high to reduce the pressure on the teeth. I understand that the life-time of these bevels is not long. If made of a grade of steel, as previously described, their diameter and speed could be considerably reduced and prolonged endurance would be realized.

About the same time No. 63 was installed a similar application was made to another mill, the gear having a different speed ratio, and the angular velocity being lower. See Fig. 64.

	Pinion, per cent.	Wheel, per cent.
Carbon	0.90	0.60
Manganese	0.64	0.64

A much larger set had been previously employed, transmitting about 2400 horse-power at 750 feet per minute peripheral speed, involving a pressure per inch of face of 3500 pounds. This latter pair were 4 feet and 8 feet respectively, 7½-inch pitch, 30-inch face, cut with involute teeth of 14 degrees obliquity. See Fig. 65.

	Pinion	Gear
Carbon	0.52	0.42
Manganese	0.55	0.73
Silicon	0.107	0.279
Phosphorus	0.022	0.078
Sulphur	0.02	0.05

These gears have all rendered excellent service, and to-day are apparently as good as at the beginning.

As considerable expense is involved in cutting large gears of hard steel, it is sometimes practicable to rough-cut the gear after it is made as soft as possible by slow cooling, a higher degree of hardening being imparted before final finishing by air hardening or rapid cooling from the refining heat. This is not infrequently done in the case of screws and gears of moderate dimensions. In this event it is desirable to have the ratio of manganese low—say, not over 0.5 or 0.6 per cent.—as a high manganese content seems to impart a permanent hardness that is not reduced by slow cooling.

It appears to be practicable to maintain sliding surfaces of steel if one of the

surfaces is hard, even if the other is comparatively soft, but for steel gearing for ordinary purposes I would suggest the use of steel not less than 0.4 carbon. If the speeds and pressures are unusually high, a much harder grade of steel becomes necessary. When a small pinion engages with a large wheel, the former alone can be made of high grade steel approaching to a carbon content of 1 per cent. When extreme speeds and pressures become necessary, the best results will be found by using in both wheels steel having a carbon content approaching 1 per cent., or an equal hardness, obtained by lower carbon and high manganese or other desirable hardening addition. With gearing accurately cut from steel of this character and securely mounted, it is believed that reasonable endurance will be obtained when the product of speed and pressure, divided by pitch, each within certain limits, does not exceed 1,000,000: for example, a speed of 3,000 feet per minute and 1,600 pounds per inch of face, or *vice versa* for gear of 5-inch pitch, assuming, so far as we know, a maximum speed of 5000 feet per minute for gear of any pitch, and permissible pressure to be proportional to the pitch.

This statement that speeds and pressures are reciprocal, or as one is increased the other must be reduced, in a fixed ratio, may not strictly be a rational one, but in a broad and general sense it is correct within the usual limits of practice.

It will be understood that such a generalization as herein stated would apply to pinions having a liberal and not the minimum number of teeth.

In the discussion of Mr. Christie's paper, Mr. E. Graves gave particulars of three duplicate sets of cast-steel bevel wheels. The pinions are the drivers and are 57.39-inch pitch diameter and have 36 teeth, 5-inch pitch, 20-inch face. The wheels are 74.8 inches diameter and have 47 teeth. The teeth are carefully cut to involute lay-out and are 3.43 inches high. The normal speed of the pinion is 360 revolutions per minute, giving 200 revolutions per minute to the wheel and nearly 4000 feet circumferential speed on the pitch line. The horse-power transmitted is 1300. Assuming the entire load to be distributed along the outer end of one tooth, the fiber strain would be 2,100 pounds per square inch at the root of the tooth.

The pinion is mounted on the upper end of a 10-inch shaft, 148 feet long, with a turbine wheel at the lower end. Both shafts extend through the gears and are supported in a massive bridge casting with adjustable bearings. The gears are enclosed in a casing and are lubricated with oil fed under pressure through several jets applied just in front of the teeth as they mesh together.

The gears have been in service for five years, but have not been entirely satisfactory. Their wearing power in the sense of resisting abrasion is satisfactory, but the teeth break. This breakage is confined to the pinion, the

nature of the break being the same in all cases, beginning at the large end, cracking around the root and following along the tooth. The quality of the steel in castings is the ordinary commercial article. The widest variation in analysis observed is, in one instance:

Silicon	0.25
Sulphur	0.036
Phosphorus	0.071
Manganese	0.74
Carbon	0.31

Another:

Silicon	0.27
Sulphur	0.03
Phosphorus	0.032
Manganese	0.80
Carbon	0.23

As is to be expected, the softer metal has resisted breaking the longer. In two sets of these gears the resisting work is of a varying nature with sudden and wide fluctuations; in the third instance the working is more constant. This variation of conditions does not seem to have influenced failure, as the teeth have broken in all the sets.

One of the practical difficulties in operating bevel gears of the nature described is the difficulty of holding them so that they will be in proper contact; longitudinal motion in either shaft throws them out of pitch. The most serious problem, however, is in securing and maintaining shafts so that the extended axis lines of same pass through a common point. The effect of power transmission from pinion to gear is to put these axis lines out of position, moving them in opposite directions and resulting in end contact of teeth and concentrated load instead of evenly distributing the load along the whole length of tooth. In this particular the question of maintaining bevel gears is decidedly more of a problem than that of spur gears. In this latter case small end motions of carrying shafts produce no effect, while the wearing of bearings is only the shifting of pitch line, and, as it occurs slowly, it will, within reasonable limits, adjust itself.

As a matter of further interest, I will mention that in this same room with these gears are three other sets of bevel gear having cut-steel pinions and mortise wheel with cast-iron rims. The diameters and ratios of these—speeds, mountings, and service—are practically the same as those described but the transmission of power is 1,100 instead of 1,300 horse-power. The

pinions have 33 teeth, $5\frac{1}{2}$-inch pitch, with 20-inch face, the teeth being planed down to $2\frac{1}{2}$-inch thickness on pitch-line. The wheels have 43 teeth. These gears have been in service some seven years. None of the pinions has ever given way; the wooden teeth in the wheels, however, last only from six weeks to two months, an extra rim being kept on hand for refilling and replacing.

Mr. Lewis, in continuing the discussion, said that in regard to the pressures carried by gear teeth, Mr. Christie seems to lay down a rule making the product of speed and pressure constant. This would reduce the load in proportion to the speed, and it seems to me an open question whether that should be adhered to or not. I do not think it has been demonstrated how the pressure of the teeth should vary with the speed. Some experiments, I think, should be made which would indicate that more clearly than has heretofore been done. It is interesting to note his remarks regarding the influence of the hardness of the metal upon the pressures carried, and instead of reckoning the pressure by the inch as so much per inch of face, it seems to me the pitch should also be included, because the face of a gear tooth is very much like a roller, and the pressure carried by a roller varies with its diameter as well as with the face. Some authorities seem to think that it should vary with the square root of the diameter, others directly with the diameter, and I am inclined to the latter opinion. If gear teeth are proportioned for strength, they are also proportioned for wearing pressure and surface to carry the load.

Mr. W. Trinks said: I wish to call attention to an article on high speed gearing in the November and December numbers of the "Zeitschrift des Vereins deutscher Ingenieure," 1899, by the chief engineer of the General Electric Company, at Berlin, Germany. The experiments show that there is no rule for the relation between pressure and speed, it depends upon accuracy; the load on the teeth may be the higher the more accurately the gears are made. A remarkable method of manufacturing gears was the outcome of the experiment. The curves are laid out on paper three or four times the size of the real tooth, reduced to proper size by photography, transferred on sheet steel, and etched in. Thus the highest degree of accuracy is obtained.

It was found that neither cycloidal nor involute curves gave the best results. Another curve was developed with a view to reducing the sliding motion between the teeth. The article contains very interesting diagrams on this point. By dividing the length of two working teeth into an equal number of parts, the amount of sliding action can be determined and the fact shown that it is reduced to a minimum by these methods.

Another thing shown by the paper is never to place a flywheel close to a gear. If possible, have a good length of shaft between. Slight inaccuracies

in the pitch of the wheel require acceleration or retardation of the mass, and in order to do this force is needed. This force causes a hammering on the teeth which may break them—in other words, plenty of elastic material should be between the inertia masses and the gears. I feel pretty sure that all engineers will be much interested in the article; it is a valuable treatise on highspeed gearing.

Mr. Christie added: The bevel gears described by Mr. Graves are very interesting and useful as a record. It is much more difficult to obtain satisfactory results with bevels than with plain spurs, as any deviation from correct alinement is fatal to correct tooth action in the former. In this instance, while speed is very high, the pressure on the teeth is comparatively low—about 750 pounds mean pressure per inch of face. Thus the products of speed and pressure in relation to the pitch are considerably below the quantity assumed as a safe maximum.

Regarding the quality of the material, the manganese is too high. While steel of this composition would be moderately hard and wear fairly well, it would be somewhat brittle. It is not surprising to learn that some teeth gave way by fracture. If the relative proportions of carbon and manganese in the steel were reversed, it would be a much better material for the purpose.

A SPUR GEAR ANGLEMETER *

In the design of spur gears it is desired to give to the teeth such a form that as the gears mesh with each other, the relative motion of the two will be the same as that of two cylinders whose diameters have the same ratio one to the other as have the diameters of the two pitch circles. By the aid of kinematics gear teeth can be so designed as to give exactly this relative motion between the gears. However, in the manufacture of gears, factors enter which make the form of the teeth of the gears as they come from the shop somewhat different from that developed by kinematics. This variation is quite marked in the case of rough-cast gears. Here several errors enter. Because of the difficulty and time required in developing a tooth outline according to kinematics, arcs of circles which approximate the correct outline are used in laying out the gears in the drafting room. From these slightly inaccurate drawings the patternmaker makes the patterns, which are apt to vary slightly in form and the spacing of the teeth from the drawing furnished by the draftsman. These inaccurate patterns then go to the foundry, and from them the molds are made. In making the mold, in order to draw the pattern it is rapped loose, so that the mold is slightly larger than the pattern and still more inaccurate

* W. M. Wilson, American Machinist, April 13, 1905.

in outline. The casting is poured, and in cooling is warped out of shape because of the cooling stresses leaving a gear with a final error in the form of its teeth made up of several smaller errors, as just enumerated.

A realization of the presence of these errors suggested that a knowledge of the final inaccuracy in the forms of the teeth of rough-cast gears would be of interest and perhaps not without value. With this idea in mind, an anglemeter for determining the variation in the angular velocity ratio of gears was designed and built.

The anglemeter consists mainly of a responsive frame carrying a drum on which is wrapped a card. If these gears operate without vibration the pointer would draw a straight line around the card. Figs. 66 and 67 will be practically self-explanatory. The pointer was found to multiply the actual variations 9.4 times.

TEST OF A PAIR OF CAST GEARS

The instrument was connected to a pair of spur gears, No. 12020, each having a circular pitch of 1½ inches and 20 teeth. Each gear was mounted on a $2\frac{15}{16}$-inch shaft. As the two gears and the two shafts were the same size, no reducing motion was needed. The guides of the instrument were not long enough to allow a complete revolution of the gears, so the teeth were numbered from 1 to 20 and one card drawn for teeth 1 to 11 and another for teeth from 11 to 1. The points on the curves corresponding to the time when these teeth came into contact were marked and then the cards were cut to these marks and pasted end to end as shown in Fig. 66.

The details of the design of this pair of gears were not known, but judging from the shape of the teeth they were modifications of involutes and the outlines of the teeth were probably laid out according to some empirical method in general use in the manufacture of rough-cast gears. Before the test the gears had been run long enough for the bearing surface to become smooth.

Cards were taken when the gears were adjusted at different distances apart. If the gears had been true involutes the velocity ratio should be constant and the same for the different distances between the centers. Whether the velocity ratio were constant or not is readily seen from the curves in Fig. 66. Due to the uncertainty of the shrinkage of castings, the diameter of the gears was not exactly equal to the computed diameter, but a trifle greater. Judging from the clearance at the ends of the teeth the proper distance between the centers of the gears was 9⅝ inches. Card *A*, Fig. 66, gives a curve corresponding to this distance between centers. Two curves were drawn before the card was removed and the ability of the instrument to duplicate a curve is considered as evidence of its accuracy. It is seen from the curve that the

velocity ratio instead of being constant is quite erratic in its variations. While in general the variations in the curve are not related to each other in any way, yet for a portion of the card at least the waves in the curve correspond roughly with the teeth on the gears. This is especially true of the portion of the curve corresponding to the teeth from 11 to 20. The curves on Card *B* were drawn when the distance between centers was $9\frac{13}{16}$ inches. In this case there is a more marked relation between the waves in the curves and the teeth on the

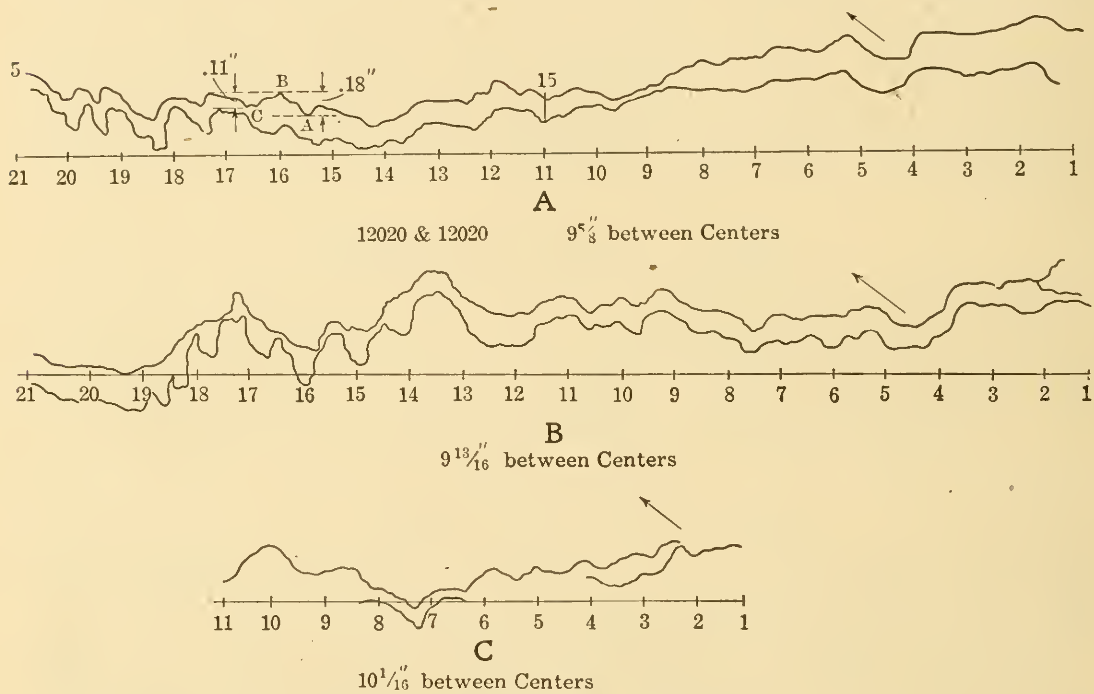


FIG. 66. CURVES FROM CAST GEARS.

gear than in the former case. For Card *C* the distance between centers was $10\frac{1}{16}$ inches, and the regularity of the waves is still more marked. In this case the card was taken for half a revolution only.

For all the cards the arrows above the different curves indicate the general direction of the tangent to the curves which give the angular acceleration of the driven as compared with the driver.

From the curves shown the following conclusions have been drawn relative to the forms of the teeth:

(1) The fact that the relation between waves in the curves and the teeth of the gears is increased indicates that all the teeth are subject to a common error due to the empirical method in which the outlines of the teeth were developed, also that this error causes a greater variation in the velocity ratio when the distance between the centers of the gears is increased. (This is in

accordance with the empirical method presented in Kent's hand book and attributed to Molsworth. By using the method referred to, the resulting tooth outline would be wider below and narrower above the pitch circle than true involute teeth. When the distance between centers is normal these errors annul each other almost completely, but as the distance is increased the result or the errors is more apparent.)

(2) Irregularities in the curves indicate errors peculiar to the individual teeth, which evidently are involved in the progress of patternmaking and molding.

(3) A shifting of the general vertical position of the curves on the cards, as shown at teeth 4 and 14, indicates errors in the spacing of the teeth.

An effort has been made to analyze the curve in Card *A*. The portion of this curve *A B C* has been chosen as being a wave whose relation to an individual tooth is the most evident. The abscissa *A C* represents the angular space occupied by one tooth on the gear, and *A B* and *B C* each represent one-half of that angular space. The ordinate between *A* and *B* measures 0.18 inch, and that between *B* and *C* measures 0.11 inch. Dividing by 9.4, the constant for the instrument, it is found that the driven gear gains on the driver by an angle whose arc measured on the circumference of the shaft is 0.18 divided by 9.4 equals 0.019 inch, while the gear turns through an angle equal to $\frac{360}{20} \times \frac{1}{2} = 9$ degrees.

An angle whose tangent is

$$\frac{.019}{1.46885 \text{ (radius of shaft)}} \text{ is } 45 \text{ minutes.}$$

Computing in the same manner the angle corresponding to the ordinate *B C* is found to be 30 minutes. That is, while the tooth No. 16 is in action the driven gains relative to the driver by an angle of 45 minutes while the latter is turning through an angle of 9 degrees, and then loses by an amount of 30 minutes in the same space. An error of 0.019 inch measured on the circumference of the shaft corresponds to 0.06 inch measured on the pitch circle.

TEST OF SPECIAL GEARS

The second pair of gears to which the instrument was attached consisted of a No. 12016 pinion having 1½ inches circular pitch and 16 teeth and a No. 12050 gear having 50 teeth. The gears were designed to mesh properly when the distance between centers varies by an amount equal to ½ inch. The shrinkage of the casting was not as much as was expected, so that the normal distance between centers is 0.20 inch more than the sum of the com-

puted radii. If the gears were true involutes the velocity ratio should be constant when the distance between centers of the gears varies from 15.7 to 16.2 inches.

The outlines of the teeth of the gears are involutes having an angle of obliquity of 23½ degrees when the distance between centers of the gears is normal. The mold for No. 12016 was made from a pattern, while for No. 12050 it was made from a tooth-block as used in a Walker gear-molding machine. The pattern and tooth-block were both made from drawings laid out in the drafting room. The tooth outlines for these drawings were developed according to kinematics, so that for the exception of any small error which the patternmaker might make, the teeth on the pattern and tooth-blocks are correct involutes.

How near the castings approached to involute gears is shown by the curves in Fig. 67. As the two gears were not of the same size, it was necessary in taking the cards to use some means of making the average velocity of the two carriages of the instrument the same. To do this the hub of the gear was turned to a smooth surface and the end of the shaft of the pinion was turned down until its diameter bore the same ratio to the diameter of the hub of the gear that the diameter of the pitch circle of the pinion bore to the diameter of the pitch circle of the gear. This gave a rigid and accurate reducing motion. The method of taking the cards was the same as for the other pair of gears except that a curve for a complete revolution of the pinion was obtained upon a single card. The distance between centers of the gears for the different cards is as indicated in Fig. 67. Cards *D*, *E*, and *F* cover the range of variation in the distance between centers for which the gears were designed. From these curves it is seen that there are no waves on the curves corresponding to the individual teeth, the curves for the most part are smooth, and are practically the same for the different distances between centers of the gears. For Card *G* the distance between centers was made 16.45 inches, or .75 inch more than the least distance between centers at which the gears were supposed to run. The remarks made in regard to curves *D*, *E*, and *F* apply equally well to this curve also. For cards *H* and *I* the distance between centers was so great that the point of contact of the teeth did not follow the line of obliquity throughout the angle of contact but lay outside of this line for the latter portion of the period of action of the teeth.

This is clearly brought out in the forms of the curves in which waves corresponding to individual teeth are quite evident.

It will be noticed that for all of the curves for the second pair of gears there is an extended wave near the middle of the card and also near each end, so that if several cards were placed end to end so as to form a continuous curve

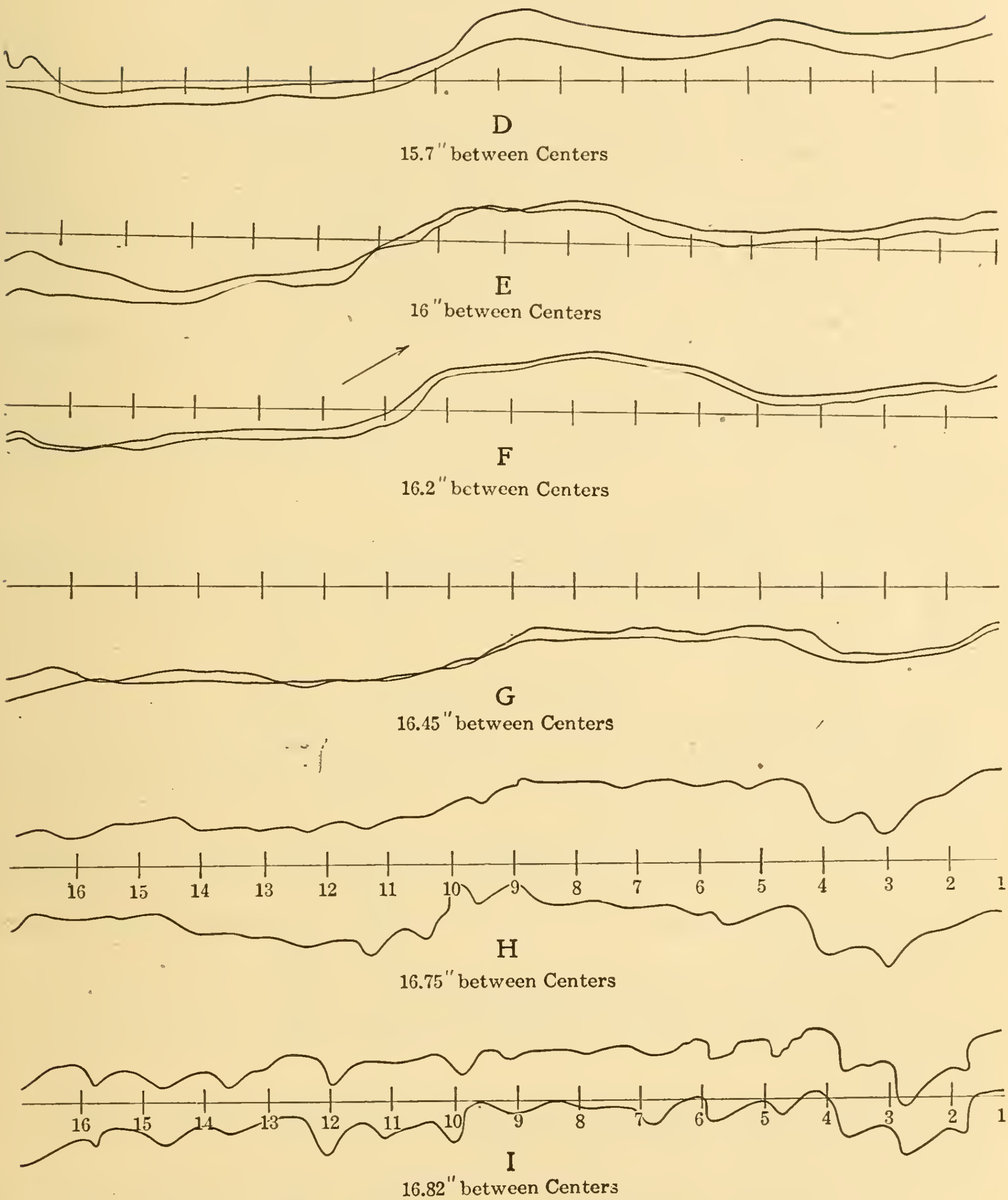


FIG. 67. CURVES FROM SPECIAL GEARS.

there would be two complete waves on this curve per revolution of the pinion. This indicates that the pinion instead of being exactly round is oval in form. This deformation can be traced to the molder in the foundry who, in order to draw the pattern, raps it loose from the sand and unless great care is taken increases the diameter more in one direction than in the other. The wave in the curve is traced to the pinion instead of to the gear because the spacing of the teeth for the gear was done by means of a molding machine and any errors which might occur would be peculiar to the individual teeth instead of being common to a group of teeth and gradually increasing and decreasing from zero up to a maximum and then back again to zero, as indicated on the cards.

From the curves in Fig. 67 the following conclusions are drawn relative to this pair of gears:

1. The outlines of the individual teeth are true involutes.

2. The spacing of the individual teeth on both the gear and the pinion is accurate.

3. The pinion is slightly oval in form.

4. For the gears used the angular velocity ratio is constant through a range in the variations in the distance between centers equal to 0.75 inches.

GENERAL CONCLUSIONS

If one would be allowed to draw conclusions from the tests of two pairs of gears only, the following statements might be made in regard to the errors in the teeth of rough-cast gears:

1. Of all the errors the greatest is due to laying out the teeth in the drafting room.

2. Given an accurate drawing the patternmaker can make a pattern very accurate in form.

3. In using the patterns the molder is apt to make the mold slightly oval in form, but in using a molding machine the error induced in the foundry is very small.

4. The surfaces of the teeth of cast gears are so smooth as not to affect the angular velocity ratio.

The difference in the appearance of the curves for the two pairs of gears is very much in accordance with the manner in which they meshed, the second pair running with very much less noise than the first pair. At a speed of 250 revolutions per minute the first pair almost threatened to shake the testing machine to pieces, while at the same speed the second pair ran with but little vibration.

Gears Lubricated						Gears Not Lubricated			
Tangential force at pitch circle, 194 lbs.		Tangential force at pitch circle, 330 lbs.		Tangential force at pitch circle, 585 lbs.		Tangential force at pitch circle, 444 lbs.		Tangential force at pitch circle, 148 lbs.	
R. P. M.	Efficiency, %	R. P. M.	Efficiency, %	R. P. M.	Efficiency, %	R. P. M.	Efficiency, %	R. P. M.	Efficiency, %
80	92.00	76	97.33	81	94.97	59	88.58	105	91.40
84	94.05	89	96.66	92	96.10	56	91.40	106	91.50
119	93.55	123	97.75	113	95.28	62	89.17	129	91.80
112	93.31	115	95.99	114	95.45	88	91.18	129	93.50
162	93.60	150	96.27	132	96.28	102	91.03	165	91.00
149	92.57	155	96.57	140	95.29	126	90.21	167	90.80
187	91.56	182	95.81	149	94.81	126	89.29	233	93.30
172	91.08	181	95.05	177	94.57	159	90.60	241	93.50
177	92.04	209	96.16	173	93.87	159	89.94		Av. 92.10
215	91.36	192	95.50		Av. 95.20	202	88.72		
215	91.19	157	95.18				Av. 90.00		
	91.00	92	97.03						
			Av. 96.30						
Efficiency of Gears alone, 92.60%		Efficiency of Gears alone, 98.00%		Efficiency of Gears alone, 96.80%		Efficiency of Gears alone, 91.50%		Efficiency of Gears alone, 94.80%	

Table 17—Gear No. 12020 Driving 12020—Standard Involutes

It might be said that the patterns for the second pair of gears were completed before there had been any thought of subjecting the gears to a test, which fact eliminates the possibility of unusual care having been taken in making the patterns.

GEAR EFFICIENCY

No thorough investigation has yet been made of the efficiency of gears, and very few data of any sort has been published, therefore little can be said. One thing is tolerably well known: the efficiency of a gear drive varies with its ratio—that is, a reduction of 2 to 1 will be more efficient than one of 10 to 1, other features governing the efficiency being the accuracy with which the teeth are formed, and spaced, the arc and obliquity of action, and the condition of the engaging surfaces. It is generally conceded that the length of face does not affect the efficiency.

22½ degree involutes. 16″ between centers. Tangential force at pitch line, 361 lbs.		24¼ degree involutes. 16.2″ between centers. Tangential force at pitch line, 355 lbs.		19½ degree involutes. 15.7″ between centers. Tangential force at pitch line, 353 lbs.	
R. P. M.	Efficiency, %	R. P. M.	Efficiency, %	R. P. M.	Efficiency, %
88	92.40	116	91.00	110	90.00
123	90.00	68	96.90	69	94.40
140	89.90	90	91.50	127	86.80
156	91.30	140	89.90	147	88.90
172	91.30	163	90.10	162	88.80
188	92.30	177	90.00	196	88.90
216	92.30	197	99.90	218	89.20
229	92.40	208	90.00		Av. 89.60
	Av. 91.50	214	90.30		
			Av. 91.20		
Efficiency of Gears alone, 92.30%		Efficiency of Gears alone, 92.10%		Efficiency of Gears alone, 90.50%	

Table 18—Gear No. 12016 Driving 12050—Lubricated

This loss comes mainly through the sliding action of the teeth when entering and leaving contact; it would seem, therefore, that the arc of action should be as short as possible, carrying the load of that portion of the tooth where it can be best borne, also, as there is less friction in the arc of recess than in the arc of approach, it will increase the efficiency to lengthen the addendum of the driving gear. The contact, however, should be for an equal distance above and below the pitch line to avoid extreme sliding friction. If the entire action was rolling contact, there would be little loss.

Tables 17 and 18 were originally published in American Machinist, by W. M. Wilson. They give the result of experiments with the same case tooth

spur gears mentioned on pages 99 and 101. The gears in Table 18 were made with extra long teeth, the various angles of obliquity as given were obtained by adjusting the gear centers.

The result of these tests is summed up as follows:

1. The efficiency of rough gray iron spur gears is independent of the speed of the gears within the range covered by this report.

2. There is no indication from the limited number of tests that the amount of power transmitted affects the efficiency to any great extent.

3. The use of a heavy grease on the teeth increases the efficiency slightly (average from tests 1.7 per cent.).

EFFICIENCY OF LARGE GEARS

Many of us know things that are not so, and with some of us a part of this useless knowledge may be in regard to the efficiency of large gears.

We believe that we are well within the bounds of truth in saying that the majority believe that large gears are very inefficient.

This misinformation or lack of information is easily explained. The data that are available in regard to the efficiency of gears of any size are few at best and apply to pinions and small gears. Large gears have not been extensively tested, for there are but few technical schools and factories equipped with appliances for gear testing and capable of absorbing several thousand or even several hundred horse-power. Again we do not always distinguish between a large gear with cast teeth and a similar gear with cut teeth.

In the early days of factory engineering, gear drives were common for transmitting power from shaft to shaft throughout the plant. As time went on these drives were replaced—in some cases by belting, in other cases by a change from mechanical transmission to electrical distribution of power. It has been very easy to assume that the reason for discarding the gears was because of their inefficiency. Enthusiastic advocates of electric drive have time and again referred to the substitution of motors for mill gearing with elation and have either stated or implied that the change brought about a great saving of power.

In further support of the common belief that gearing in large sizes is inefficient we quote the following from the presidential address of Mr. Denny to the Institution of Marine Engineers, in England, on October 5, 1908: "It has frequently been suggested that if some inspired engineer could evolve a system of gearing that would be lasting and reliable, not too noisy, and would not absorb in friction more than, say, 10 per cent. of the power, turbine engines would be capable of application to any speed of vessel and to any size of propeller." Here Mr. Denny gives expression to an oft-repeated suggestion,

that if large gearing could be made having an efficiency of 90 per cent., a big step forward would be made.

With direct bearing on this belief we have the record of performance of the largest gears in point of horse-power ever made and tested—the Melville-Macalpine reduction gear, through which 6000 horse-power has been transmitted.

To show the measure of the performance we quote a paragraph from an article by George Westinghouse in the January number of *The Electric Journal:* "Considering the important bearing of the question of efficiency on the ultimate success or failure of the gear, it is peculiarly gratifying to have found by repeated trial and careful measurement that the transmission loss hoped for by Mr. Denny has been divided by seven. To be exact, the efficiency surpasses the more than satisfactory figure of 98.5 per cent."

The hope was for an efficiency of 90 per cent., the fulfilment was an efficiency of 98.5 per cent.

Large bevel-gear drives have perhaps been especially condemned; it is, therefore, of interest to read the following from a paper presented by Prof. C. M. Allen to the American Society of Mechanical Engineers on the testing of water wheels: "The total horse-power delivered to the generator was approximately 700. The driving gear was of the ordinary wood-mortise type, outside diameter 6 feet 5 inches approximately, with 68 teeth 14 inches wide, meshing with a cast-iron pinion which had 48 teeth with planed-tooth outline. At full load the loss of the gear was 3.5 per cent. and 3.4 per cent. for two separate units, or the efficiency of the horizontal-shaft vertical-wheel gear drive was about 96.5 per cent. The gears were well lubricated with a thick grease.

"About nine months later it was necessary to test one of these same units in exactly the same manner. The loss in gears this time was a trifle less, the test giving 3.1 per cent.

"All of the information obtained concerning the loss due to bevel-gear drives leads the writer to conclude that if gears are properly designed, set up, and operated, and are not overloaded intermittently or continuously or left to care for themselves, they should show an efficiency of from 95 to 97 per cent."

Are not these bevel-gear drives efficient compared with the majority of mechanical devices?

As a matter of fact, do not the above figures agree with common sense? The quantity of heat generated by the absorption of one horse-power for an hour is 2545 British thermal units. If we try to give an expression to the statement that large gears are inefficient by assuming a loss of 10 per cent., in the case cited by Professor Allen, 70 horse-power would be dissipated in heat. A multiplication will show what this means in British thermal units

per hour, and the presumption is strong that the gears would heat, cut, and wear under such a condition.

The same reasoning will probably apply to many large-gear drives concerning which there has been speculation in regard to efficiency. Had they been extremely inefficient, they would not have operated satisfactorily for a long period of time.

These figures show that large-gear drives are not necessarily inefficient, but, on the contrary, may be decidedly the reverse.

SECTION IV

Gear Proportions and Details of Design

These formulas should not be used indiscriminately, as one design will not meet all conditions. No attempt has been made to proportion arms or rim to the power to be transmitted, as all proportions are derived from the pitch and face of the gear, with the double object of obtaining equal strength and sound castings.

As the smallest section of a casting is, per square inch, its strongest part, this fact should enter more into the question of proportion than has been the custom in the matter of gears. To illustrate the value of this, a test piece cut from the point of a cast gear tooth will often be as much as 30 per cent. stronger than a similar piece cut from its root. Hence the rim of a gear is made thinner than has been the practice, and the central rib deeper, to secure the necessary

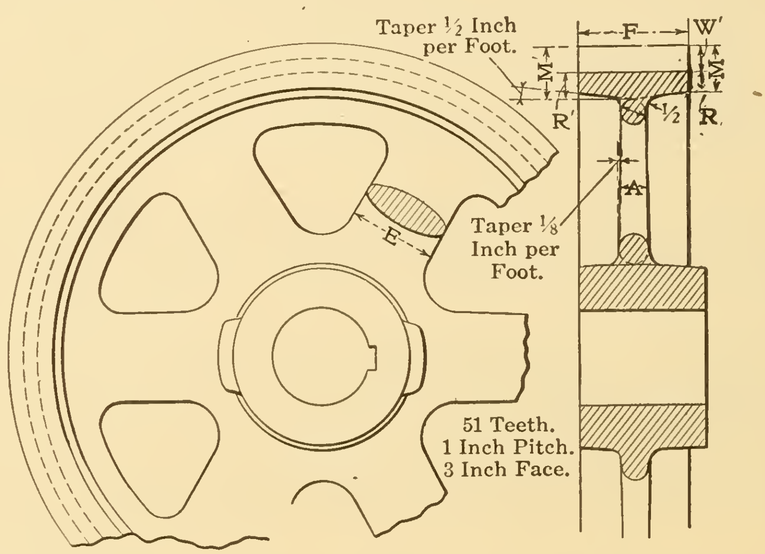


FIG. 68. PLAN AND SECTION OF A SPUR GEAR, SHOWING NOTATION.

section and thus obtain a stronger casting with the same weight of material. This same rule applies to the hub, the outside diameter being reduced and a deeper rim added, and reinforcements placed over keyways, or, better still, two reinforcements directly opposite, especially if the gear is to be balanced; in fact, this is imperative even at relatively low velocities. For the same

reason the teeth should be cored whenever it is possible to do so, as the rim of a casting for a cut gear has always the heaviest section, and, therefore, most subject to blow holes, especially at the junction of the arms and rim. It follows, then, if this be true, that the more uniform the section throughout, the sounder and stronger the casting.

Cored teeth, however, should be machine spaced, as uneven spacing will render it difficult, if not impossible, to cut the teeth in the usual manner. To secure accurate spacing when cutting, it is absolutely necessary that the cutter have an equal amount of stock to remove from each side of the tooth space. Also exposure to hard scale and core sand quickly destroy the cutter. When the teeth are to be planed this point is not of so much importance, as a little unevenness of stock can be readily taken care of, but the cut should be always under the scale if time is of any importance.

FORMULAS

These formulas are based on Brown & Sharpe standard 14½-degree involute tooth.

Thickness of Rim, $M = \frac{3.927}{p}$, or $1.25\ p'$

Mean Thickness of Rim, $M' = \frac{5.026}{p}$, or $1.60\ p'$

Mean Thickness of Rim under Tooth, $R' = \frac{2.868}{p}$, or $0.913\ p'$

Whole Depth of Tooth, $W' = \frac{2.157}{p}$, or $0.6866\ p'$

Minimum Thickness under Tooth, $R' = \frac{1.769}{p}$, or $0.563\ p'$

Area of Rim Total, $MF' = M'F$

Area of Rim under Tooth $= R'F$

Average Area of Arm, $Æ = t\,F\ 1.3$, or $MF\ 0.52$

Average Thickness of Arm, $A = \sqrt{\frac{Æ\ 1.27}{3}}$

Average Width of Arm, $E = 3\ A$

Outside Diameter of Hub = Bore $+ \frac{2}{3} \sqrt[3]{NF}$

Number of Arms = 4, 6, 8, 10, etc., according to design and diameter of gear.

p = Diametral pitch.
p' = Circular pitch.
t = Thickness of tooth at pitch line.

DISCUSSION OF FORMULAS

Thickness of Rim, M—The thickness of rim should be equal to 3.927 divided by the diametral pitch, or 1.25 multiplied by the circular pitch. When the gear is small and accurately made it is often good practice to make the dimension 1.12 of the circular pitch, and so secure the same section and additional strength by adding 50 per cent. to the depth of the central rib.

Mean Thickness of Rim, M'—By mean thickness is meant the thickness of one side of a parallelogram necessary to contain the actual area of the rim. This will take care of the central rib and fillets, so that by multiplying the width of the face of the gear by dimension M' the entire area of rim may be obtained. This will be found necessary also for estimating the weight.

Mean Thickness of Rim under Tooth, R'—This dimension $\left(\frac{2.868}{p}\right)$ multiplied by the width of the face will give the area of the entire section of the rim and rib under the teeth.

Minimum Thickness of Rim under Teeth, R—This determines the thickness of the rim under the tooth measured at the edge of the rim.

Whole Depth of Tooth, W'—This gives the whole depth of the tooth as per Brown & Sharpe standard $= 0.6866\ p'$, or $\frac{2.157}{p}$.

Total Area of Rim—The total area of the rim is found by multiplying the mean thickness M' by face of gear F.

Area of Rim under Tooth—The area of the rim under the tooth is determined by multiplying the mean thickness R' by the face F.

Average Area of Arm, $Æ$—The average area of the arm is that area midway between the inside of the rim and the outside of the hub, and is found by adding 30 per cent. to the area of the tooth at the pitch line, or the thickness of tooth at pitch line $t \times F \times 1.3$. The same result may be reached by taking 0.52 of $M' \times F$, although the foregoing is simpler. Taper of arm to be ⅛ inch per foot above and below this point.

Average Thickness of Arm—The average thickness of arm (A) may be determined by dividing the quotient of the area of arm $Æ \times 1.27$ by 3, and extracting the square root. If the arm was made in the form of a parallelogram it would not be necessary to multiply the area by 1.27, but as it is to be elliptical, this is essential to insure sufficient section, as 27 per cent. of the area of the parallelogram is lost when inscribing an ellipse therin.

Note.—If width of arm is desired, 2 or 2½ times the thickness instead of 3 times, as given, 2 or 2½ is to be substituted.

Average Width of Arm, *E*—To determine the width of an arm multiply its thickness by three.

Outside Diameter of Hub—As a rule the outside diameter of a gear hub is made double that of its bore, but when keyways are reinforced, or the gear is to carry a load less than proportional to the diameter of its shaft, the hub diameter may be less than this rule prescribes. Thus, if a gear of 30-inch pitch diameter was mounted on a shaft 15 inches in diameter, the hub diameter should only be increased sufficiently to maintain its section and strength

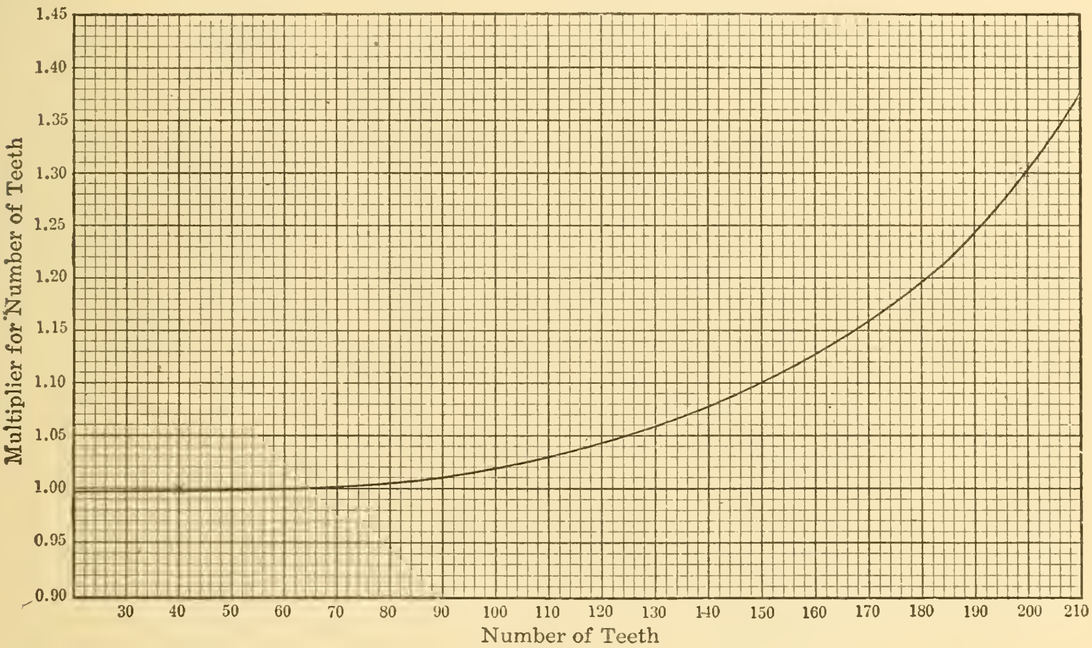


CHART 5. MULTIPLIER FOR INCREASED NUMBER OF TEETH OF SPUR GEARS.

proportional to the gear, not to the shaft. The formula given will proportion the hub to easily carry the entire load applied to the gear but should be used with discretion. Generally, however, when the bore is small or proportioned to the diameter of the gear, the outside diameter of the hub may be taken as 1.75 times the diameter of the bore with reinforcement for keyways.

From the foregoing it is obviously almost as important that the hub should not be disproportionately heavy as that it should be heavy enough. But if for any reason a materially heavier hub is required, it should be split, by means of thin cores, into as many equal, radial sections as there are arms in the gear, thus obviating blow-holes, and strains caused by shrinking. Fill the cored spaces with babbitt or lead before machining, and shrink steel bands on the hub for a grip on the shaft. See Fig. 69.

Number of Arms—There is no definite rule for this, as this point depends

almost entirely upon the judgment of the designer. In general, however, gears up to 60 inches are either webbed or with four or six arms to suit conditions; over 60 inches eight arms are generally used and over 80 inches in

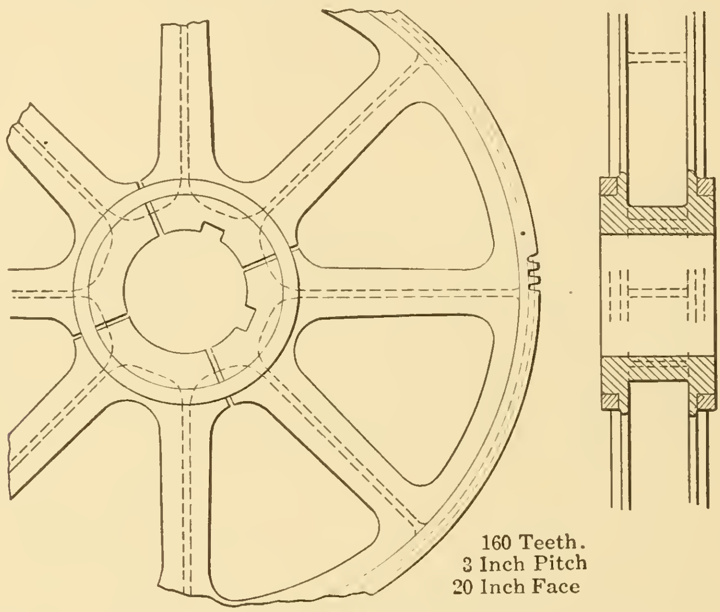


FIG. 69. PLAN AND SECTION OF SPUR GEAR WITH SPLIT HUB.

diameter 10 arms. In no case should the greatest distance between the arms exceed the length of the arm measured from the center of the gear to its intersection with the rim.

Width of the Face—The face of spur gear is generally estimated at about three times its circular pitch, as follows:

1	diametral pitch..............	9 inches face
1¼	diametral pitch..............	7½ inches face
1½	diametral pitch..............	6 inches face
1¾	diametral pitch..............	5½ inches face
2	diametral pitch..............	5 inches face
2½	diametral pitch..............	4 inches face
3	diametral pitch..............	3 inches face
4	diametral pitch..............	2 inches face
5	diametral pitch..............	1¾ inches face
6	diametral pitch..............	1½ inches face
8	diametral pitch..............	1¼ inches face
10	diametral pitch..............	1 inches face
12	diametral pitch..............	¾ inches face
14	diametral pitch..............	⅝ inches face
16	diametral pitch..............	½ inches face
18	diametral pitch..............	⅜ inches face
20	diametral pitch..............	⅜ inches face

It is becoming better understood, however, that a wider face is more efficient ("increasing the face does not increase the friction of the teeth in proportion"),* and as the wear of the teeth is governed by the diameter of the gear, or rather by the combination of diameters and the width of the face, the face, therefore, should be amply wide, and the pitch just sufficient to resist fracture.

Street-railway gears are made 3-pitch, 5-inch face, with good results. A gear face of five times its circular pitch is now generally considered to be good proportion.

WEBBED SPUR GEARS

No definite rule can be laid down for the design of webbed gears. See Fig. 70. Generally the thickness of the web is made equal to R'', which is the thickness of the rim at its thickest part.

Core holes H tend to make a sounder casting, and furnish means to secure the gear while machining. When these holes are made large, or are shaped to follow outlines of the arms and rim, ribs are added on each side. Care should

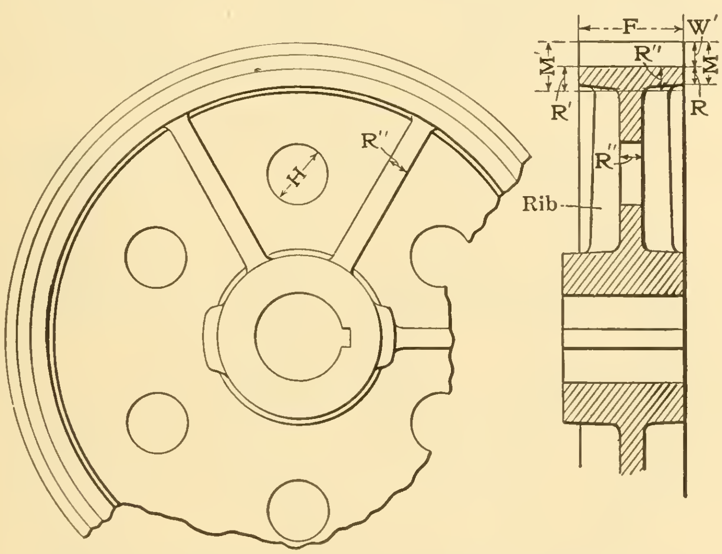


FIG. 70. PLAN AND SECTION OF WEBBED SPUR GEAR.

be exercised, however, not to make the arm too light, for when the hub is heavy the light section connecting the rim and hub will cool too rapidly, setting up serious shrinkage strains, and causing flaws in the casting that cannot be remedied by annealing. Sharp corners, small fillets, and narrow ribs should be avoided for the same reason.

* George B. Grant.

SPLIT SPUR GEARS

It is not good practice to split a gear between the arms, but when this is necessary, the following points should be kept in mind (see Fig. 71):

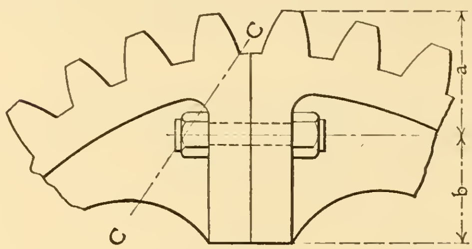


FIG. 71

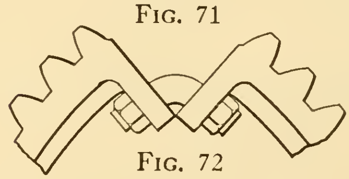


FIG. 72

DIAGRAM ILLUSTRATING SPLITTING OF GEARS AT THE RIM.

Bolts should be placed as close to the rim as possible.

The dimension *b* must in no case be less than the dimension of *a;* otherwise the bolts will be subject to other than the direct tensile stress tending to spread the gear.

Section *C-C* should be stiff enough to resist any strain tending to bend the lugs. By placing the bolt close in the corner of the rim and the lug, the length of the lug may be reduced, as this lug need only be long enough to counteract leverage on bolt.

The bolts should be sufficiently heavy to carry the load applied at the pitch line of gear tooth, not neglecting the initial stress set up by the tightening of the nut, which is generally neglected, sometimes disastrously. When a

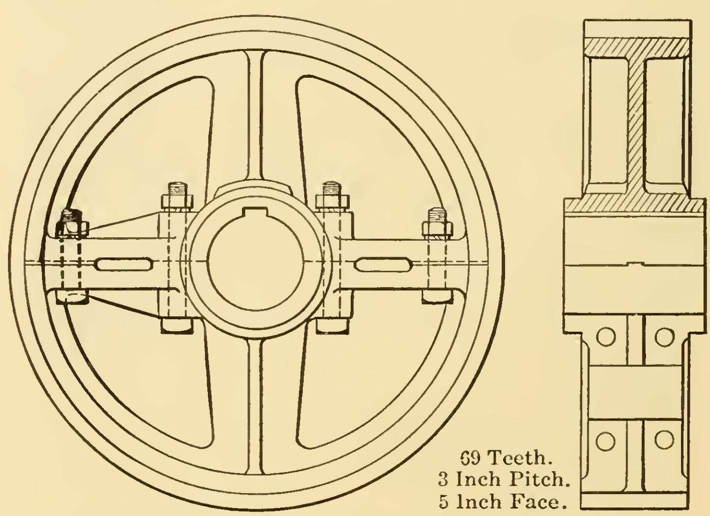


FIG. 73. PLAN AND SECTION OF AVERAGE DESIGN OF SPLIT RAILWAY GEAR.

bolt is placed close in the corner it is necessary to use a stud bolt, drawing up the nuts as the gear halves are brought together.

When the dimension *b* is shorter than *a*, Fig. 71, as is generally the case, the load on teeth of gear will cause a fracture of the rim, as illustrated in Fig. 72.

The best method of splitting gears is through the arm, as illustrated in Fig. 73, which is a cut of the type used on street railways. When the gear is large, Fig. 74 illustrates a good average design.

When splitting a gear of an odd number of teeth, the split should be made ¼ of the circular pitch off the center line. This will bring the split through center of two tooth spaces. It is good practice to spline the adjoining sur-

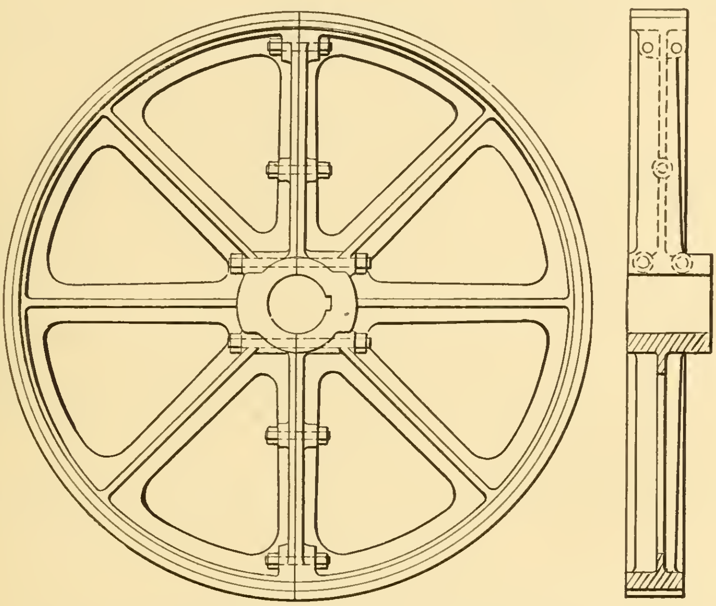


FIG. 74. PLAN AND SECTION OF AVERAGE DESIGN OF LARGE SPLIT GEAR.

faces, as illustrated in Fig. 73, instead of using fitted bolts or depending on dowel pins. A spline ¾ inch wide and ⅛ inch high will answer for any but the largest gears.

One point that must be considered in designing split gears for high speed is the fact that weight at any point or part of the rim, and not integral to the rim proper (as lugs for bolting, see Fig. 71), locates the bending moment, and if safe speed is exceeded to the moment of fracture, it will occur at or near such weight.

This fact is pointed and explained by Charles H. Benjamin, in an article in the AMERICAN MACHINIST of December 26, 1901, entitled "The Bursting of Small Cast-iron Flywheels."

I-SHAPED ARMS

For gears such as are shown in Fig. 69, the proportion of rim, arms, and hub may be determined according to formulas given above, the area of arms, of

course, being contained in an I instead of an elliptical section. For gears of this size, however, there is more variation in the design of the gear, and the values given cannot be followed so closely.

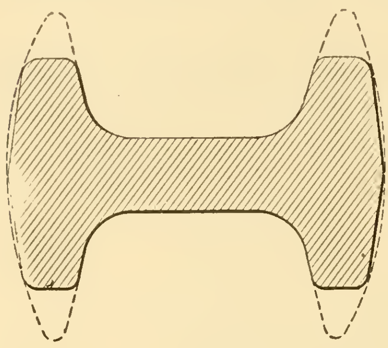


FIG. 75. SUGGESTED CONNECTING-ROD SECTION.

The I-section arm is much more desirable in heavy gears because it distributes the metal contained in a section of the arm over a greater surface of the rim at their intersection than does the elliptical arm, and, therefore, lessens liability of blow holes at this point. Aside from this, it gives better support to rim in case the face is wide, and makes a stronger section than the elliptical arm of the same weight.

For the above reasons I am disposed to advocate a connecting-rod section, such as is illustrated by Fig. 75, for smaller gears.

CONNECTING-ROD-ARM SPUR GEAR

This design could be applied to gears of all sizes: As the face increased, the section could be changed—as per dotted lines. This, however, would allow a central rib instead of two ribs on each side of the rim as when the ⊢⊣ is turned

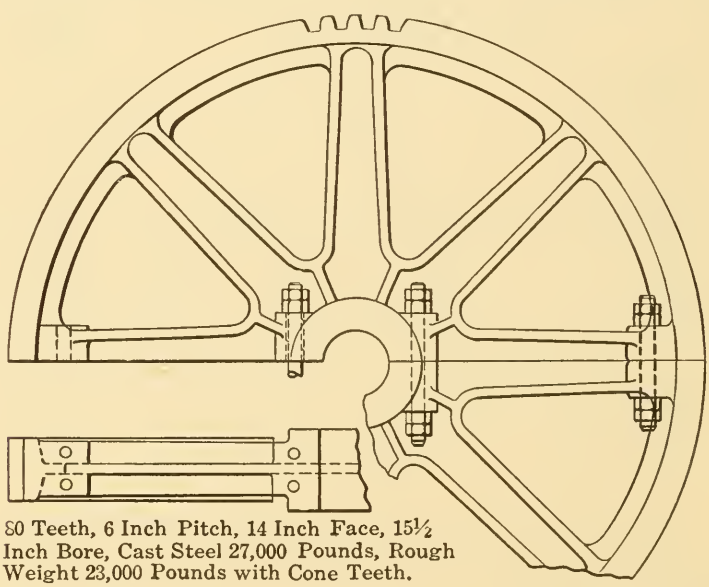


FIG. 76. PLAN AND SECTION OF LARGE SPUR GEAR WITH CONNECTING-ROD ARMS.

the other way (see Fig. 69), but would allow the use of bolts instead of links when a split gear had no hub projections. This design is illustrated by Fig. 76.

For gears of an extremely wide face it will be found that the formula for arm

will give a section that cannot be contained in the space between hub and rim. This practically means making a web gear. However, when the face is wide it would seem better to use a light web, say, according to dimension on Fig. 70 and extending ribs toward the side, making a cross-sectioned arm, or using the section shown by Fig. 69. This section is generally the most desirable, but this depends upon conditions.

The formulas given here will form a basis for the design of worm and bevel gears, although I believe that the + or cross-shaped section is superior to the oval arm for worm gears on account of the side strain encountered.

FOR CALCULATING WEIGHT

The accompanying Chart 6 gives a rapid, approximate method of calculating the weight of a cast-iron spur gear blank for a cut gear, designed according to the above formulas.

This table was derived from a formula by Reuleaux, which gives the weight of a cast-tooth gear from the combined product of a constant for the number of teeth, face and square of the circular pitch, as follows:

"The approximate weight of gear wheels, W, may be obtained from the following:

"$W = 0.0357\, b\, c^2\ (6.25\, N + 0.04\, N^2)$, where b = face, c = circular pitch, N = number of teeth, and W = weight of gear.

"The following table will facilitate the application of the formula; it gives the value $\frac{W}{b\,c^2}$ for the number of teeth which may be given, and the weight may be readily found by multiplying the value in the table by $b\,c^2$:"

N	0	2	4	6	8
20	5.04	5.60	6.18	6.77	7.38
30	7.99	8.61	9.24	9.89	10.52
40	11.09	11.90	12.59	13.30	14.02
50	14.74	15.48	16.23	17.00	17.77
60	18.55	19.35	20.15	20.97	21.80
70	22.65	23.50	24.36	25.24	26.12
80	27.02	27.93	28.85	29.79	30.73
90	31.69	32.66	33.63	34.62	35.63
100	36.63	37.67	38.70	39.75	40.81
120	47.40	48.54	49.69	50.85	52.03
140	59.30	60.56	61.82	63.10	64.27
160	72.35	73.73	75.10	76.39	77.90
180	86.54	88.03	89.52	91.02	92.54
200	101.88	103.48	104.98	106.70	108.34
320	118.36	120.08	122.15	123.52	125.27

Example: A gear 50 teeth 2 inches pitch, 4 inches face, we have: $b\,c^2$ 14.74 = $4 \times 2^2 \times$ 14.74 = 235.84, say, 236 pounds.

WEIGHT OF CAST-IRON GEARING. *Reuleaux.*

In endeavoring to apply this table to practice it was found that the square of the pitch (C^2) was not a correct factor, and a separate table was made up for it.

After repeated trials and corrections in the value of the constant for the number of teeth, it was noticed (when close results were finally obtained) that

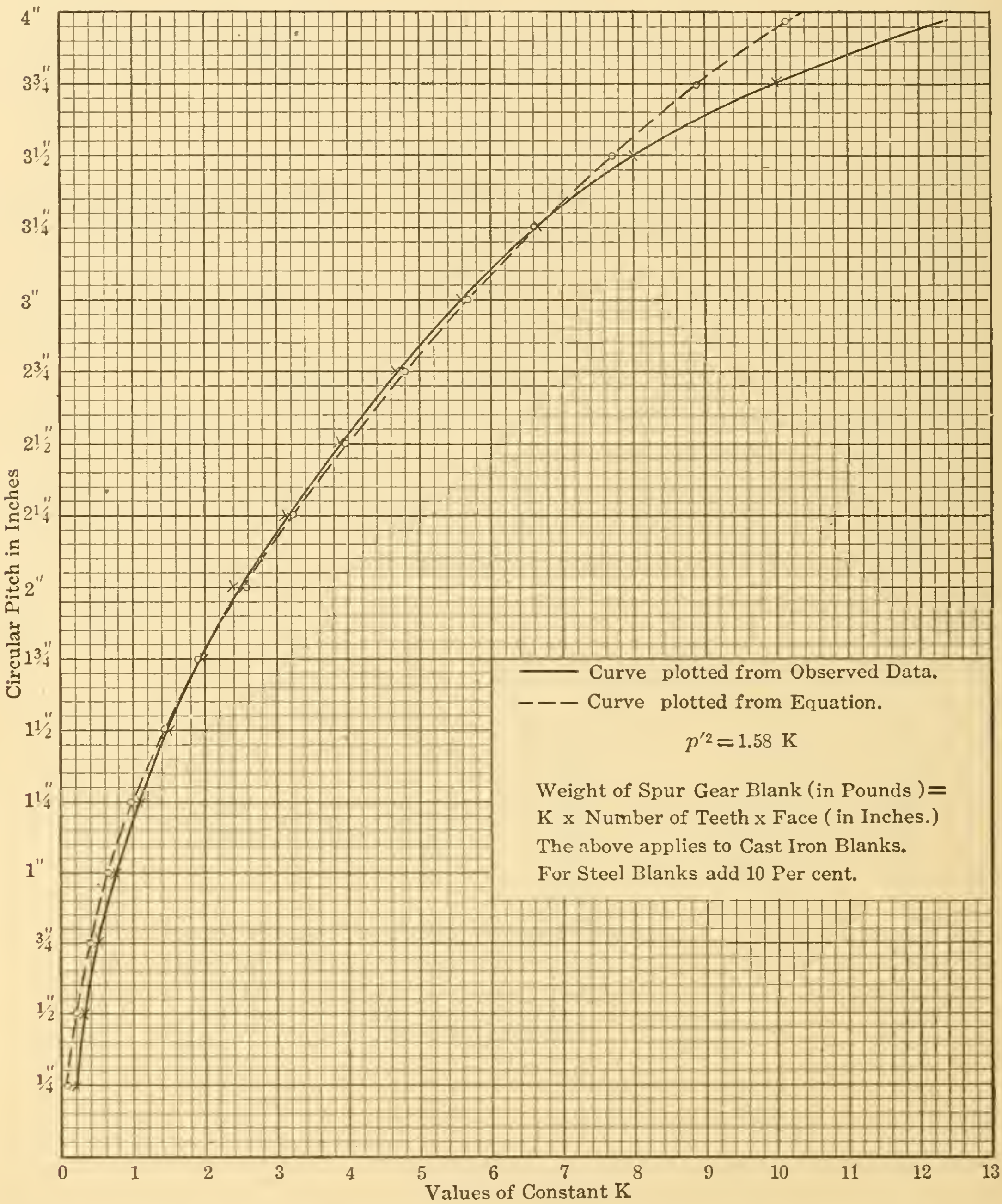


CHART 6. RELATIONSHIP BETWEEN CIRCULAR PITCH AND FACTOR K USED IN ESTIMATING WEIGHTS OF SPUR GEAR BLANKS.

the values ran parallel with the number of teeth, and were, therefore, dropped, changing the value of the constants to the square of the pitch, so called, to obtain like results.

To find the weight, therefore, it is only necessary to find the combined product of the number of teeth, face, and a constant given for the pitch:

Weight = number of teeth $\times F \times K$.

The accompanying Chart 6 gives the value of K.

This formula has its limitation; it cannot be used for low numbers of teeth, varying with the pitch, also for large gears there is a variation in design that cannot well be covered by one constant. However, the proper constant may be readily determined by trial for different constructions. In general it must be used with discretion, but is invaluable as a check. The finished weight is found by deducting 30 per cent.

The following equation seems to give a close approximation to the curve of Chart 6 throughout the ordinary working range up to 3½ inches circular pitch: $p'^2 = 1.58\,K$,

in which p' is the circular pitch and K the constant previously referred to. Transposing we have $K = \dfrac{p'^2}{1.58}$, or substituting in equation for weight of spur gears,

$$\textit{Weight} = \frac{p'^2}{1.58} \times \textit{number of teeth} \times \textit{face}.$$

Beyond 3½ inches circular pitch there is considerable variation between the curves from the actual data and from the equation. However, as stated, the great variations in the design of large gears cannot be cared for by a single constant. In using the formula its limitations should be carefully understood. As a matter of interest, it might be stated that for 6 inches circular pitch the curves are again practically in agreement.

ARMS FOR SPUR GEARS *

In deducing a formula for gear arms it is assumed that the thickness of the rim is sufficient to distribute the load between the arms; this assumption is quite justified, as such a depth is necessary to prevent bending of the rim between adjacent arms. By equating the expressions for the tooth strength and that of a beam supported at one end and loaded at the other, the general expression arrived at is

$$Z = \frac{p'^3\,R\,(N-7)}{50\,A} \text{ for circular pitch, and}$$

$$Z = \frac{\pi^3\,R\,(N-7)}{50\,p^3\,A} \text{ for diametral pitch,}$$

* Henry Hess.

when

Z = modulus resistance of arm cross-section.
p' = circular pitch.
p = diametral pitch.
R = ratio of face width to circular pitch $= \frac{F}{p'}$.
F = face width.
N = number of gear teeth.
A = number of arms.

If it is preferred to use the face width itself, instead of its ratio to the circular pitch, then

$$Z = \frac{p'^2 F (N-7)}{50 A} \text{ for circular pitch.}$$

$$Z = \frac{F (N-7)}{50 p^2 A} \text{ for diametral pitch.}$$

By these formulas the dimensions of a gear arm of any section whatever can be determined.

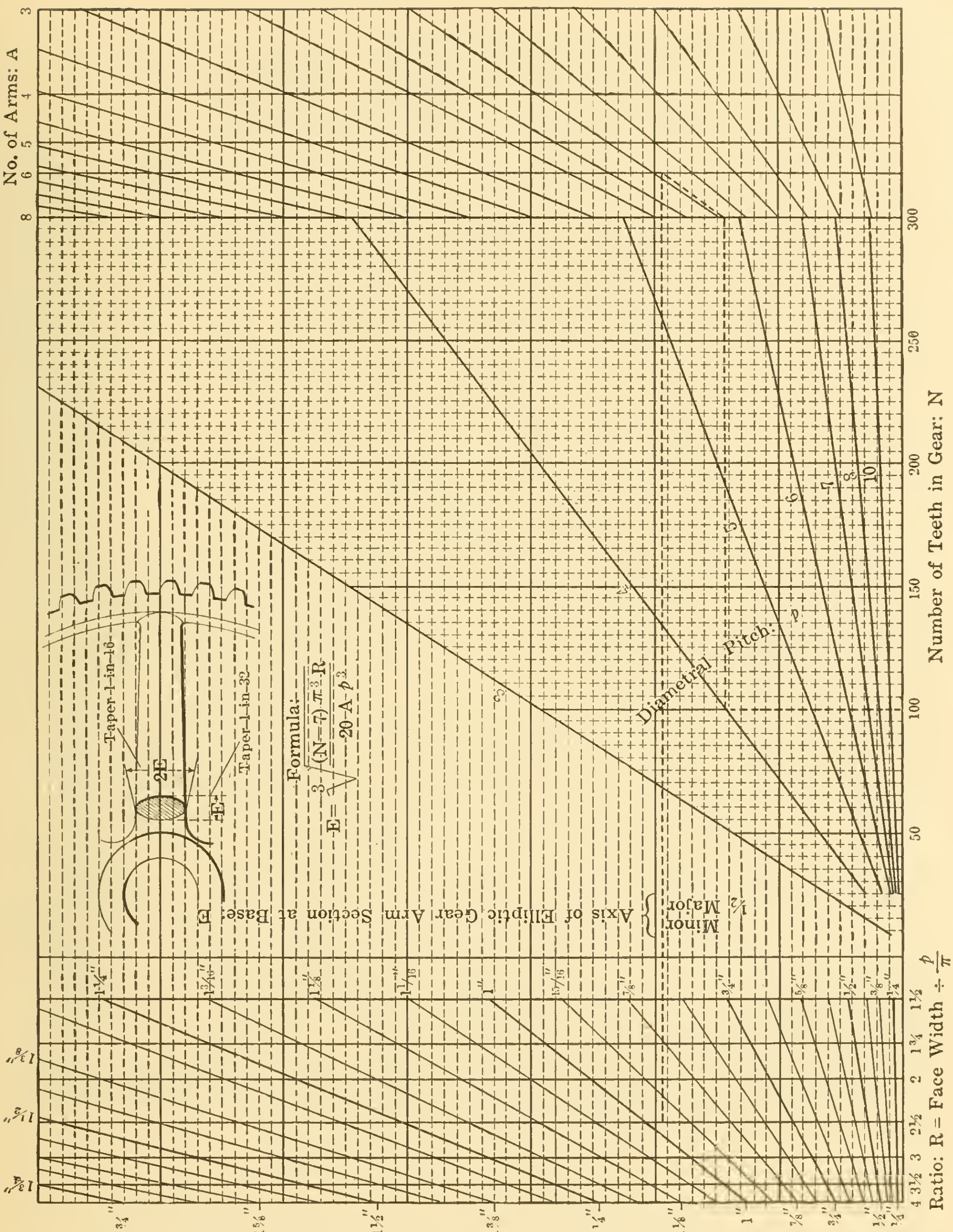
As by far the great majority of cast gear arms are of elliptical cross-section, these expressions are reduced by inserting the terms of the modulus of resistance of an ellipse in which the major axis is double the minor, and the formulas become, when E = the thickness of the arm at its base, and $2E$ = the width of the arm at its base;

$$E = \sqrt{\frac{(N-7) p'^3 R}{20 A}} = \sqrt{\frac{(N-7) p'^2 F}{20 A}} \left.\right\} \text{ for circular pitch;}$$

$$E = \sqrt[3]{\frac{(N-7) \pi^3 R}{20 A p^3}} = \sqrt[3]{\frac{(N-7) \pi^2 F}{20 A p^2}} \left.\right\} \text{ for diametral pitch.}$$

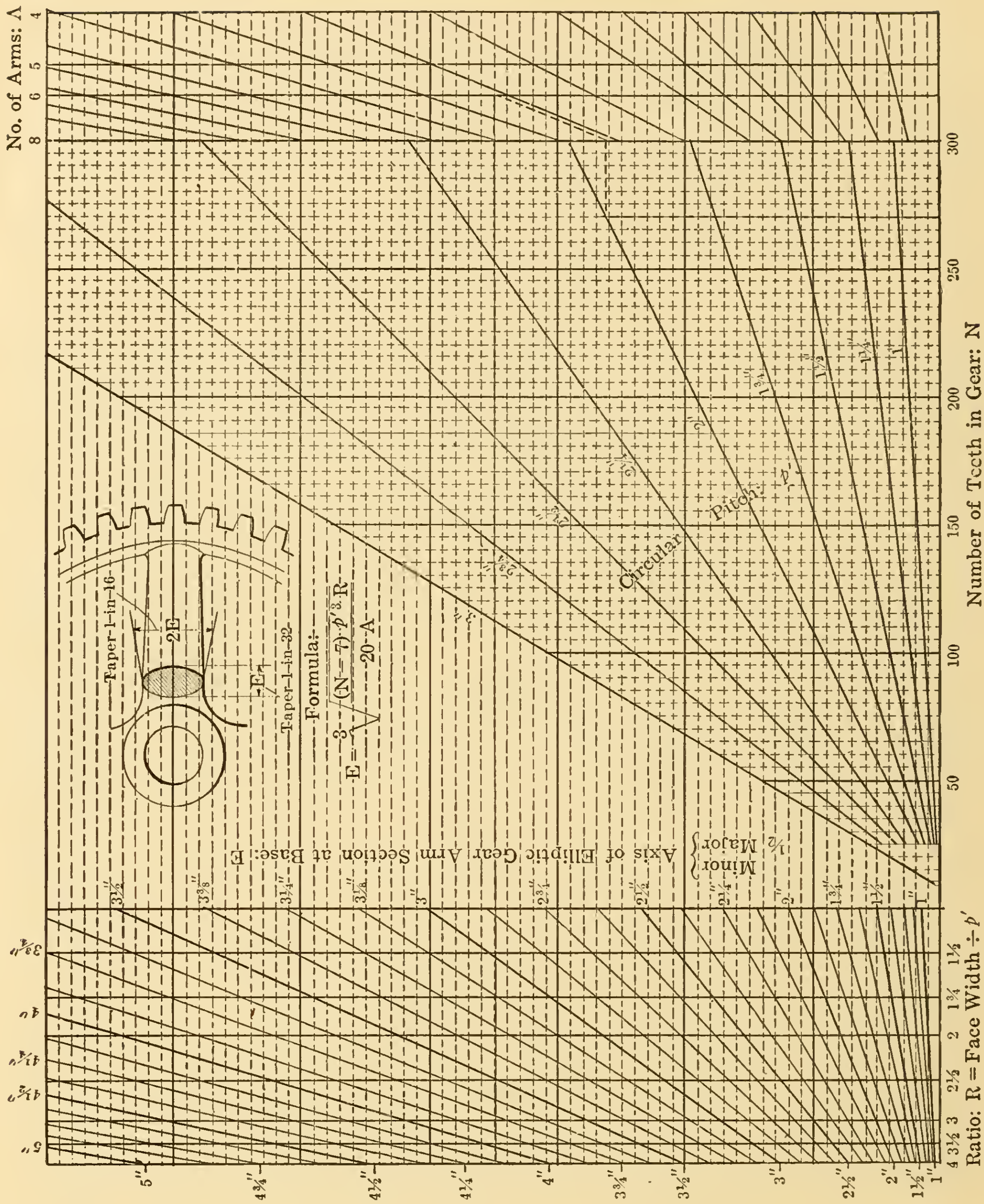
To reduce the labor involved by the mathematical solution, Charts 7 and 8 have been constructed, one for diametral pitches ranging from 10 to 3, and the other for circular pitches ranging from 1 to 3 inches; as 3 diametral pitch is very nearly equal to 1-inch circular pitch, the second chart extends the range of the first without a break, so that, between the two, any case likely to arise will be taken care of. Diametral pitch is given in Chart 7, as the more general practice uses that for small and medium-sized gears, while for large work, circular pitch is generally employed, and is therefore used as the basis of Chart 8. Two charts are required, as the inclination of the pitch diagonals toward the end values would become too slight to admit of accurate reading on a single one.

By tracing the number of teeth from the bottom scale to the pitch diagonal



DIRECTIONS: Trace up from the Number of Teeth to the Pitch Diagonal, then Horizontally to the Vertical above 300 Teeth, then Parallel with the Nearest Diagonal to the Vertical Headed with the Arm Number *E;* then Trace to the left Horizontally to the Vertical *R* representing the Ratio of Face Width to Circular Pitch *R*. Take the Nearest Diagonal as the Arm Base Thickness *E*.

CHART 7. PROPORTIONS OF GEAR ARMS FROM DIAMETRAL PITCH.



DIRECTIONS: Trace up from the Number of Teeth to the Pitch Diagonal, then Horizontally to the Vertical above 300 Teeth, and Trace Parallel to the Nearest Diagonal to the Vertical Headed with Number of Arms; then Trace to the left Horizontally to the Vertical *R* representing the Ratio of Face Width to Circular Pitch. Take Nearest Diagonal as Base Thickness of Arm *E*.

CHART 8. PROPORTIONS OF GEAR ARMS FROM CIRCULAR PITCH.

in the main portion of the circular pitch chart and referring the intersection to the vertical scale under 8, the value is found of

$$\frac{(N-7)\,p'^3}{20\,A} \text{ for } A = 8 \text{ arms.}$$

For any other number of arms this is modified by employing the auxiliary portion of the chart at the right, referring the value just found along or between the nearest slant lines to intersection with that vertical representing the number of arms actually used. By now tracing this hight horizontally to the left to that vertical R representing the particular ratio of face width to circular pitch employed, and taking a reading from the nearest slant line crossing this vertical, the value first found is multiplied by R and the cube root extracted.

Past practice gives a face width between two and three times the circular pitch, but as the tendency is now toward a wider face, ratios from 1½ to 4 are given.

Concise directions are printed with the charts. Dotted trace lines of the following examples are also drawn in:

Example 1. Given a gear of 100 teeth, 4 diametral pitch, 6 arms and ratio of face width to circular pitch = 2½.

Trace on Chart 7 100 teeth up to diagonal for 4 pitch, horizontally to number of arms 8, slantwise up to number of arms 6, horizontally to the left to ratio 2½, which is intersected between 15/16 inch and 1 inch; therefore thickness of arm is to be taken as 1 inch and width as 2 inches. By calculation the dimensions are 0.96 inch and 1.98 inch.

Example 2. Given a gear of 270 teeth, 2-inch circular pitch, 6 arms and ratio of face width to circular pitch = 2.

Trace on Chart 8 as before and find 3¼ inches full as thickness of gear arm at base, and 6½ inches full as width. The calculated dimensions 3.27 inch and 6.54 inch agree almost absolutely with the much more quickly obtained values by the chart.

In large arms the designer will frequently prefer a cored section. A satisfactory one will be that of Fig. 77, in which major and minor axes of both core and arm are relatively as 2 to 1. By equating the moduli of resistance for solid and hollow elliptical sections of these proportions, it is found that $E^3 = \frac{D^4 - d^4}{D}$, in which E is the thickness of the solid arm as obtained by chart or formula; d and D are dimensions of the cored arm. See Figs. 77 and 78.

In order to lessen the work of making the core box by substituting flat surfaces for curved ones, an approximation like Fig. 78 will add but slightly to the weight, as is shown by the ellipse dotted in for comparison.

The ellipse outlines are formed of circular arcs struck from four centers, which will approximate very closely to the true ellipse. The construction of the core sides is readily apparent from the sketch.

The arm taper is stated as 1 in 32 and 16, respectively, for the arm thickness and width; this gives a pleasing appearance for a moderately long arm,

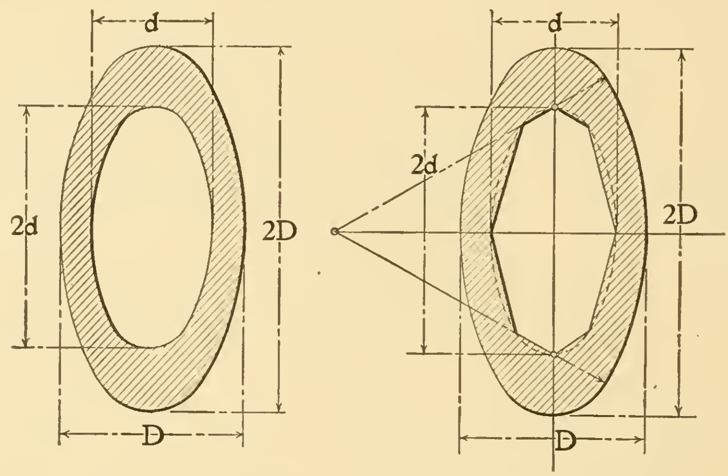


FIG. 77. PROPORTIONS OF HOLLOW ARMS. FIG. 78.

but it is not a hard-and-fast rule, as a greater or lesser taper may be employed to suit the designer's fancy without affecting the strength of the arm, unless the taper is made so excessive as to bring the dimensions at the rim down to one half of these at the base.

As the tooth and arm are of the same material, the method is satisfactory for all cast gears, but this must not be interpreted to mean that this or any other formula will prevent shrinkage strain due to relatively large hubs or very heavy rims; where these occur, great care must be exercised in the foundry, and it will also not be amiss to add a generous amount of metal to the arms.

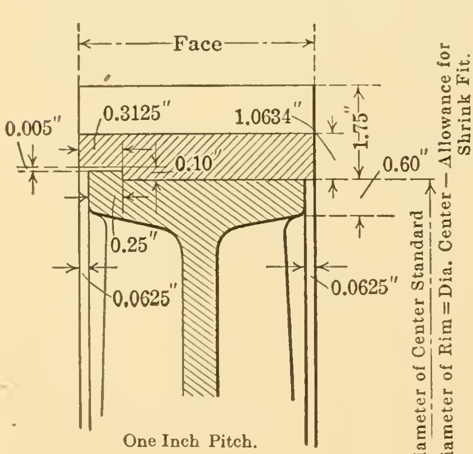


FIG. 79. PROPORTIONS OF RIM GEARS.

RIM GEAR PROPORTIONS

Where steel castings prove inefficient for the work intended, forged steel rims, designed somewhat as illustrated in Fig. 79, are used. The center is made of cast steel of a heavy pattern, the forged rims being shrunk thereon. As this rim material may be obtained of a higher grade of steel than is possible in the casting, and is free from hidden flaws, also as the

rim is renewable, it is a much better and, in the end, a cheaper proposition. The rims are made of a forged billet, which is first pierced and then rolled into shape by the same process employed for locomotive tires.

There are many variations of this design, but it is thought best to make the face of the center narrower than the rim on account of possible unevenness in fitting, and turn a shoulder on center to bring rim up true, instead of depending upon parallels or surface plates. Also it will be desirable in many cases to replace rim without removing gear from the shaft. This is accomplished by rapidly heating rim by means of a circular gas or oil burner made to suit diameter of gear and protected by asbestos cover to localize the heat.

DESIGN OF BEVEL GEARS

The rules for the design of spur gears may be applied to helical, herringbone, worm, and spiral gears, using the circular pitch as a basis. Also for bevel gears, taking the proportions from the large end of the tooth. The average design of bevel gears is shown in Fig. 80. The hub should be carried well back of the face and connected to the rim by ribs, 4, 6, 8, or 10 in number, depending upon the diameter of the gear. The hub should not be carried too far in the front, or small end, of the gear, as a long hub will make it impossible in many cases to cut the teeth, for if this hub is carried too far it will interfere with the operation of the machine. See Fig. 81. A small hub, however, should always be put on the front end of the gear, otherwise it will be necessary to counterbore to secure a finished bearing.

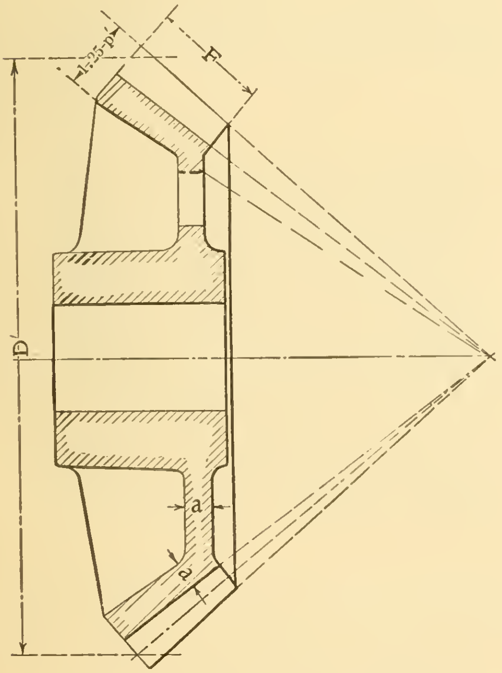


FIG. 80. BEVEL GEAR PROPORTIONS

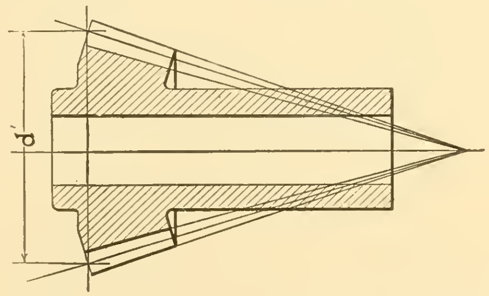


FIG. 81. BEVEL GEAR WITH LONG FRONT HUB EXTENSION.

RAWHIDE GEARS

Rawhide gears are commonly made with brass flanges on either side of the face to hold the rawhide in position and to engage the mating gear, unless rawhide contact alone is desired. It is practice to speak of the face of the rawhide gear as including these flanges. See Fig. 82. There is no great gain in

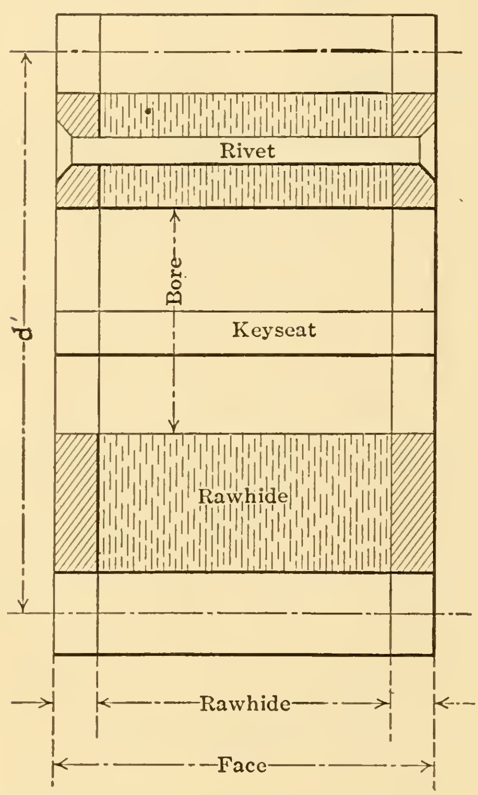


FIG. 82. RAWHIDE GEAR.

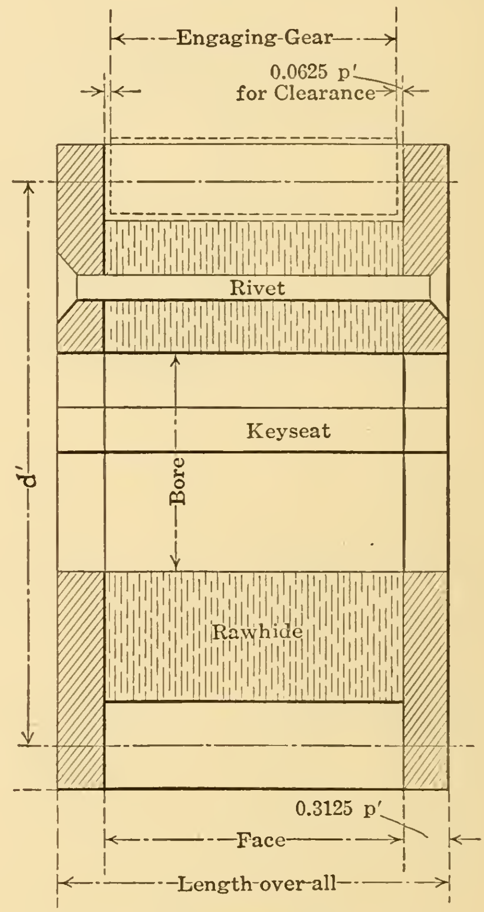


FIG. 83. SHROUDED RAWHIDE GEAR.

preventing the flanges from coming into contact, and to make a rawhide gear without cutting through the flanges, as per Fig. 83, is unnecessary and expensive.

There is no reason why steel flanges cannot be used in place of brass, especially for the larger sizes; boiler plate will be found excellent for this. The thickness of the flange is something that varies greatly, although 5/16 of the circular pitch is a fair average.

For the larger rawhide gears it is recommended that bolts instead of rivets be used, as it is impossible to otherwise draw up a wide-face rawhide gear.

The bolt head may be countersunk so that one side of the gear will be flush. It is also sometimes possible to put the nut in a counterbore; this depends, of course, on the design of gear.

Rawhide gears to run loose on the shaft should be bushed. When it is necessary to move a rawhide gear on a spline one of the flanges should be made as

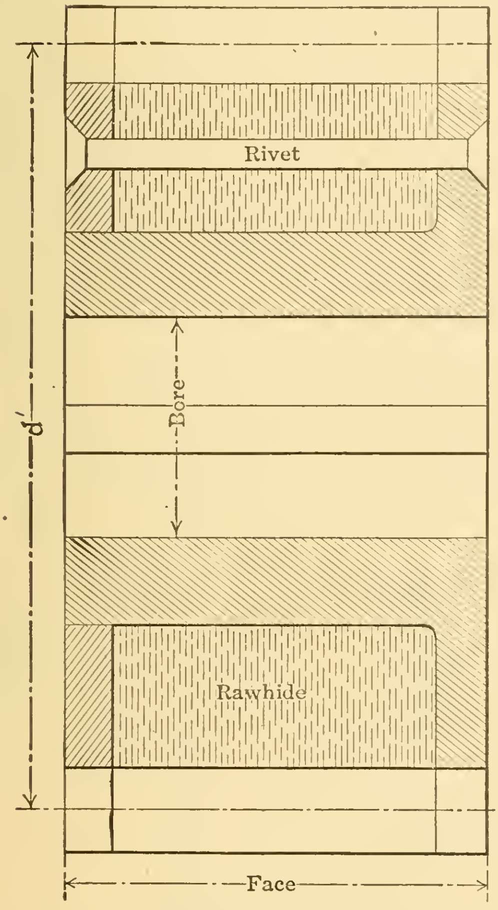


FIG. 84. BUSHING AND FLANGE IN ONE PIECE.

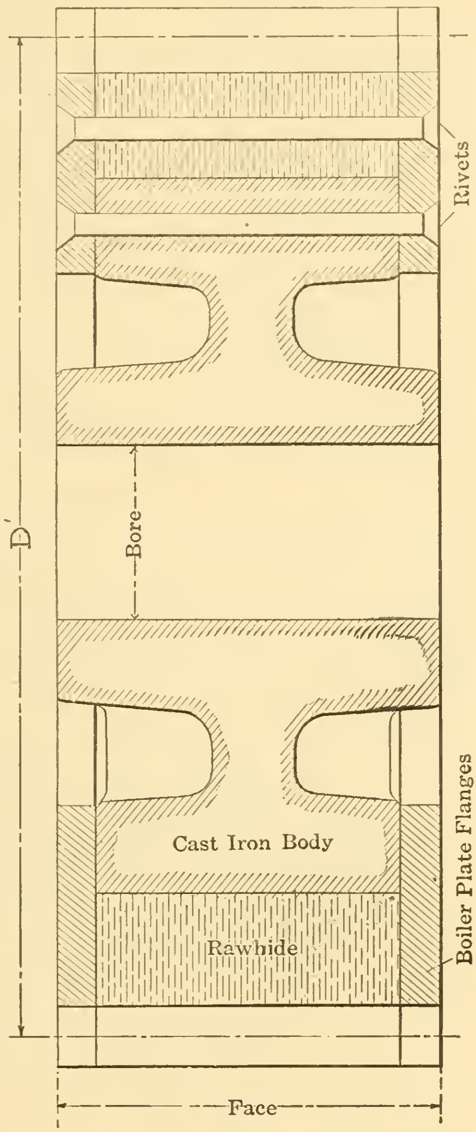


FIG. 85. DESIGN OF RAWHIDE GEAR.

part of the bush, as per Fig. 84. The usual design of the larger size rawhide gear is shown in Fig. 85.

Rawhide bevel gears are designed similar to Fig. 86. Both ends of the teeth must be flanged to facilitate the cutting of the teeth.

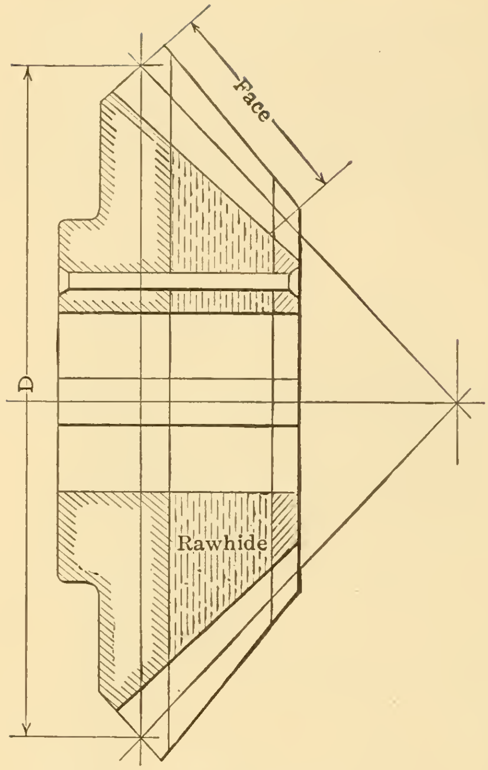


FIG. 86. RAWHIDE BEVEL GEAR.

Fiber is often used in place of rawhide, but is usually more brittle and has a tendency to wear the engaging gear, although made of a harder material. Fiber gears are ordinarily furnished without flanges. Gears are also made of laminations of rawhide and bronze, or fiber and bronze, but not to any great extent.

A fiber stress of 5000 pounds per square inch is amply safe for calculating the strength of rawhide gears.

MORTISE GEARS

The mortise gear is composed of a cast-iron rim, containing cored slots, into which wooden teeth are driven. See Fig. 87. These gears cannot compare either in efficiency or cost to a properly cut cast-iron gear, but are still used in many places where excessive noise is prohibitive. The wooden teeth are made either of apple or maple, treated in linseed oil and cut to the proper form after being inserted in the cored rim; otherwise the spacing of the teeth would be gov-

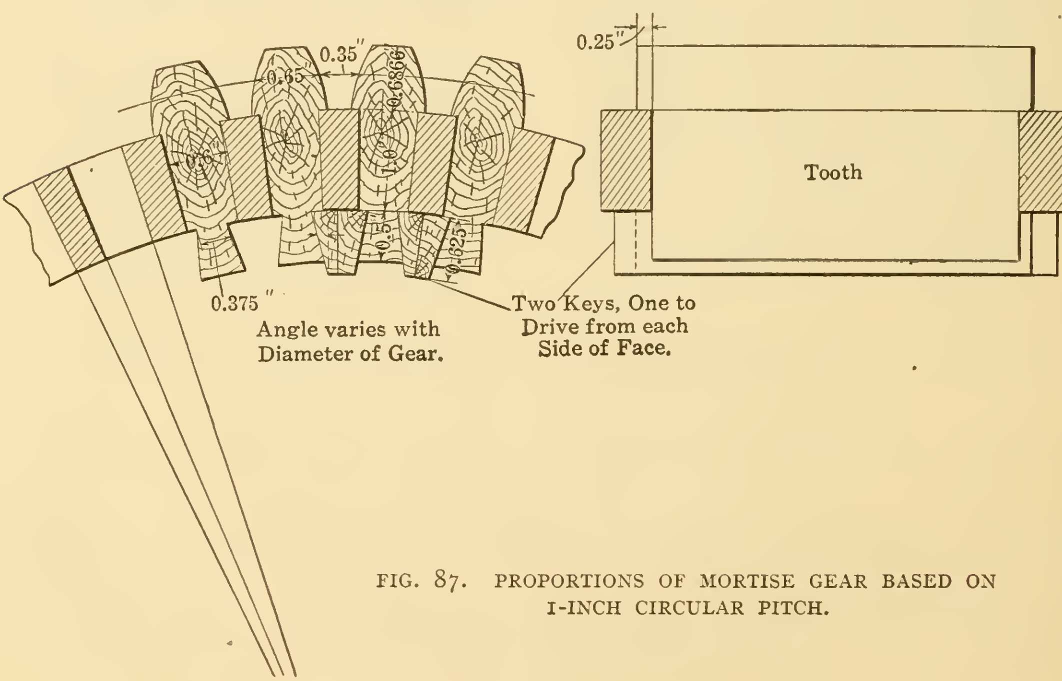
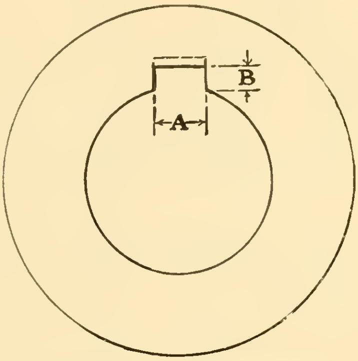


FIG. 87. PROPORTIONS OF MORTISE GEAR BASED ON 1-INCH CIRCULAR PITCH.

erned by the spacing of the cores, which can never be very accurate. Replaced teeth must be fitted and shaped by hand. The strength of the mortise gear is governed by the thickness of the iron teeth in the engaging gear, which are made 0.35 of the pitch instead of 0.5 pitch, as for cut gears. The outside diameter of the cored rim is turned to the dedendum diameter that the teeth will be set an equal distance from the center.

KEYSEATS



The commonly accepted keyseat standard is that of Jones and Laughlin, as per Table 19. In this the width of the key approximates one-quarter the shaft diameter. A square key is used, one half being in the gear and one half in the shaft. Dimensions for taper keys are included in this table. A taper keyseat, however, is nothing more than a means of tightening up a poor fit; a gear properly fitted to the shaft will

BORE	KEYSEAT TAPER 1/8 INCH PER FOOT		STRAIGHT KEYSEAT	
	WIDTH "A"	SMALLEST HIGHT "B"	WIDTH "A"	HIGHT "B"
IN. IN.	IN.	IN.	IN.	IN.
8 to 7 7/8	2	21/32	2	1
7 3/4 " 7 3/8	1 7/8	5/8	1 7/8	15/16
7 1/4 " 6 7/8	1 3/4	19/32	1 3/4	7/8
6 3/4 " 6 3/8	1 5/8	17/32	1 5/8	13/16
6 1/4 " 5 7/8	1 1/2	1/2	1 1/2	3/4
5 3/4 " 5 3/8	1 3/8	29/64	1 3/8	11/16
5 1/4 " 4 7/8	1 1/4	27/64	1 1/4	5/8
4 3/4 " 4 7/16	1 1/8	3/8	1 1/8	9/16
4 3/8 " 4 3/16	1 1/16	11/32	1 1/16	17/32
4 1/8 " 3 15/16	1	11/32	1	1/2
3 7/8 " 3 11/16	15/16	5/16	15/16	15/32
3 5/8 " 3 7/16	7/8	19/64	7/8	7/16
3 3/8 " 3 3/16	13/16	17/64	13/16	13/32
3 1/8 " 2 15/16	3/4	1/4	3/4	3/8
2 7/8 " 2 11/16	11/16	15/64	11/16	11/32
2 5/8 " 2 7/16	5/8	13/64	5/8	5/16
2 3/8 " 2 3/16	9/16	3/16	9/16	9/32
2 1/8 " 1 15/16	1/2	11/64	1/2	1/4
1 7/8 " 1 11/16	7/16	9/64	7/16	7/32
1 5/8 " 1 7/16	3/8	1/8	3/8	3/16
1 3/8 " 1 3/16	5/16	7/64	5/16	5/32
1 1/8 " 1	1/4	3/32	1/4	1/8

TABLE 19—STANDARD KEYSEATS

not work loose if a straight key is used with clearance on the top. The best fit can be spoiled by a little carelessness in driving a taper key, as it is sure to make the gear run out if driven the least bit too tight. The taper key is applicable to only the heaviest work where the mass of metal will prevent any such distortion.

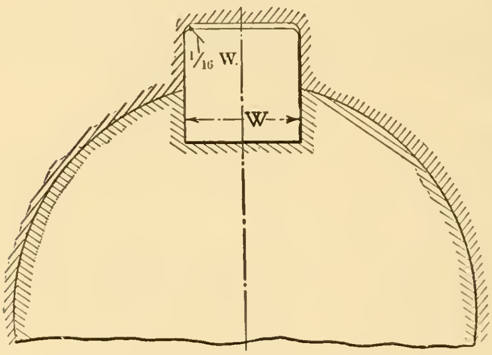
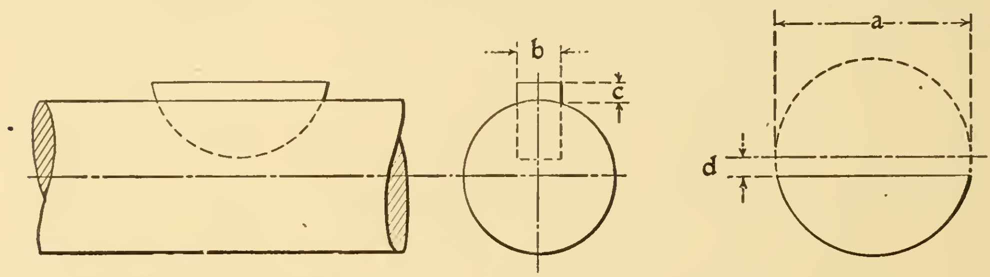


FIG. 88. FILLETED KEYWAY.

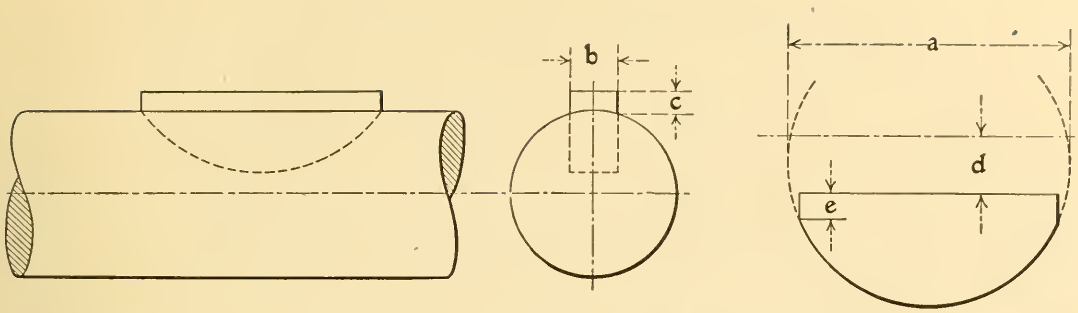
For gears that are to be hardened it is important that there be a fillet in the corners, the top of the key being beveled off to suit, otherwise a crack is very liable to start from the sharp corner of the keyway. For that matter, this would be an excellent plan to adopt for all keyways. See Fig. 88.

For machine tools and automobiles the Woodruff key is generally used.



NO. OF KEY	DIAMETER OF KEY	THICKNESS OF KEY	DEPTH OF KEYWAY	CENTER OF STOCK, FROM WHICH KEY IS MADE, TO TOP OF KEY	NO. OF KEY	DIAMETER OF KEY	THICKNESS OF KEY	DEPTH OF KEYWAY	CENTER OF STOCK, FROM WHICH KEY IS MADE, TO TOP OF KEY
	a	*b*	*c*	*d*		*a*	*b*	*c*	*d*
1	1/2	1/16	1/32	3/64	*B*	1	5/16	5/32	1/16
2	1/2	3/32	3/64	3/64	16	1 1/8	3/16	3/32	5/64
3	1/2	1/8	1/16	3/64	17	1 1/8	7/32	7/64	5/64
4	5/8	3/32	3/64	1/16	18	1 1/8	1/4	1/8	5/64
5	5/8	1/8	1/16	1/16	*C*	1 1/8	5/16	5/32	5/64
6	5/8	5/32	5/64	1/16	19	1 1/4	3/16	3/32	5/64
7	3/4	1/8	1/16	1/16	20	1 1/4	7/32	7/64	5/64
8	3/4	5/32	5/64	1/16	21	1 1/4	1/4	1/8	5/64
9	3/4	3/16	3/32	1/16	*D*	1 1/4	5/16	5/32	5/64
10	7/8	5/32	5/64	1/16	*E*	1 1/4	3/8	3/16	5/64
11	7/8	3/16	3/32	1/16	22	1 3/8	1/4	1/8	3/32
12	7/8	7/32	7/64	1/16	23	1 3/8	5/16	5/32	3/32
A	7/8	1/4	1/8	1/16	*F*	1 3/8	3/8	3/16	3/32
13	1	3/16	3/32	1/16	24	1 1/2	1/4	1/8	7/64
14	1	7/32	7/64	1/16	25	1 1/2	5/16	5/32	7/64
15	1	1/4	1/8	1/16	*G*	1 1/2	3/8	3/16	7/64

TABLE 20—WOODRUFF STANDARD KEYS



NUMBER OF KEY	DIAMETER OF KEY	THICKNESS OF KEY	DEPTH OF KEYWAY	CENTER OF STOCK, FROM WHICH KEY IS MADE, TO TOP OF KEY	WIDTH OF FLAT	NUMBER OF KEY	DIAMETER OF KEY	THICKNESS OF KEY	DEPTH OF KEYWAY	CENTER OF STOCK, FROM WHICH KEY IS MADE, TO TOP OF KEY	WIDTH OF FLAT
	a	*b*	*c*	*d*	*c*		*a*	*b*	*c*	*d*	*c*
26	2 1/8	3/16	3/32	17/32	3/32	31	3 1/2	7/16	7/32	13/16	3/16
27	2 1/8	1/4	1/8	17/32	3/32	32	3 1/2	1/2	1/4	13/16	3/16
28	2 1/8	5/16	5/32	17/32	3/32	33	3 1/2	9/16	9/32	13/16	3/16
29	2 1/8	3/8	3/16	17/32	3/32	34	3 1/2	5/8	5/16	13/16	3/16
30	3 1/2	3/8	3/16	13/16	3/16						

TABLE 21—WOODRUFF SPECIAL KEYS

DIAMETER OF SHAFT	NUMBER OF KEYS	DIAMETER OF SHAFT	NUMBER OF KEYS	DIAMETER OF SHAFT	NUMBER OF KEYS
5/16–3/8	1	7/8–5/16	6, 8, 10	1 3/8–1 7/16	14, 17, 20
7/16–1/2	2, 4	1	9, 11, 13	1 1/2–1 5/8	15, 18, 21, 24
9/16–5/8	3, 5	1 1/16–1 1/8	9, 11, 13, 16	1 11/16–1 3/4	18, 21, 24
11/16–3/4	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1 1/4–1 5/16	12, 14, 17, 20	2 1/16–2 1/2	25

WOODRUFF STANDARD KEYS TO USE WITH VARIOUS DIAMETER SHAFTS.

For heavy work the ordinary key will not answer: it is often necessary to put in two keys diametrically opposite, and for extremely heavy work, what is known as the "Kennedy" key, Fig. 89, is used. This is the only key that will answer the requirements of rolling-mill work. At the armor-plate mill of the Carnegie plant this type of key is used in the 22-inch shaft of rolls that reverse on an average of 20 times per minute. No other key would stand up to this work. These keys are made approximately one-quarter of the shaft diameter, and located in the gear so that the corners of the keys intersect the bore. It is not necessary for the bottoms of the keys to be on a vertical line. The keys are made to a taper of 1/8 inch per foot on the top for a driving fit,

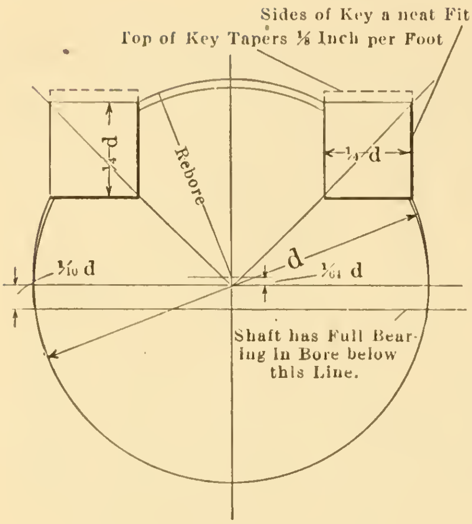


FIG. 89. THE KENNEDY KEY.

the sides being just a neat fit. The shaft is first bored for a press fit, then rebored about 1/64 of the shaft diameter off center; the keyways are cut in the eccentric side. That portion of the bore opposite the keys remains as originally bored to within one-tenth of the shaft diameter below the center line.

The "Kennedy" key is especially desirable where it is necessary to move the gear for any considerable distance on the shaft before securing.

For street-railway work the gear is often pressed on the axle, not using a key of any description.

Where a sliding gear is used for heavy work, three keys, having radial sides as illustrated by Fig. 90, are generally employed. An example of this is the

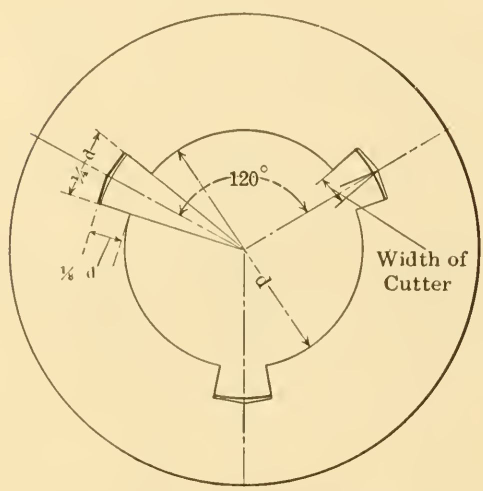


FIG. 90. KEYWAYS FOR HEAVY SLIDING GEAR.

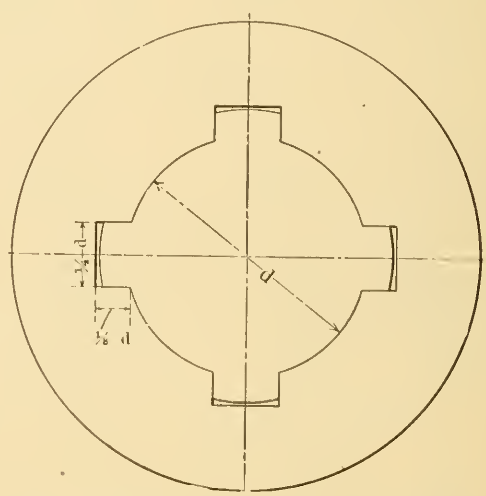


FIG. 91. FOUR-KEYED SLIDING GEAR.

gear drive for vertical rolls. This style of key has a distinct advantage over the four-key type with parallel sides as illustrated by Fig. 91.

BORE

The bore of a gear is supposed to be standard, any allowance for a fit being made in the shaft. This follows out the practice of the manufacturers of cold-rolled shafting, that is, to make the shaft enough under size for a sliding fit in a gear or pulley which is bored standard.

An exception to this is when press or shrink fits are desired. In this case the allowance is made in the gear, the shaft being turned standard.

PRESSURES AND ALLOWANCES FOR FORCE FITS

The Lane & Bodley Company of Cincinnati, O., furnishes the following data. For several years this firm has been keeping a record of observations on press fits with a view to making an analysis of them when a sufficient body of data had been accumulated, and thus obtaining a guide for future practice. Hundreds of cases of such press fits have been recorded, forming a body of data which is probably unequaled. See Chart 9.

MEASUREMENTS AND PRESSURE READINGS

In these records the measurements have been made with great thoroughness. Both plug and hole have been measured on two diameters and at both ends, the average of these micrometer readings being taken as the true diameter. The pressures have been read at the beginning, middle, and end of the length of the fit; the material of both plug and ring, the length of the fit, the radial thickness of the hub, the areas and volumes of the fitted surfaces, and some other minor points have been noted, 24 entries being regularly made for each case. The resulting chart will thus be seen to have a very broad foundation.

In ordinary cases the quantities which are fixed by the conditions are the nominal diameter and length of the fit, the radial thickness of the hub, and the material. With these given it is required to find the press allowance for a given pressure to force the plug home.

HOW THE PRESSURE VARIES

Regarding the influence of these various factors, the pressure varies:

1. Directly as the surface of the fit for a given diameter.

2. Directly as the press-fit allowance, this allowance being such as not to stretch the metal beyond the elastic limit.

3. As some function of the radial thickness of the hub, which, while not determined mathematically, is shown in the chart.

4. With the materials used in a manner not yet determined owing to insufficient data. The chart is for steel or iron shafts and cast-iron cranks. Cast-steel cranks require much heavier pressures for the same press-fit allowances, but how much heavier cannot at present be said.

5. With other conditions, which cannot be formulated, and which lead to erratic results in the observations make it impossible to formulate a rule or construct a chart which shall give other than approximate results.

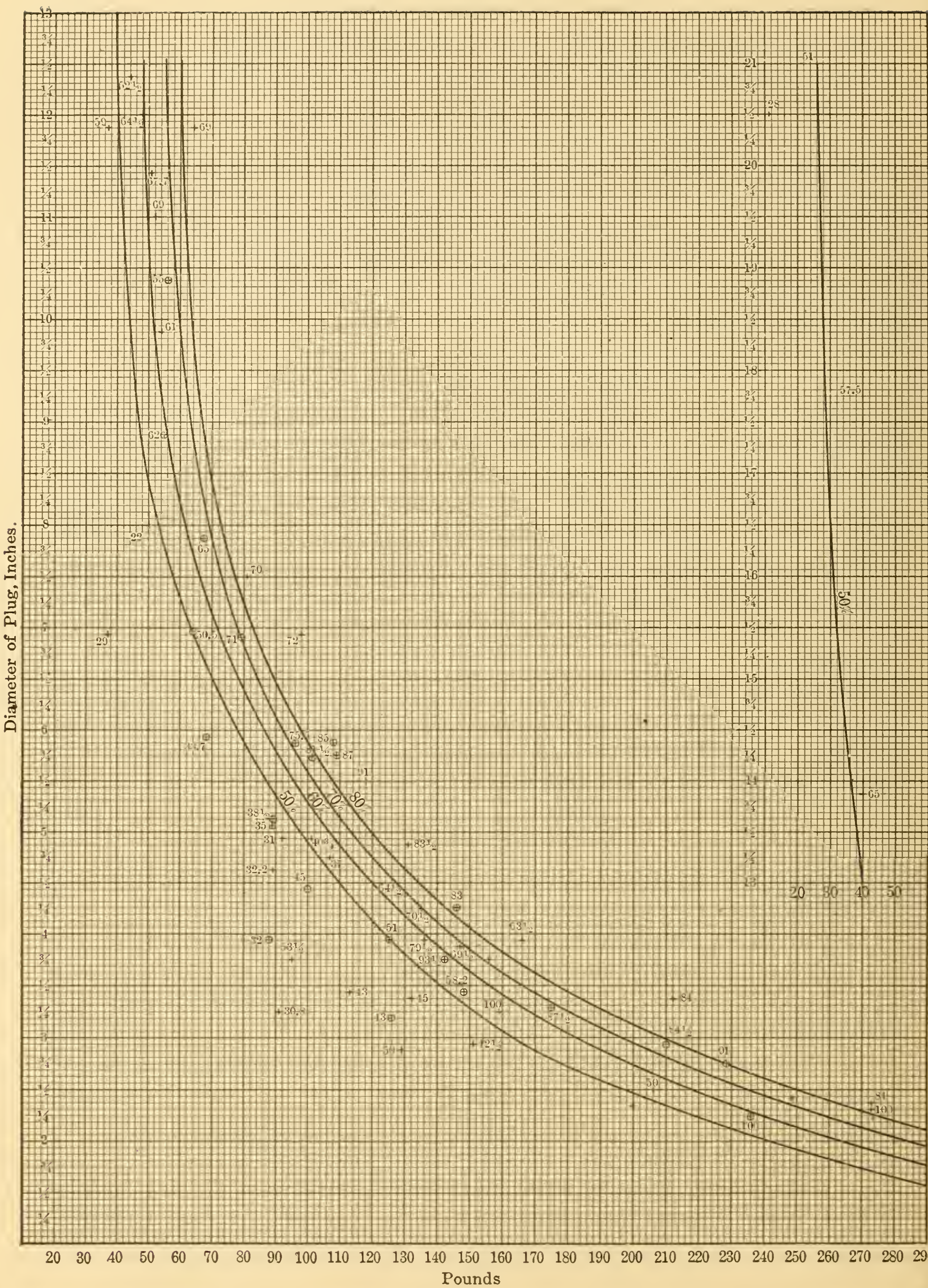


CHART 9. FORCE-FIT RELATIONS SHOWING LANE & BODLEY PRACTICE.

VARYING CONDITIONS

Among these are the nature of the surfaces as regards smoothness, the varying character of materials going under the same name, the shape of the crank and the speed with which the work is done. The term cast iron includes materials of widely varying hardness and other properties, and it is apparent that the web of a disk crank would have an influence not expressed by the radius of the hub. If a counterbalance were cast in the disk, this and the crank arm would naturally produce an effect on the effective thickness of the hub which would be different from the effect of the arm alone on a plain crank. Again, with a plain crank, the arm, being taper, would reinforce a greater arc of the pin eye than of the shaft eye.

STILL ANOTHER FACTOR

Another factor, which no doubt introduces some of the discrepancies of the diagram, is that while most of the shafts were of steel, some were of wrought iron, and no discrimination between these materials has been made in the analysis. These considerations explain the erratic results obtained, but it is nevertheless plain that the observations follow the general direction of the curves in a very marked manner.

AVERAGES SHOWN BY THE DIAGRAM

The plotted observations, it should be remarked, are in most cases the averages of many. The figures attached give the percentage of radial hub thickness to plug diameter. It will be observed in this connection that the discrepancies grow less as the diameters increase. This is doubtless chiefly due to the fact that the percentage of error is always greater with small experiments than with large. Its effect is to give increased value to the diagram when used with large sizes where it is most needed.

THE PROBLEM SOLVED BY THE CURVES

Of course, the holding power of these fits is the real thing desired, but it is obvious that the probability of adequate experiments being undertaken to determine this in large sizes is slight. The problem as it presents itself in the shop calls for the determination of the press-fit allowance to give a required pressure in forcing the parts home, and this the present diagram solves with a degree of accuracy sufficient for most purposes.

In order to reduce the size of the diagram, the portion applying to diameters above 13 inches has been detached and placed at the right. For these large

sizes the observations are too few to justify the drawing of more than one curve. The lubricant used in all cases was linseed oil.

DIRECTIONS FOR USE

Select the curve which gives the ratio of the radial thickness of the hub divided by the diameter of the plug. Below the point of intersection of the plug diameter line with the selected curve, read pounds. Multiply this reading by the area of the fitted surface in square inches, and by the number of thousandths of an inch allowed for the press fit. The result will be the pressure in pounds carried to force the plug home.

AN EXAMPLE

The following example illustrates the use of the diagram: Diameter of plug, 8 inches, length of fit, 6 inches, diameter of hub, 16 inches, press-fit allowance, 0.020 inch. Required, the pressure to force the parts together.

$$\frac{\textit{Radial thickness of hub} = 4 \textit{ inches}}{\textit{Diameter of plug} = 8 \textit{ inches}} = 0.5.$$

Finding a point on the 50 per cent. curve opposite 8 inches, and tracing downward, we find 52 pounds. Area of fitted surface = 8 × 3.1416 × 6 = 150 square inches, 52 × 150 × 20 = 156,000 pounds = 78 tons.

SECTION V

Bevel Gears

This discussion is based on the notations and formulas found on pages 143 and 144. Also see Fig. 93.

The tangent of the center angle of a bevel gear is found by dividing the number of teeth in the gear by the number of teeth in the pinion. Since the sum of the center angles must be 90 degrees, the angle of pinion is 90 degrees less the center angle of the gear.

The face angle is found by adding the angle increment to the center angle. This result is the angle from the center line of the bore. If the gears are to be turned in a lathe this angle should be subtracted from 90 degrees, as the opposite, or complement of this angle is required. See Fig. 92. When the face angle is not shown on the sketch, it is always understood to be given in this way. Most all reference tables give the face angle in the same manner.

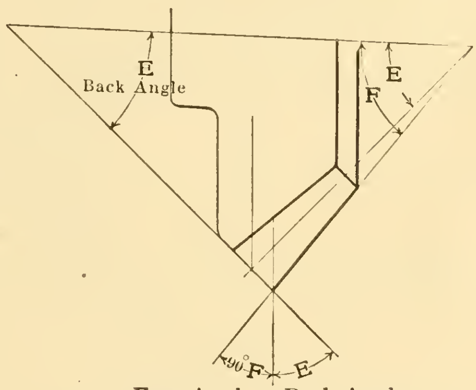


FIG. 92. LOCATION OF FACE AND BACK ANGLES.

When bevel gears are to be milled, it is common practice to obtain the cutting angle by subtracting the angle increment instead of the angle decrement. This makes the face angle of the gear the same as the cutting angle of the pinion, and makes the clearance the same at both ends of the tooth. The back and center angles are the same. These terms are used for turning the gears.

The cutting angle is found by subtracting the angle decrement from the center angle.

The tangent of the angle increment is found by dividing twice the sine of the center angle by the number of teeth. This formula, however, can only be used for gears having an addendum of $\frac{1}{p}$, or 0.3183 p' (Brown & Sharpe standard). When special teeth are used the angle increment must be found

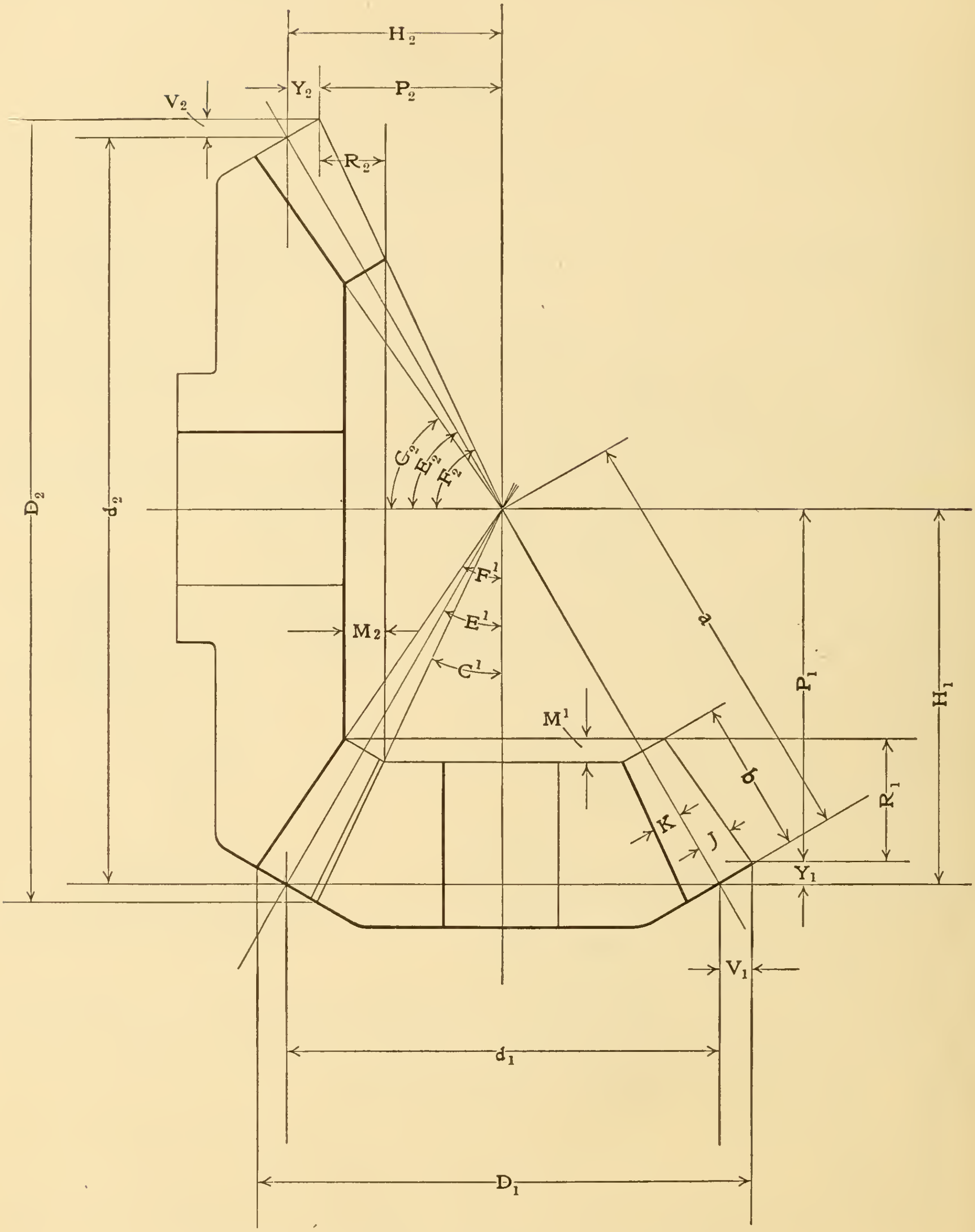


FIG. 93. DIAGRAM AND NOTATION FOR BEVEL GEAR

by dividing the addendum by the apex distance $\left(\frac{s}{a}\right)$. The angle increment and angle decrement are the same in both gears of a pair.

The angle decrement is equal the angle increment plus the clearance $\left(\frac{s-f}{a}\right)$. For gears having an addendum of $\frac{1}{p}$ the angle decrement is found by dividing the product of 2.314 and the sine of the center angle by the number of teeth.

The diameter increment is found by multiplying the addendum by the sine of the center angle. This gives the amount to be added to the pitch diameter *on one side.*

The distance from the pitch line to the point of the tooth is called the backing. This amount added to the distance from the back of the hub to the pitch line will give the distance from the back of the hub to the point of the tooth, which is required in order to correctly turn the gear.

The outside diameter of a bevel gear is found by adding to the pitch diameter twice the value of the diameter increment. It will be noted that the diameter increment of the gear is also the backing of the pinion, and that the backing of the gear is the diameter increment of the pinion, and *vice versa.* This will save time in making these calculations.

The apex distance is found by dividing the pitch diameter by twice the sine of the center angle. It is the same for both gears of a pair.

The distance P from the axis of the mating gear to the point of tooth at large end of tooth is found by subtracting the backing V from one half the pitch diameter of the mating gear.

When turning gears in large quantities, considerable time may be saved if the length of the face is given measured parallel with the center line of the bore. This distance R is found by multiplying the face by the cosine of the face angle.

The distance H is simply one-half the pitch diameter of the mating gear.

Time will be saved in turning bevel gears if the depth at the front end can be given. This will allow the workman to finish the face, front end, and bore of the gear during the first operation while it is held in the chuck by the back hub. The factor 0.2 in the formula will give a space of 0.2 inch per inch circular pitch at the small end below the bottom of the tooth. This may be varied to suit different requirements by changing the constant 0.2 in the formula.

This does not hold good if the length through the hub as well as the backing is specified, as the hub may in some cases extend beyond the points of the

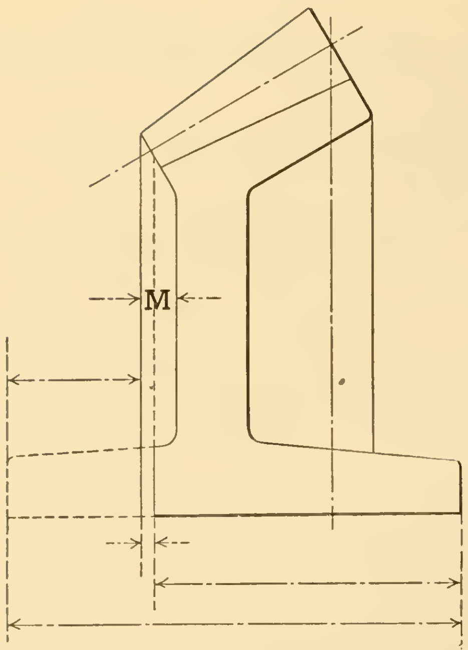


FIG. 94. DEPTH OF FRONT OF RIM.

teeth at small end. But M should never be less than the distance given by this formula, otherwise there will not be room for a full tooth at the small end. See Fig. 94.

The pitch diameter at the small end of teeth may be found by subtracting from the pitch diameter at the large end twice the product of the face and the sine of the center angle.

The outside diameter at the small end is found by subtracting from the outside diameter at the large end twice the product of the face and the sine of the face angle.

The number of teeth for which the cutter should be selected is found by dividing the number of teeth in the gear or pinion by the cosine of its center angle.

The thickness of the tooth at the small end is found by dividing the product of the thickness of the tooth at the large end and the difference between the apex distance and the face by the apex distance. Or:

$$a : t :: a-b : t'. \quad \text{Therefore } t' = \frac{t\,(a-b)}{a}.$$

All dimensions for the small end of the tooth may be determined in this proportion.

NOTATION AND FORMULAS FOR MITRE GEARS

Center angle.	$E = 45$ degrees.
Face angle.	$F = 45$ deg. $+ J$.
Cutting angle.	$C = 45$ deg. $- K$.
Angle increment.	$Tan.\ J = \frac{s}{a}$, or $\frac{1.414}{N}$ when $s = \frac{1}{p}$.
Angle increment.	$Tan.\ K = \frac{s+f}{a}$, or $\frac{1.636}{N}$ when $s = \frac{1}{p}$.
Diameter increment.	$V = s\ 0.70711$.
Backing.	$Y = s\ 0.70711$.
Outside diameter.	$D = d + 2V$.
Apex distance.	$a = \frac{d}{0.1414}$, or $d\ 0.70711$.
Distance from apex to point of tooth—large end. Parallel with axis.	$P = \frac{d}{2} - Y$.

Face, measured parallel with axis. $R = b \text{ cosine } F.$

Depth of rim at small end. $M = \dfrac{W + 0.2\, p'\, (a-b)}{a}\, 0.70711.$

Pitch diameter at small end. $d^{s\prime} = d - 2\,(b\ 0.70711).$

Outside diameter at small end. $D^s = D - 2\,(b \text{ sine } F).$

Number of teeth for which cutter should be selected. $= \dfrac{N}{0.70711}.$

Thickness of tooth at small end. $t' = \dfrac{t\,(a-b)}{a}.$

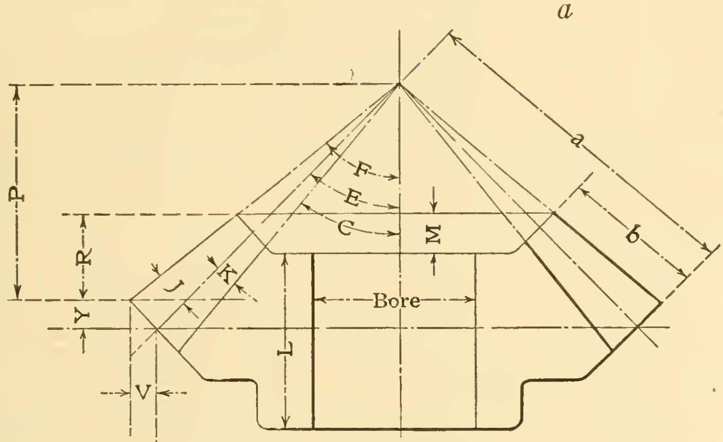


FIG. 95. MITRE GEAR.

NOTATION FOR BEVEL GEARS

AXES AT 90 DEGREES

E = center angle.
F = face angle.
C = cutting angle.
J = angle increment.
K = angle decrement.
V = diameter increment.
Y = backing, or distance from point of tooth to pitch line.
d = pitch diameter.
D = outside diameter.
p' = circular pitch.
p = diametral pitch.
d^s = pitch diameter at small end.
D^s = outside diameter at small end.
t = thickness of tooth at largest pitch diameter.
t' = thickness of tooth at smallest pitch diameter.
a = apex distance.
H = distance from pitch line to apex.

GEAR		PINION		REMARKS
REQ.	FORMULA	REQ.	FORMULA	
E_2	$Tan.\ E_2 = \frac{N_2}{N_1}$	E_1	$90° - E_2$	
F_2	$E_2 + J$	F_1	$E_1 + J$	Subtract from 90° for use in lathe.
C_2	$E_2 - K$	C_1	$E_1 - K$	
J	$Tan.\ J = \frac{2\ sine\ E_2}{N_2}$	J	$Tan.\ J = \frac{2\ sine\ E_1}{N_1}$	Same for both gear and pinion.
K	$Tan.\ K = \frac{2.314\ sine\ E_2}{N_2}$	K	$Tan.\ K = \frac{2.314\ sine\ E_1}{N_1}$	Same for both gear and pinion.
V_2	$S\ cos.\ E_2$	V_1	$S\ cos.\ E_1$	V_1 or pinion same as Y_2 for gear.
Y_2	$S\ sine\ E_2$	Y_1	$S\ sine\ E_1$	Y_1 for pinion same as V_2 for gear.
D_2	$d_2 + 2V_2$	D_1	$d_1 + 2V_1$	
a	$\frac{d_2}{2\ sine\ E_2}$	a	$\frac{d_1}{2\ sine\ E_1}$	Same per both gear and pinion.
P_2	$\frac{d_1}{2} - Y_2$	P_1	$\frac{d_2}{2} - Y_1$	
R_2	$b\ cos.\ F_2$	R_1	$b\ cos.\ F_1$	
H_2	$\frac{d_1}{2}$	H_1	$\frac{d_2}{2}$	
M_2	$\frac{W' + 0.2\ p'\ (a - b)}{a}\ sine\ E_2$	M_1	$\frac{W' + 0.2\ p'\ (a - b)}{a}\ sine\ E_1$	
d_2^s	$d_2 - 2\ (b\ sine\ E_2)$	d_1^s	$d_1 - 2\ (b\ sine\ E_1)$	
D_2^s	$D_2 - 2\ (b\ sine\ F_2)$	D_1^s	$D_1 - 2\ (b\ sine\ F_1)$	
Cutter$_2$	$\frac{N_2}{cos.\ E_2}$	Cutter$_1$	$\frac{N_1}{cos.\ E_1}$	
t'	$\frac{t\ (a - b)}{a}$			Same for both gear and pinion.

FORMULAS FOR BEVEL GEARS, SHAFTS AT 90 DEGREES

P = distance from point of tooth to axis of mating gear.
R = face measured parallel with axis.
M = depth of rim at front end.
b = face.
s = addendum.
W' = whole depth.
N = number of teeth.

FORMULAS FOR BEVEL GEARS AT ANY ANGLE

Many formulas have heretofore been published for determining the angles and dimensions of bevel gears not at right angles, all of which are more or less confusing. It is simply a matter of finding the center angles; all other angles and dimensions are obtained as for ordinary bevel gears at right angles, each gear being figured separately. This also applies to the use of tables, the table being entered for each gear separately, according to its center angle and independent of its mate.

When the axes are not at right angles there are four other combinations; axes less than 90 degrees, see Fig. 96; axes greater than 90 degrees, Fig. 97; crown bevel gears, Fig. 98; and internal bevel gears, Fig. 99. The center angles for these gears are found as follows:

N_2 = number of teeth in gear.
N_1 = number of teeth in pinion.

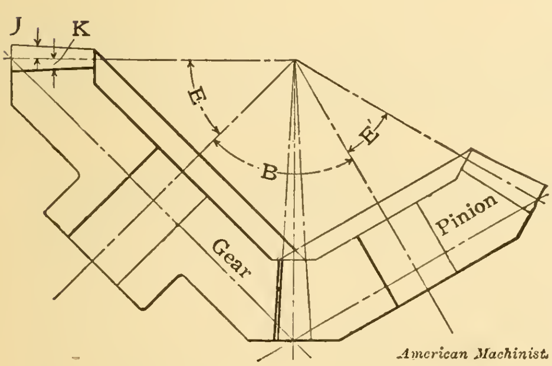


FIG. 96. BEVEL GEARS WITH AXES LESS THAN 90 DEGREES.

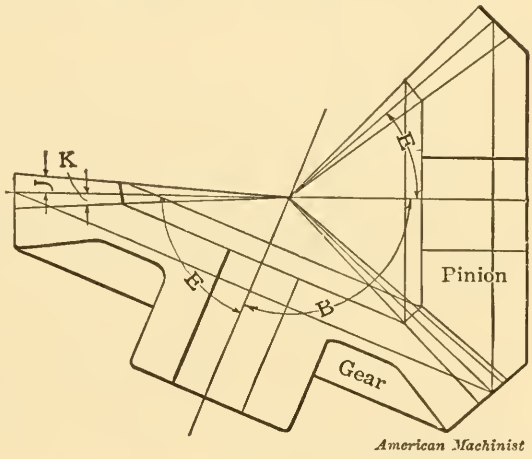


FIG. 97. BEVEL GEARS WITH AXES GREATER THAN 90 DEGREES.

$$Tan.\ E = \frac{sine\ B}{\frac{N_1}{N_2} + cos.\ B}.$$

$$E' = B - E.$$

$$Tan.\ E = \frac{sine\ (180 - B)}{\frac{N_1}{N_2} - cos.\ (180 - B)}.$$

$$E' = B - E.$$

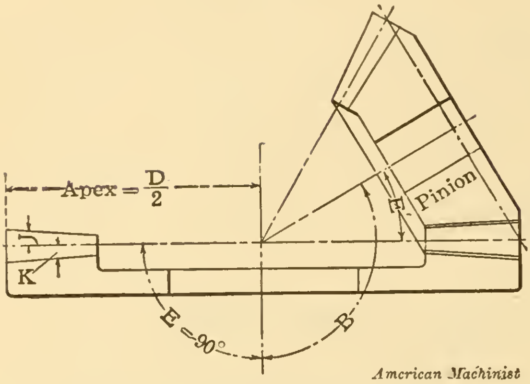


FIG. 98. CROWN BEVEL GEARS.

$$E = 90°.$$
$$E' = B - 90.$$

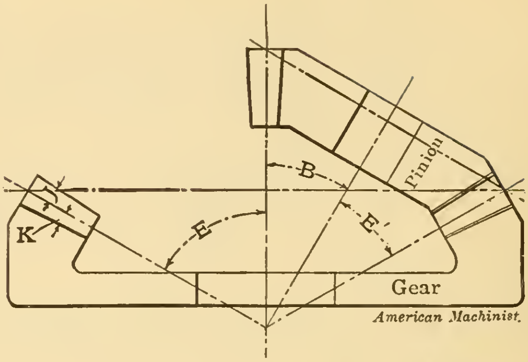


FIG. 99. INTERNAL BEVEL GEARS.

$$Tan.\ E = \frac{sine\ B}{sine\ B - \frac{N_1}{N_2}}.$$
$$E' = E - B.$$

USE OF BEVEL GEAR TABLE

To use the bevel gear table, Table 23, divide the number of teeth in pinion by the number of teeth in gear; the quotient will equal the tangent of center angles. Find the nearest number in the table of tangents and on the same line on the left will be found the degrees, and at the top of the column hundredth degrees for the center angle of pinion. On the same line of the right will be found the degrees, and at the bottom of the column hundredth degrees for gear.

On the same line on the left will be found the angle increase, which, when divided by the number of teeth in the pinion, will give as a quotient the angle increase for either wheel.

To obtain the face angles add the angle increase to the center angle, and to obtain the cutting angle subtract the angle decrement from the center angle. Now on the same line on the left will be found the diameter increase for the pinion, and on the same line on the right will be found the diameter increase for the gear. These when divided by the required diametral pitch, equal the diametral increase for that pitch, which, added to the pitch diameters, give the outside diameters.

AN ILLUSTRATIVE EXAMPLE

In a pair of bevel gears, 24 and 72 teeth, 8 diametral pitch; 24 divided by 72 = 0.3333, which is the tangent of the center angles. The nearest tangent in the table is 0.3346, which gives:

Center angle of pinion, 18.50 degrees.
Center angle of gear, 71.50 degrees.

On the same line at the left will be found the angle increase, 36, which divided by the number of teeth in the pinion will give the angle increase $\frac{36}{24} = 1.5$ degrees. This angle added to the center angle will give the face angle.

The cutting angle is found by subtracting this angle, plus 16 per cent., from the center angle, which in this example would be $1.50 \times 0.16 = 0.24$, therefore the angle increment would be $1.50 + 0.24 = 1.74$ degrees.

Face angle of pinion $= 18.50 + 1.50 = 20.00$ degrees.
Cutting angle of pinion $= 18.50 - 1.74 = 16.76$ degrees.
Face angle of gear $= 71.50 + 1.50 = 73.00$ degrees.
Cutting angle of gear $= 71.50 - 1.74 = 69.76$ degrees.

On the same line to the left will be found the diameter increase for the pinion, 1.90, which divided by the pitch, or $\frac{1.90}{8}$, $= 0.237$ inches. On the right of this same line will be found the diameter increase for the gear, 0.65, which divided by the pitch, or $\frac{0.65}{8}$, $= 0.081$ inches.

The pitch diameter of the pinion is $\frac{24}{8} = 3$ inches. The pitch diameter of the gear is $\frac{72}{8} = 9$ inches.

Outside diameter of gear $= 9 + 0.081 = 9.081$ inches.
Outside diameter of pinion $= 3 + 0.237 = 3.237$ inches.

For gears not at right angles, first determine the center angle, and enter the table for each gear separately.

Number of Teeth in Wheel

		42	41	40	39	38	37	36	35	34	33	32	31	30	29	28
12	J	2° 38′	2° 42′	2° 45′	2° 48′	2° 53′	2° 57′	3° 2′	3° 6′	3° 11′	3° 15′	3° 21′	3° 27′	3° 33′	3° 38′	3° 45′
	K	3° 2′	3° 6′	3° 10′	3° 15′	3° 19′	3° 24′	3° 29′	3° 35′	3° 40′	3° 46′	3° 52′	3° 59′	4° 6′	4° 13′	4° 21′
13	J	2° 37′	2° 40′	2° 44′	2° 47′	2° 52′	2° 56′	3° 0′	3° 4′	3° 9′	3° 14′	3° 19′	3° 24′	3° 29′	3° 36′	3° 43′
	K	3° 1′	3° 5′	3° 9′	3° 13′	3° 18′	3° 23′	3° 28′	3° 33′	3° 38′	3° 44′	3° 50′	3° 56′	4° 3′	4° 10′	4° 17′
14	J	2° 35′	2° 38′	2° 43′	2° 46′	2° 50′	2° 54′	2° 58′	3° 3′	3° 7′	3° 12′	3° 17′	3° 22′	3° 27′	3° 33′	3° 39′
	K	2° 59′	3° 4′	3° 7′	3° 12′	3° 16′	3° 21′	3° 26′	3° 31′	3° 36′	3° 42′	3° 47′	3° 54′	4° 0′	4° 7′	4° 13′
15	J	2° 35′	2° 38′	2° 41′	2° 44′	2° 48′	2° 53′	2° 56′	3° 1′	3° 4′	3° 10′	3° 14′	3° 19′	3° 24′	3° 30′	3° 36′
	K	2° 58′	3° 2′	3° 6′	3° 10′	3° 14′	3° 19′	3° 24′	3° 29′	3° 34′	3° 39′	3° 45′	3° 51′	3° 57′	4° 4′	4° 10′
16	J	2° 34′	2° 37′	2° 40′	2° 43′	2° 47′	2° 51′	2° 55′	2° 58′	3° 3′	3° 7′	3° 12′	3° 17′	3° 22′	3° 27′	3° 33′
	K	2° 57′	3° 1′	3° 4′	3° 8′	3° 13′	3° 17′	3° 22′	3° 26′	3° 31′	3° 37′	3° 42′	3° 48′	3° 54′	4° 0′	4° 6′
17	J	2° 33′	2° 35′	2° 38′	2° 42′	2° 45′	2° 49′	2° 53′	2° 57′	3° 1′	3° 5′	3° 9′	3° 14′	3° 18′	3° 24′	3° 29′
	K	2° 56′	2° 59′	3° 3′	3° 7′	3° 11′	3° 15′	3° 20′	3° 24′	3° 27′	3° 34′	3° 39′	3° 45′	3° 50′	3° 56′	4° 2′
18	J	2° 31′	2° 33′	2° 37′	2° 40′	2° 43′	2° 47′	2° 51′	2° 55′	2° 58′	3° 3′	3° 7′	3° 12′	3° 16′	3° 21′	3° 26′
	K	2° 54′	2° 58′	3° 1′	3° 5′	3° 9′	3° 13′	3° 17′	3° 22′	3° 26′	3° 31′	3° 36′	3° 42′	3° 47′	3° 53′	3° 57′
19	J	2° 30′	2° 32′	2° 35′	2° 38′	2° 42′	2° 45′	2° 49′	2° 53′	2° 57′	3° 0′	3° 4′	3° 9′	3° 13′	3° 18′	3° 23′
	K	2° 52′	2° 56′	2° 59′	3° 3′	3° 7′	3° 12′	3° 16′	3° 19′	3° 24′	3° 29′	3° 33′	3° 38′	3° 44′	3° 49′	3° 55′
20	J	2° 29′	2° 30′	2° 33′	2° 37′	2° 40′	2° 43′	2° 47′	2° 50′	2° 54′	2° 58′	3° 2′	3° 6′	3° 10′	3° 15′	3° 19′
	K	2° 50′	2° 54′	2° 58′	3° 1′	3° 5′	3° 9′	3° 14′	3° 17′	3° 21′	3° 26′	3° 31′	3° 35′	3° 40′	3° 45′	3° 51′
21	J	2° 28′	2° 28′	2° 32′	2° 35′	2° 38′	2° 42′	2° 45′	2° 48′	2° 52′	2° 56′	3° 0′	3° 3′	3° 8′	3° 12′	3° 17′
	K	2° 49′	2° 52′	2° 56′	2° 59′	3° 3′	3° 7′	3° 11′	3° 15′	3° 19′	3° 23′	3° 27′	3° 32′	3° 36′	3° 41′	3° 47′
22	J	2° 26′	2° 27′	2° 30′	2° 33′	2° 37′	2° 40′	2° 43′	2° 46′	2° 49′	2° 53′	2° 57′	3° 0′	3° 4′	3° 8′	3° 13′
	K	2° 48′	2° 51′	2° 54′	2° 58′	3° 1′	3° 5′	3° 8′	3° 12′	3° 16′	3° 20′	3° 24′	3° 29′	3° 32′	3° 38′	3° 43′
23	J	2° 25′	2° 26′	2° 28′	2° 32′	2° 35′	2° 38′	2° 42′	2° 44′	2° 47′	2° 51′	2° 54′	2° 58′	3° 2′	3° 5′	3° 9′
	K	2° 46′	2° 49′	2° 52′	2° 56′	2° 59′	3° 2′	3° 6′	3° 10′	3° 14′	3° 18′	2° 22′	3° 26′	3° 30′	3° 34′	3° 39′
24	J	2° 24′	2° 24′	2° 27′	2° 30′	2° 33′	2° 36′	2° 39′	2° 42′	2° 45′	2° 48′	2° 52′	2° 55′	2° 58′	3° 2′	3° 6′
	K	2° 44′	2° 47′	2° 51′	2° 54′	2° 58′	3° 1′	3° 4′	3° 7′	3° 11′	3° 15′	3° 19′	3° 23′	3° 27′	3° 31′	3° 35′
25	J	2° 22′	2° 23′	2° 26′	2° 28′	2° 31′	2° 34′	2° 37′	2° 40′	2° 43′	2° 46′	2° 49′	2° 53′	2° 56′	2° 59′	3° 3′
	K	2° 43′	2° 46′	2° 49′	2° 52′	2° 56′	2° 58′	3° 1′	3° 4′	3° 8′	3° 12′	3° 16′	3° 20′	3° 24′	3° 28′	3° 32′
26	J	2° 20′	2° 22′	2° 24′	2° 26′	2° 29′	2° 32′	2° 35′	2° 38′	2° 40′	2° 43′	2° 47′	2° 49′	2° 53′	2° 56′	3° 0′
	K	2° 41′	2° 44′	2° 46′	2° 49′	2° 54′	2° 56′	2° 59′	3° 2′	3° 6′	3° 9′	3° 13′	3° 16′	3° 20′	3° 24′	3° 28′
27	J	2° 19′	2° 19′	2° 22′	2° 24′	2° 27′	2° 30′	2° 33′	2° 36′	2° 38′	2° 41′	2° 44′	2° 47′	2° 50′	2° 53′	2° 57′
	K	2° 39′	2° 42′	2° 44′	2° 47′	2° 51′	2° 53′	2° 57′	3° 0′	3° 3′	3° 6′	3° 10′	3° 13′	3° 17′	3° 20′	3° 24′
28	J	2° 17′	2° 18′	2° 21′	2° 23′	2° 26′	2° 28′	2° 31′	2° 33′	2° 36′	2° 39′	2° 42′	2° 44′	2° 47′	2° 51′	2° 53′
	K	2° 37′	2° 40′	2° 43′	2° 45′	2° 49′	2° 51′	2° 54′	2° 58′	3° 0′	3° 3′	3° 7′	3° 10′	3° 14′	3° 17′	3° 21′
29	J	2° 16′	2° 17′	2° 19′	2° 21′	2° 24′	2° 26′	2° 29′	2° 31′	2° 34′	2° 37′	2° 39′	2° 42′	2° 44′	2° 47′	—
	K	2° 36′	2° 38′	2° 41′	2° 44′	2° 47′	2° 49′	2° 52′	2° 54′	2° 57′	3° 1′	3° 5′	3° 7′	3° 10′	3° 14′	
30	J	2° 14′	2° 15′	2° 17′	2° 19′	2° 22′	2° 24′	2° 27′	2° 29′	2° 32′	2° 34′	2° 36′	2° 39′	2° 42′	—	
	K	2° 34′	2° 36′	2° 39′	2° 42′	2° 44′	2° 46′	2° 49′	2° 52′	2° 55′	2° 58′	3° 1′	3° 4′	3° 7′		
31	J	2° 13′	2° 13′	2° 16′	2° 18′	2° 20′	2° 22′	2° 24′	2° 27′	2° 29′	2° 32′	2° 34′	2° 37′	—		
	K	2° 32′	2° 34′	2° 37′	2° 39′	2° 42′	2° 44′	2° 47′	2° 50′	2° 53′	2° 56′	2° 58′	3° 1′			
32	J	2° 11′	2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 23′	2° 25′	2° 27′	2° 29′	2° 32′	—			
	K	2° 31′	2° 33′	2° 35′	2° 37′	2° 40′	2° 43′	2° 45′	2° 48′	2° 51′	2° 53′	2° 56′				
33	J	2° 10′	2° 11′	2° 12′	2° 14′	2° 16′	2° 18′	2° 21′	2° 23′	2° 25′	2° 27′	—				
	K	2° 29′	2° 31′	2° 33′	2° 36′	2° 38′	2° 40′	2° 42′	2° 45′	2° 48′	2° 51′					
34	J	2° 9′	2° 8′	2° 11′	2° 12′	2° 14′	2° 17′	2° 18′	2° 21′	2° 23′	—					
	K	2° 27′	2° 29′	2° 31′	2° 34′	2° 36′	2° 37′	2° 40′	2° 42′	2° 45′						
35	J	2° 7′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	—						
	K	2° 25′	2° 27′	2° 30′	2° 32′	2° 34′	2° 35′	2° 38′	2° 40′							
36	J	2° 6′	2° 6′	2° 8′	2° 9′	2° 11′	2° 13′	2° 15′	—							
	K	2° 24′	2° 26′	2° 28′	2° 30′	2° 32′	2° 34′	2° 36′								
37	J	2° 5′	2° 4′	2° 6′	2° 8′	2° 9′	2° 12′	—								
	K	2° 22′	2° 24′	2° 26′	2° 28′	2° 30′	2° 32′									
38	J	2° 3′	2° 2′	2° 4′	2° 6′	2° 7′	—									
	K	2° 20′	2° 22′	2° 24′	2° 25′	2° 27′										
39	J	2° 1′	2° 1′	2° 3′	2° 5′	—										
	K	2° 18′	2° 20′	2° 22′	2° 23′											
40	J	1° 59′	1° 59′	2° 1′	—											
	K	2° 17′	2° 18′	2° 20′												
41	J	1° 58′	1° 58′	—												
	K	2° 15′	2° 16′													
42	J	1° 57′	—													
	K	2° 13′														

TABLE 22—ADDENDUM AND DEDENDUM ANGLES FOR BEVEL GEARS. ANGLE BETWEEN

Number of Teeth in Wheel

Number of Teeth in Pinion	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
12	3° 53′	4° 0′	4° 7′	4° 16′	4° 25′	4° 34′	4° 43′	4° 55′	5° 5′	5° 16′	5° 30′	5° 42′	5° 56′	6° 10′	6° 27′	6° 43′
	4° 29′	4° 37′	4° 46′	4° 56′	5° 6′	5° 16′	5° 28′	5° 40′	5° 53′	6° 6′	6° 22′	6° 36′	6° 52′	7° 9′	7° 27′	7° 46′
13	3° 49′	3° 57′	4° 3′	4° 12′	4° 20′	4° 29′	4° 38′	4° 48′	4° 58′	5° 8′	5° 20′	5° 32′	5° 45′	5° 57′	6° 12′	
	4° 25′	4° 33′	4° 42′	4° 51′	5° 0′	5° 10′	5° 21′	5° 32′	5° 43′	5° 57′	6° 10′	6° 24′	6° 39′	6° 54′	7° 10′	
14	3° 46′	3° 54′	3° 59′	4° 7′	4° 15′	4° 24′	4° 32′	4° 40′	4° 50′	5° 0′	5° 11′	5° 22′	5° 37′	5° 45′		
	4° 21′	4° 29′	4° 37′	4° 46′	4° 54′	5° 4′	5° 14′	5° 25′	5° 36′	5° 48′	6° 0′	6° 11′	6° 24′	6° 43′		
15	3° 43′	3° 49′	3° 55′	4° 2′	4° 9′	4° 18′	4° 26′	4° 34′	4° 43′	4° 52′	5° 2′	5° 12′	5° 22′			
	4° 16′	4° 24′	4° 32′	4° 41′	4° 49′	4° 58′	5° 7′	5° 17′	5° 27′	5° 38′	5° 50′	6° 2′	6° 14′			
16	3° 39′	3° 45′	3° 52′	3° 58′	4° 5′	4° 14′	4° 20′	4° 28′	4° 36′	4° 45′	4° 53′	5° 3′				
	4° 13′	4° 21′	4° 27′	4° 36′	4° 44′	4° 52′	5° 0′	5° 10′	5° 19′	5° 29′	5° 40′	5° 50′				
17	3° 35′	3° 41′	3° 47′	3° 53′	4° 0′	4° 8′	4° 13′	4° 22′	4° 30′	4° 37′	4° 45′					
	4° 9′	4° 16′	4° 23′	4° 30′	4° 37′	4° 45′	4° 54′	5° 2′	5° 12′	5° 21′	5° 30′					
18	3° 32′	3° 37′	3° 43′	3° 49′	3° 55′	4° 2′	4° 8′	4° 15′	4° 22′	4° 29′						
	4° 4′	4° 10′	4° 18′	4° 25′	4° 32′	4° 39′	4° 47′	4° 55′	5° 3′	5° 12′						
19	3° 28′	3° 33′	3° 38′	3° 44′	3° 50′	3° 56′	4° 2′	4° 8′	4° 15′							
	4° 1′	4° 7′	4° 13′	4° 19′	4° 26′	4° 33′	4° 40′	4° 48′	4° 55′							
20	3° 24′	3° 29′	3° 34′	3° 40′	3° 45′	3° 51′	3° 56′	4° 3′								
	3° 56′	4° 2′	4° 8′	4° 14′	4° 20′	4° 27′	4° 34′	4° 41′								
21	3° 21′	3° 25′	3° 30′	3° 35′	3° 40′	3° 46′	3° 52′									
	3° 53′	3° 58′	4° 3′	4° 8′	4° 14′	4° 21′	4° 27′									
22	3° 17′	3° 22′	3° 26′	3° 31′	3° 36′	3° 41′										
	3° 48′	3° 52′	3° 58′	4° 4′	4° 9′	4° 15′										
23	3° 14′	3° 18′	3° 22′	3° 27′	3° 32′											
	3° 44′	3° 49′	3° 54′	3° 59′	4° 4′											
24	3° 10′	3° 14′	3° 18′	3° 22′												
	3° 39′	3° 44′	3° 49′	3° 54′												
25	3° 7′	3° 10′	3° 14′													
	3° 36′	3° 40′	3° 45′													
26	3° 3′	3° 7′														
	3° 32′	3° 36′														
27	3° 0′															
	3° 28′															
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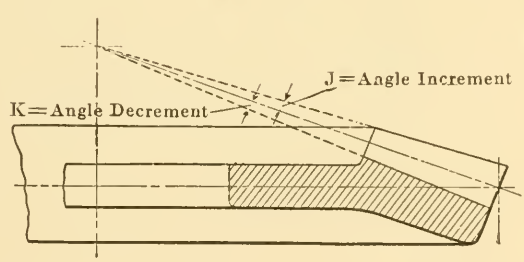


Axes 90°, Tooth Proportions Brown & Sharpe Standard. By F. Withers

Number of Teeth in Wheel

		72	71	70	69	68	67	66	65	64	63	62	61	60	59
12	J	1° 34′	1° 35′	1° 37′	1° 38′	1° 40′	1° 41′	1° 43′	1° 45′	1° 46′	1° 48′	1° 50′	1° 51′	1° 53′	1° 55′
	K	1° 49′	1° 50′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′	2° 0′	2° 2′	2° 4′	2° 6′	2° 8′	2° 10′	2° 12′
13	J	1° 34′	1° 35′	1° 37′	1° 38′	1° 40′	1° 41′	1° 43′	1° 44′	1° 46′	1° 48′	1° 50′	1° 51′	1° 53′	1° 55′
	K	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 2′	2° 4′	2° 5′	2° 7′	2° 9′	2° 12′
14	J	1° 34′	1° 35′	1° 37′	1° 38′	1° 39′	1° 41′	1° 43′	1° 44′	1° 45′	1° 47′	1° 50′	1° 50′	1° 52′	1° 54′
	K	1° 48′	1° 50′	1° 51′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′	2° 7′	2° 9′	2° 11′
15	J	1° 34′	1° 35′	1° 36′	1° 38′	1° 39′	1° 40′	1° 42′	1° 44′	1° 45′	1° 47′	1° 49′	1° 50′	1° 51′	1° 54′
	K	1° 48′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 57′	1° 59′	2° 0′	2° 3′	2° 5′	2° 7′	2° 9′	2° 11′
16	J	1° 33′	1° 35′	1° 36′	1° 37′	1° 38′	1° 40′	1° 42′	1° 43′	1° 45′	1° 46′	1° 48′	1° 50′	1° 51′	1° 53′
	K	1° 47′	1° 49′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′	2° 0′	2° 2′	2° 4′	2° 6′	2° 8′	2° 10′
17	J	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 40′	1° 41′	1° 43′	1° 45′	1° 45′	1° 48′	1° 50′	1° 50′	1° 53′
	K	1° 47′	1° 49′	1° 50′	1° 51′	1° 53′	1° 55′	1° 57′	1° 58′	2° 0′	2° 2′	2° 4′	2° 6′	2° 8′	2° 10′
18	J	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 39′	1° 41′	1° 42′	1° 44′	1° 45′	1° 47′	1° 49′	1° 50′	1° 52′
	K	1° 47′	1° 49′	1° 50′	1° 51′	1° 53′	1° 55′	1° 56′	1° 58′	1° 59′	2° 1′	2° 3′	2° 5′	2° 7′	2° 9′
19	J	1° 33′	1° 34′	1° 35′	1° 36′	1° 37′	1° 39′	1° 40′	1° 42′	1° 43′	1° 45′	1° 47′	1° 48′	1° 50′	1° 52′
	K	1° 47′	1° 48′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 57′	1° 59′	2° 1′	2° 2′	2° 4′	2° 6′	2° 8′
20	J	1° 32′	1° 33′	1° 34′	1° 36′	1° 37′	1° 38′	1° 40′	1° 41′	1° 42′	1° 45′	1° 46′	1° 48′	1° 49′	1° 51′
	K	1° 46′	1° 48′	1° 49′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 0′	2° 2′	2° 4′	2° 6′	2° 8′
21	J	1° 32′	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 40′	1° 41′	1° 42′	1° 44′	1° 45′	1° 47′	1° 49′	1° 50′
	K	1° 46′	1° 47′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′	2° 7′
22	J	1° 32′	1° 33′	1° 34′	1° 35′	1° 36′	1° 38′	1° 39′	1° 40′	1° 42′	1° 44′	1° 45′	1° 47′	1° 48′	1° 50′
	K	1° 46′	1° 47′	1° 48′	1° 49′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 1′	2° 2′	2° 4′	2° 6′
23	J	1° 31′	1° 33′	1° 33′	1° 35′	1° 36′	1° 37′	1° 39′	1° 40′	1° 41′	1° 43′	1° 45′	1° 46′	1° 48′	1° 50′
	K	1° 45′	1° 46′	1° 48′	1° 49′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′	2° 0′	2° 2′	2° 4′	2° 5′
24	J	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 39′	1° 41′	1° 42′	1° 44′	1° 45′	1° 47′	1° 49′
	K	1° 45′	1° 46′	1° 48′	1° 49′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′
25	J	1° 30′	1° 32′	1° 33′	1° 34′	1° 35′	1° 36′	1° 38′	1° 39′	1° 40′	1° 42′	1° 43′	1° 45′	1° 46′	1° 48′
	K	1° 44′	1° 45′	1° 47′	1° 48′	1° 50′	1° 51′	1° 52′	1° 54′	1° 56′	1° 57′	1° 59′	2° 0′	2° 2′	2° 4′
26	J	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 40′	1° 41′	1° 43′	1° 44′	1° 45′	1° 47′
	K	1° 44′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	1° 59′	2° 1′	2° 3′
27	J	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 39′	1° 40′	1° 42′	1° 43′	1° 45′	1° 46′
	K	1° 44′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 51′	1° 53′	1° 55′	1° 56′	1° 58′	1° 59′	2° 1′	2° 2′
28	J	1° 29′	1° 30′	1° 31′	1° 33′	1° 33′	1° 35′	1° 36′	1° 37′	1° 38′	1° 40′	1° 41′	1° 43′	1° 44′	1° 45′
	K	1° 43′	1° 44′	1° 45′	1° 46′	1° 48′	1° 49′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′	2° 0′	2° 1′
29	J	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 36′	1° 38′	1° 39′	1° 40′	1° 42′	1° 44′	1° 45′
	K	1° 42′	1° 43′	1° 45′	1° 46′	1° 47′	1° 48′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 57′	1° 59′	2° 1′
30	J	1° 28′	1° 29′	1° 30′	1° 32′	1° 33′	1° 34′	1° 35′	1° 35′	1° 37′	1° 38′	1° 40′	1° 41′	1° 43′	1° 45′
	K	1° 42′	1° 42′	1° 44′	1° 45′	1° 47′	1° 48′	1° 50′	1° 51′	1° 53′	1° 54′	1° 55′	1° 56′	1° 58′	2° 0′
31	J	1° 28′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 35′	1° 35′	1° 37′	1° 38′	1° 40′	1° 40′	1° 42′	1° 44′
	K	1° 41′	1° 42′	1° 44′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	1° 59′
32	J	1° 27′	1° 28′	1° 30′	1° 30′	1° 32′	1° 33′	1° 34′	1° 34′	1° 36′	1° 38′	1° 39′	1° 40′	1° 41′	1° 43′
	K	1° 41′	1° 42′	1° 43′	1° 44′	1° 45′	1° 47′	1° 48′	1° 50′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′
33	J	1° 27′	1° 28′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 37′	1° 38′	1° 40′	1° 40′	1° 42′
	K	1° 40′	1° 41′	1° 43′	1° 44′	1° 45′	1° 46′	1° 48′	1° 49′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 57′
34	J	1° 26′	1° 27′	1° 29′	1° 30′	1° 30′	1° 32′	1° 33′	1° 33′	1° 35′	1° 36′	1° 38′	1° 39′	1° 40′	1° 41′
	K	1° 40′	1° 41′	1° 42′	1° 43′	1° 45′	1° 46′	1° 47′	1° 48′	1° 50′	1° 51′	1° 52′	1° 53′	1° 55′	1° 56′
35	J	1° 26′	1° 27′	1° 28′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 35′	1° 35′	1° 37′	1° 38′	1° 39′	1° 40′
	K	1° 39′	1° 40′	1° 42′	1° 43′	1° 44′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 56′
36	J	1° 25′	1° 26′	1° 28′	1° 28′	1° 30′	1° 30′	1° 32′	1° 32′	1° 34′	1° 35′	1° 36′	1° 37′	1° 38′	1° 40′
	K	1° 39′	1° 40′	1° 41′	1° 42′	1° 43′	1° 44′	1° 46′	1° 47′	1° 48′	1° 49′	1° 51′	1° 52′	1° 54′	1° 55′
37	J	1° 25′	1° 26′	1° 27′	1° 28′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 36′	1° 38′	1° 39′
	K	1° 38′	1° 39′	1° 40′	1° 41′	1° 43′	1° 44′	1° 45′	1° 46′	1° 47′	1° 48′	1° 50′	1° 51′	1° 53′	1° 54′
38	J	1° 24′	1° 25′	1° 26′	1° 27′	1° 28′	1° 30′	1° 30′	1° 31′	1° 32′	1° 33′	1° 35′	1° 35′	1° 37′	1° 38′
	K	1° 38′	1° 39′	1° 40′	1° 41′	1° 42′	1° 43′	1° 45′	1° 46′	1° 47′	1° 48′	1° 49′	1° 50′	1° 52′	1° 53′
39	J	1° 24′	1° 25′	1° 25′	1° 27′	1° 28′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 36′	1° 37′
	K	1° 38′	1° 38′	1° 39′	1° 40′	1° 41′	1° 42′	1° 44′	1° 45′	1° 46′	1° 47′	1° 48′	1° 49′	1° 51′	1° 52′
40	J	1° 23′	1° 24′	1° 25′	1° 26′	1° 27′	1° 28′	1° 30′	1° 31′	1° 32′	1° 33′	1° 33′	1° 34′	1° 35′	1° 37′
	K	1° 37′	1° 37′	1° 38′	1° 39′	1° 40′	1° 41′	1° 43′	1° 44′	1° 45′	1° 46′	1° 48′	1° 49′	1° 50′	1° 51′
41	J	1° 23′	1° 24′	1° 25′	1° 26′	1° 26′	1° 27′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′	1° 36′
	K	1° 36′	1° 37′	1° 38′	1° 39′	1° 40′	1° 41′	1° 42′	1° 43′	1° 44′	1° 45′	1° 47′	1° 48′	1° 49′	1° 50′
42	J	1° 23′	1° 24′	1° 24′	1° 25′	1° 26′	1° 27′	1° 28′	1° 29′	1° 30′	1° 31′	1° 32′	1° 33′	1° 34′	1° 35′
	K	1° 35′	1° 36′	1° 37′	1° 38′	1° 39′	1° 40′	1° 41′	1° 42′	1° 43′	1° 44′	1° 46′	1° 47′	1° 48′	1° 49′

TABLE 22—*Continued.* ADDENDUM AND DEDENDUM ANGLES FOR BEVEL GEARS.

Number of Teeth in Wheel

58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	*Number of Teeth in Pinion.*
1° 56′	1° 58′	2° 0′	2° 2′	2° 4′	2° 6′	2° 8′	2° 11′	2° 15′	2° 16′	2° 19′	2° 21′	2° 24′	2° 27′	2° 30′	2° 34′	12
2° 14′	2° 16′	2° 19′	2° 21′	2° 24′	2° 26′	2° 29′	2° 32′	2° 35′	2° 38′	2° 41′	2° 44′	2° 47′	2° 51′	2° 54′	2° 58′	
1° 56′	1° 58′	2° 0′	2° 2′	2° 4′	2° 5′	2° 8′	2° 10′	2° 14′	2° 15′	2° 18′	2° 21′	2° 24′	2° 27′	2° 30′	2° 33′	13
2° 14′	2° 16′	2° 18′	2° 21′	2° 23′	2° 26′	2° 28′	2° 31′	2° 34′	2° 37′	2° 40′	2° 43′	2° 46′	2° 50′	2° 53′	2° 57′	
1° 55′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′	2° 7′	2° 10′	2° 13′	2° 15′	2° 17′	2° 20′	2° 23′	2° 26′	2° 29′	2° 32′	14
2° 13′	2° 15′	2° 18′	2° 20′	2° 22′	2° 25′	2° 28′	2° 30′	2° 33′	2° 36′	2° 39′	2° 42′	2° 45′	2° 49′	2° 52′	2° 56′	
1° 55′	1° 57′	1° 59′	2° 0′	2° 3′	2° 5′	2° 6′	2° 9′	2° 12′	2° 14′	2° 16′	2° 19′	2° 22′	2° 25′	2° 28′	2° 31′	15
2° 13′	2° 15′	2° 17′	2° 19′	2° 21′	2° 24′	2° 27′	2° 29′	2° 32′	2° 35′	2° 38′	2° 41′	2° 44′	2° 48′	2° 51′	2° 54′	
1° 55′	1° 56′	1° 58′	2° 0′	2° 2′	2° 4′	2° 5′	2° 8′	2° 10′	2° 13′	2° 15′	2° 18′	2° 22′	2° 24′	2° 27′	2° 29′	16
2° 12′	2° 14′	2° 17′	2° 19′	2° 21′	2° 24′	2° 26′	2° 28′	2° 31′	2° 34′	2° 37′	2° 40′	2° 43′	2° 46′	2° 50′	2° 53′	
1° 54′	1° 56′	1° 57′	2° 0′	2° 1′	2° 3′	2° 5′	2° 8′	2° 10′	2° 13′	2° 15′	2° 17′	2° 21′	2° 23′	2° 25′	2° 28′	17
2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 23′	2° 25′	2° 27′	2° 30′	2° 33′	2° 36′	2° 39′	2° 42′	2° 45′	2° 49′	2° 52′	
1° 54′	1° 55′	1° 56′	1° 59′	2° 0′	2° 3′	2° 5′	2° 7′	2° 9′	2° 12′	2° 14′	2° 16′	2° 19′	2° 21′	2° 25′	2° 27′	18
2° 11′	2° 13′	2° 15′	2° 17′	2° 20′	2° 22′	2° 24′	2° 26′	2° 29′	2° 32′	2° 35′	2° 38′	2° 41′	2° 44′	2° 47′	2° 50′	
1° 53′	1° 55′	1° 56′	1° 59′	2° 0′	2° 2′	2° 4′	2° 6′	2° 8′	2° 11′	2° 13′	2° 15′	2° 18′	2° 20′	2° 23′	2° 26′	19
2° 10′	2° 12′	2° 14′	2° 16′	2° 19′	2° 21′	2° 23′	2° 25′	2° 28′	2° 31′	2° 34′	2° 37′	2° 40′	2° 43′	2° 46′	2° 49′	
1° 53′	1° 54′	1° 55′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′	2° 7′	2° 10′	2° 12′	2° 14′	2° 17′	2° 19′	2° 22′	2° 25′	20
2° 10′	2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 23′	2° 24′	2° 27′	2° 30′	2° 33′	2° 36′	2° 39′	2° 41′	2° 44′	2° 47′	
1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 0′	2° 2′	2° 4′	2° 6′	2° 9′	2° 11′	2° 13′	2° 15′	2° 18′	2° 21′	2° 24′	21
2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	2° 22′	2° 24′	2° 26′	2° 29′	2° 31′	2° 35′	2° 37′	2° 39′	2° 43′	2° 46′	
1° 51′	1° 53′	1° 54′	1° 56′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′	2° 8′	2° 10′	2° 12′	2° 15′	2° 17′	2° 19′	2° 22′	22
2° 8′	2° 10′	2° 12′	2° 14′	2° 16′	2° 18′	2° 21′	2° 23′	2° 25′	2° 28′	2° 30′	2° 33′	2° 35′	2° 38′	2° 42′	2° 45′	
1° 50′	1° 53′	1° 54′	1° 56′	1° 57′	1° 59′	2° 0′	2° 3′	2° 4′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 18′	2° 21′	23
2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 20′	2° 22′	2° 24′	2° 26′	2° 29′	2° 31′	2° 34′	2° 37′	2° 40′	2° 43′	
1° 50′	1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 2′	2° 3′	2° 6′	2° 8′	2° 10′	2° 12′	2° 15′	2° 17′	2° 19′	24
2° 7′	2° 8′	2° 10′	2° 12′	2° 14′	2° 16′	2° 19′	2° 21′	2° 23′	2° 25′	2° 28′	2° 30′	2° 33′	2° 36′	2° 39′	2° 41′	
1° 49′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 1′	2° 2′	2° 5′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 18′	25
2° 6′	2° 8′	2° 10′	2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 22′	2° 24′	2° 27′	2° 29′	2° 32′	2° 34′	2° 37′	2° 40′	
1° 49′	1° 50′	1° 51′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 1′	2° 4′	2° 6′	2° 8′	2° 10′	2° 12′	2° 14′	2° 16′	26
2° 5′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	2° 21′	2° 23′	2° 26′	2° 28′	2° 30′	2° 32′	2° 35′	2° 38′	
1° 48′	1° 50′	1° 51′	1° 53′	1° 54′	1° 55′	1° 57′	1° 59′	2° 0′	2° 3′	2° 5′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	27
2° 4′	2° 6′	2° 8′	2° 10′	2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 22′	2° 24′	2° 26′	2° 29′	2° 31′	2° 34′	2° 36′	
1° 47′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 2′	2° 3′	2° 5′	2° 8′	2° 10′	2° 12′	2° 14′	28
2° 3′	2° 5′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	2° 21′	2° 23′	2° 25′	2° 28′	2° 30′	2° 32′	2° 34′	
1° 46′	1° 48′	1° 50′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 1′	2° 3′	2° 4′	2° 6′	2° 8′	2° 10′	2° 12′	29
2° 3′	2° 4′	2° 6′	2° 8′	2° 10′	2° 11′	2° 14′	2° 15′	2° 17′	2° 19′	2° 22′	2° 24′	2° 26′	2° 28′	2° 31′	2° 33′	
1° 45′	1° 47′	1° 49′	1° 50′	1° 51′	1° 53′	1° 55′	1° 56′	1° 58′	2° 0′	2° 2′	2° 3′	2° 5′	2° 7′	2° 9′	2° 11′	30
2° 2′	2° 3′	2° 5′	2° 7′	2° 9′	2° 10′	2° 13′	2° 14′	2° 16′	2° 18′	2° 20′	2° 22′	2° 25′	2° 27′	2° 30′	2° 32′	
1° 44′	1° 46′	1° 48′	1° 50′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 0′	2° 2′	2° 4′	2° 5′	2° 8′	2° 9′	31
2° 1′	2° 2′	2° 4′	2° 6′	2° 8′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	2° 21′	2° 23′	2° 25′	2° 28′	2° 30′	
1° 44′	1° 45′	1° 46′	1° 49′	1° 50′	1° 51′	1° 53′	1° 54′	1° 55′	1° 58′	1° 59′	2° 1′	2° 3′	2° 4′	2° 6′	2° 8′	32
2° 0′	2° 2′	2° 3′	2° 5′	2° 7′	2° 8′	2° 10′	2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 22′	2° 24′	2° 26′	2° 28′	
1° 43′	1° 45′	1° 46′	1° 48′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	1° 57′	1° 58′	2° 0′	2° 1′	2° 3′	2° 5′	2° 7′	33
1° 59′	2° 0′	2° 2′	2° 4′	2° 6′	2° 7′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 18′	2° 20′	2° 22′	2° 25′	2° 27′	
1° 42′	1° 44′	1° 45′	1° 47′	1° 48′	1° 50′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 59′	2° 0′	2° 2′	2° 4′	2° 5′	34
1° 58′	1° 59′	2° 1′	2° 3′	2° 5′	2° 6′	2° 8′	2° 9′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	2° 21′	2° 23′	2° 25′	
1° 41′	1° 43′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 58′	1° 59′	2° 1′	2° 3′	2° 4′	35
1° 57′	1° 58′	2° 0′	2° 2′	2° 4′	2° 5′	2° 7′	2° 8′	2° 10′	2° 11′	2° 13′	2° 15′	2° 17′	2° 19′	2° 21′	2° 23′	
1° 40′	1° 42′	1° 44′	1° 45′	1° 46′	1° 48′	1° 49′	1° 50′	1° 52′	1° 52′	1° 54′	1° 57′	1° 58′	1° 59′	2° 1′	2° 3′	36
1° 56′	1° 57′	1° 59′	2° 1′	2° 3′	2° 4′	2° 6′	2° 7′	2° 9′	2° 10′	2° 12′	2° 14′	2° 16′	2° 18′	2° 20′	2° 22′	
1° 40′	1° 41′	1° 43′	1° 44′	1° 45′	1° 47′	1° 48′	1° 49′	1° 50′	1° 51′	1° 53′	1° 55′	1° 57′	1° 58′	2° 0′	2° 1′	37
1° 56′	1° 57′	1° 58′	2° 0′	2° 2′	2° 3′	2° 5′	2° 6′	2° 8′	2° 9′	2° 11′	2° 13′	2° 15′	2° 16′	2° 18′	2° 20′	
1° 39′	1° 40′	1° 42′	1° 43′	1° 44′	1° 46′	1° 47′	1° 48′	1° 49′	1° 50′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′	1° 59′	38
1° 55′	1° 56′	1° 57′	1° 58′	2° 0′	2° 1′	2° 3′	2° 4′	2° 6′	2° 7′	2° 9′	2° 11′	2° 13′	2° 14′	2° 16′	2° 18′	
1° 38′	1° 39′	1° 41′	1° 42′	1° 43′	1° 45′	1° 46′	1° 47′	1° 48′	1° 49′	1° 51′	1° 52′	1° 53′	1° 55′	1° 57′	1° 58′	39
1° 53′	1° 54′	1° 56′	1° 57′	1° 59′	2° 0′	2° 2′	2° 3′	2° 5′	2° 6′	2° 8′	2° 9′	2° 11′	2° 12′	2° 14′	2° 16′	
1° 38′	1° 39′	1° 40′	1° 41′	1° 42′	1° 44′	1° 45′	1° 46′	1° 47′	1° 48′	1° 50′	1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	40
1° 52′	1° 53′	1° 55′	1° 56′	1° 58′	1° 59′	2° 1′	2° 2′	2° 4′	2° 5′	2° 7′	2° 8′	2° 10′	2° 11′	2° 13′	2° 15′	
1° 36′	1° 38′	1° 39′	1° 40′	1° 41′	1° 43′	1° 44′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	41
1° 51′	1° 52′	1° 54′	1° 55′	1° 57′	1° 58′	2° 0′	2° 1′	2° 3′	2° 4′	2° 5′	2° 6′	2° 8′	2° 10′	2° 11′	2° 13′	
1° 36′	1° 37′	1° 38′	1° 39′	1° 40′	1° 42′	1° 43′	1° 44′	1° 45′	1° 46′	1° 47′	1° 49′	1° 50′	1° 52′	1° 53′	1° 55′	42
1° 50′	1° 51′	1° 53′	1° 54′	1° 56′	1° 57′	1° 59′	2° 0′	2° 1′	2° 2′	2° 4′	2° 5′	2° 7′	2° 8′	2° 10′	2° 11′	

ANGLE BETWEEN AXES 90°, TOOTH PROPORTIONS BROWN & SHARPE STANDARD.

ANGLE INCREASE. DIVIDE BY TEETH IN PINION	DIAMETER INCREASE. DIVIDE BY PITCH OF PINION	CENTER ANGLE FOR PINION	CENTER-ANGLE-HUNDREDTH-DEGREES LEFT-HAND COLUMN READ HERE							CENTER ANGLE FOR GEAR	DIAMETER INCREASE. DIVIDE BY PITCH OF GEAR
			.00	.17	.33	.50	.67	.83	1.00		
1	2.00	0	.0000	.0029	.0058	.0087	.0116	.0145	.0175	89	.03
2	2.00	1	.0175	.0204	.0233	.0262	.0291	.0320	.0349	88	.07
4	2.00	2	.0349	.0378	.0407	.0437	.0466	.0495	.0524	87	.10
6	2.00	3	.0524	.0553	.0582	.0612	.0641	.0670	.0699	86	.14
8	1.99	4	.0699	.0729	.0758	.0787	.0816	.0846	.0875	85	.17
10	1.99	5	.0875	.0904	.0934	.0963	.0992	.1022	.1051	84	.21
12	1.99	6	.1051	.1080	.1110	.1139	.1169	.1198	.1228	83	.24
14	1.98	7	.1228	.1257	.1278	.1317	.1346	.1376	.1405	82	.28
16	1.98	8	.1405	.1435	.1465	.1495	.1524	.1554	.1584	81	.31
18	1.98	9	.1584	.1614	.1644	.1673	.1703	.1733	.1763	80	.34
20	1.97	10	.1763	.1793	.1823	.1853	.1883	.1914	.1944	79	.38
22	1.96	11	.1944	.1974	.2004	.2035	.2065	.2095	.2126	78	.41
24	1.96	12	.2126	.2156	.2186	.2217	.2247	.2278	.2309	77	.45
26	1.95	13	.2309	.2339	.2370	.2401	.2431	.2462	.2493	76	.48
28	1.94	14	.2493	.2524	.2555	.2586	.2617	.2648	.2679	75	.51
30	1.93	15	.2679	.2711	.2742	.2773	.2805	.2836	.2867	74	.55
32	1.92	16	.2867	.2899	.2931	.2962	.2994	.3026	.3057	73	.58
34	1.91	17	.3057	.3089	.3121	.3153	.3185	.3217	.3249	72	.62
36	1.90	18	.3249	.3281	.3314	.3346	.3378	.3411	.3443	71	.65
37	1.89	19	.3443	.3476	.3508	.3541	.3574	.3607	.3640	70	.68
39	1.88	20	.3640	.3673	.3706	.3739	.3772	.3805	.3839	69	.71
41	1.86	21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	.75
43	1.85	22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	.78
45	1.84	23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	.81
47	1.82	24	.4452	.4487	.4522	.4557	.4592	.4628	.4663	65	.84
49	1.81	25	.4663	.4699	.4734	.4770	.4806	.4841	.4877	64	.88
50	1.79	26	.4877	.4913	.4950	.4986	.5022	.5059	.5095	63	.91
52	1.78	27	.5095	.5132	.5169	.5206	.5243	.5280	.5317	62	.93
54	1.76	28	.5317	.5354	.5392	.5430	.5467	.5505	.5543	61	.97
56	1.74	29	.5543	.5581	.5619	.5658	.5696	.5735	.5774	60	1.00
57	1.73	30	.5774	.5812	.5851	.5890	.5930	.5969	.6009	59	1.03
59	1.71	31	.6009	.6048	.6088	.6128	.6168	.6208	.6249	58	1.05
61	1.69	32	.6249	.6289	.6330	.6371	.6412	.6453	.6494	57	1.08
63	1.67	33	.6494	.6536	.6577	.6619	.6661	.6703	.6745	56	1.11
64	1.65	34	.6745	.6787	.6830	.6873	.6916	.6959	.7002	55	1.14
66	1.63	35	.7002	.7046	.7089	.7133	.7177	.7221	.7265	54	1.17
68	1.61	36	.7265	.7310	.7355	.7400	.7445	.7490	.7536	53	1.20
69	1.59	37	.7536	.7581	.7627	.7673	.7720	.7766	.7813	52	1.23
71	1.57	38	.7813	.7860	.7907	.7954	.8002	.8050	.8098	51	1.25
72	1.55	39	.8098	.8146	.8195	.8243	.8292	.8342	.8391	50	1.28
73	1.53	40	.8391	.8441	.8491	.8541	.8591	.8642	.8693	49	1.31
75	1.51	41	.8693	.8744	.8796	.8847	.8899	.8952	.9004	48	1.33
77	1.48	42	.9004	.9057	.9110	.9163	.9217	.9271	.9325	47	1.36
79	1.46	43	.9325	.9380	.9435	.9490	.9545	.9601	.9657	46	1.39
80	1.43	44	.9657	.9713	.9770	.9827	.9884	.9942	1.0000	45	1.41
81	1.41	45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44	1.43
			1.00	.83	.67	.50	.33	.17	.00		
			RIGHT-HAND COLUMN READ HERE								

TABLE 23—DIAMETERS AND ANGLES OF BEVEL GEAR—SHAFT ANGLES 90 DEGREES

Becker Milling Machine Company

MILLING BEVEL GEARS

The milling of bevel gears is something that has always been more or less of a mystery to the average mechanic, even to those connected directly with the gear business. It has been necessary to use the cut and try method, and then put in more time filing than it took to mill the teeth. The milling of bevel gears is now practically a thing of the past, except for an occasional job required for shop use, the generating of bevel gears having reached such a point that the milling machine cannot compete either in quality or time. However, this subject is still an interesting one.

PARALLEL DEPTH BEVEL GEARS

The milling of parallel depth bevel gear is perhaps the easiest explained, and, it is thought, superior to the ordinary milled bevel.

This type of gear was first described by A. D. Pentz on page 6 of the September 10, 1891, issue of AMERICAN MACHINIST, also by Walter Gribben, who suggested the use of the ordinary spur cutter, February 25, 1892. Later a practical method of cutting was published by S. K. Allen, January 14, 1909, the substance of which is used in the following:

The face and cutting angle of the gears are made the same as the center angle, the tops and bottom of the teeth being parallel with the center angle for their entire length, as shown in Fig. 100.

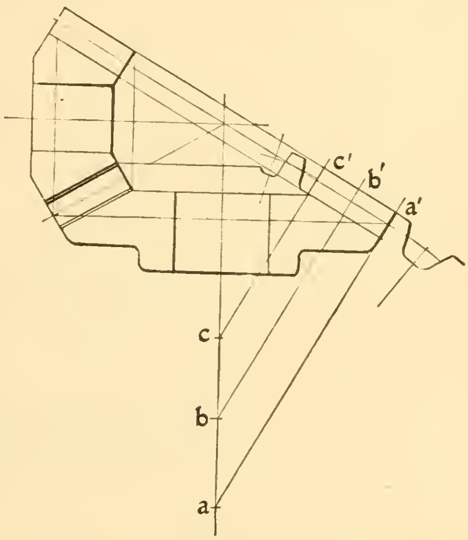


FIG. 100. PARALLEL DEPTH BEVEL GEARS.

The pitch of the cutter is determined from the small end of the gear, and the form of the cutter from the average back cone distance, as at $b - b$. This will give the average form of tooth, as it is apparent that the true form cannot be maintained the entire length of face. The only change in form is due to the reduced back cone distance from *a* to *c;* there is no change due to reducing the depth of cut, as is found in the ordinary methods of bevel gear milling, the pitch line of the bevel gear being cut, and the pitch line of the cutter coinciding during the entire cutting operation.

The milling of a parallel or, for that matter, tapered tooth bevel gears will be better understood if the pitch is considered at the small end of the tooth. When taking side cuts, this pitch alone should be kept in mind and the matter will appear in a new light.

After the first central cut has been taken as illustrated by Fig. 101 the blank is rolled to the right, bringing the pitch line of the left-hand side of space parallel with the travel of cutter, as shown by Fig. 102. This movement is

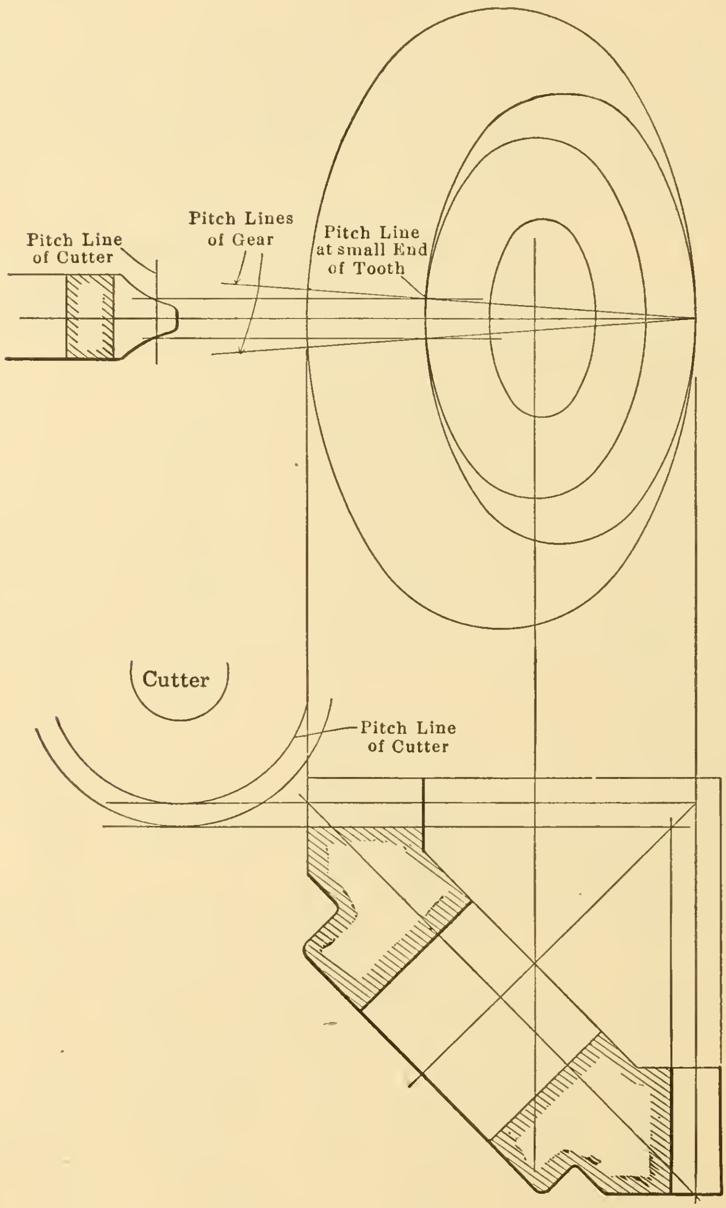


FIG. 101. FIRST CUT IN MILLING BEVEL GEARS.

accomplished by indexing the blank one-quarter as many holes in the index plate as are used altogether to space the teeth. The table is then moved toward the nose of the spindle a distance equal to one-half the tooth thickness at *small end*, or one-quarter the pitch at small end. This will bring the pitch

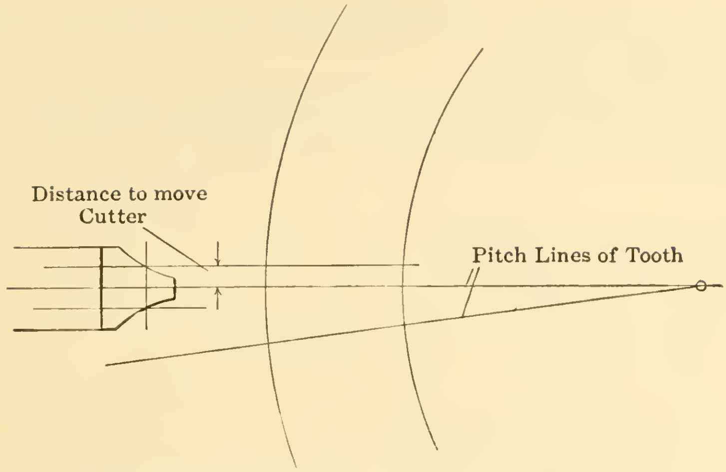


FIG. 102. POSITION OF BLANK WHEN ROTATED ONE-QUARTER OF THE INDEX.

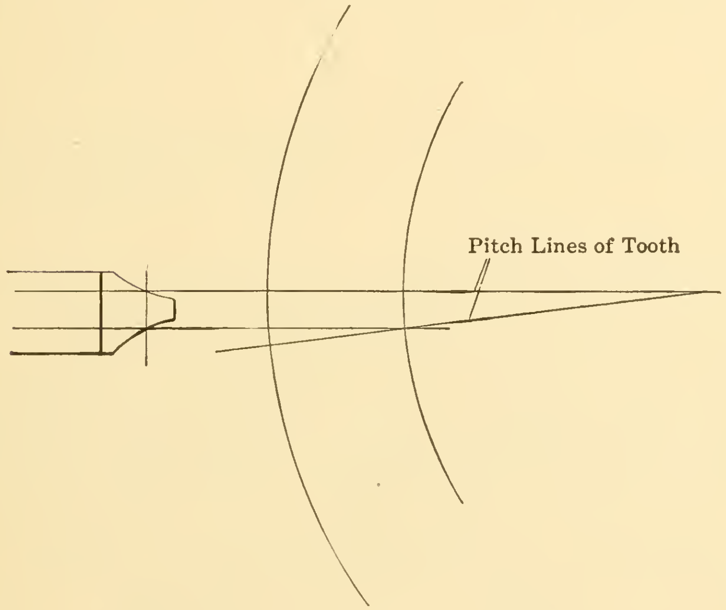


FIG. 103. CUTTER IN POSITION FOR THE FIRST SIDE CUT.

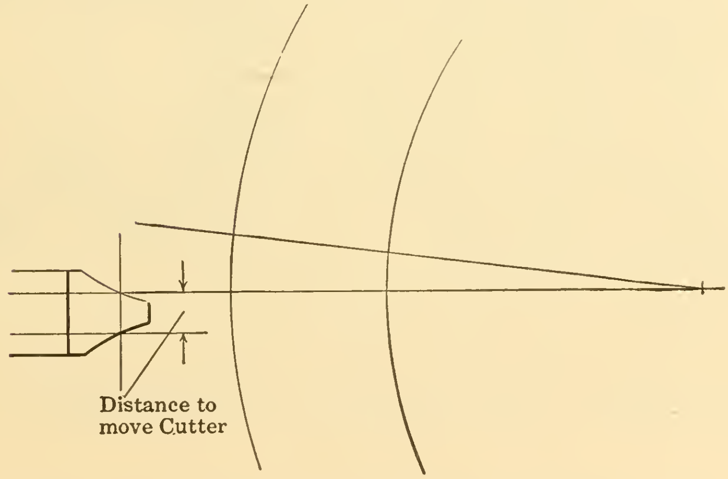


FIG. 104. BLANK IN POSITION FOR SECOND SIDE CUT.

line of gear to the pitch line of the cutter, and the blank in position for first side cut, as shown by Fig. 103.

After this cut has been made, roll the gear blank to the left, one-half as many holes in index plate as are used altogether to space the tooth, as per Fig. 104,

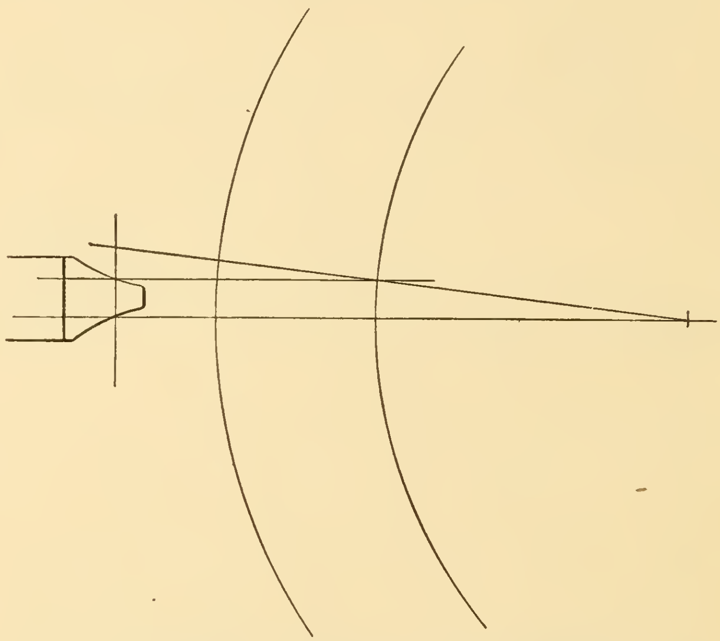


FIG. 105. CUTTER IN POSITION FOR SECOND SIDE CUT.

and move the table *away* from the nose of spindle the thickness of tooth *at small end;* this will bring the other side of the tooth into the same relative position, and the blank is in position for the second side cut, as in Fig. 105.

It will be noted that the pitch is figured at the small end of the tooth, an ordinary spur gear cutter corresponding to the pitch at this point being used.

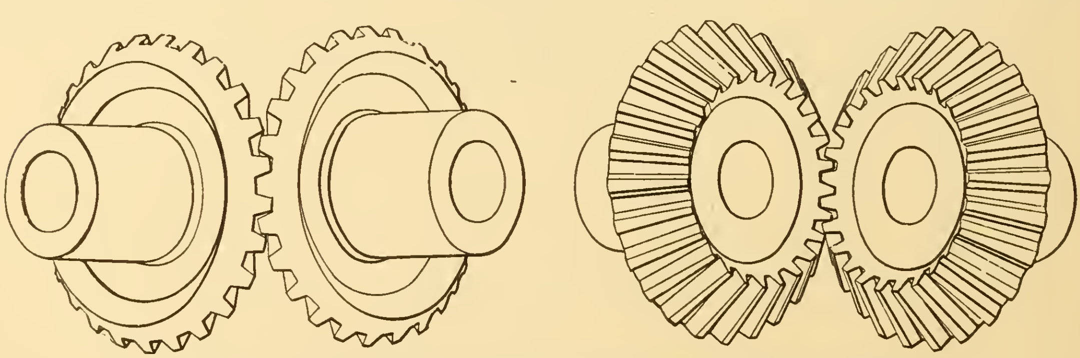


FIG. 106. A PAIR OF PARALLEL DEPTH BEVEL GEARS IN MESH.

The half tone, Fig. 106, shows a pair of parallel tooth gears, sent with W. Allen's original article. Their operation was entirely satisfactory and the teeth

gave no evidence of being filed. These samples were 25 teeth, 10 pitch at small end and one-inch face, No. 2, 10 pitch cutter used.

Referring to Figs. 101 to 105; in moving the table forward and back one-half the thickness of space at the small end there is a small error due to the

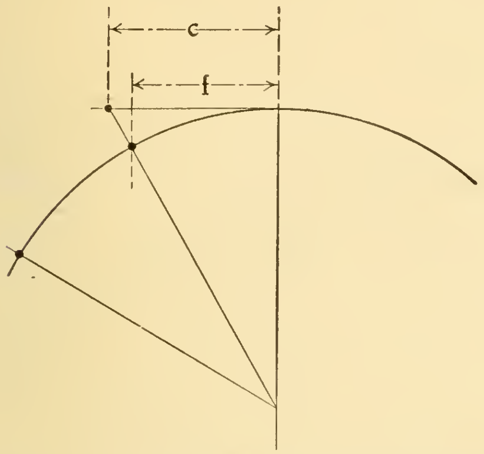


FIG. 107. DIAGRAM SHOWING ERROR IN SETTING OVER CUTTER FOR SIDE CUT ON MILLED BEVEL GEARS.

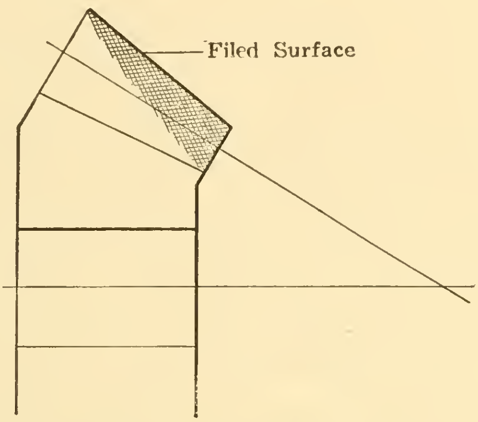


FIG. 108. THE FILED SURFACE OF MILLED BEVEL GEARS.

fact that instead of these moves being made in the direction of the pitch line they are made tangent to it, as illustrated by Fig. 107, which is exaggerated to show this clearly, *c* representing the distance actually moved and *f* the theoretical distance. This error, however, is on the safe side, so that setting the machine as directed will allow a little clearance, depending, of course, with the diameter and pitch of gear, but will not be noticed except in extreme cases.

I see no reason why the above method of rolling and setting over the blank cannot be applied to the regular milled bevel gear; all other rules or tables seen are apparently subject to the cut and try method.

In milling the taper tooth type of bevels, much depends upon the selection of the cutter. There is sure to be filing to do unless the teeth are cut deeper at the small end; which approximates the parallel depth method. This plan can only be used on narrow faces. If the shape of cutter is selected to suit the larger end of the tooth, the small end must be filed, as shown in Fig. 108. If made to suit the central back cone radius *b–b*, Fig. 100, the small end will require *less* filing and too much will be taken off the point of the teeth at the large end for proper contact. The milling of these gears is, therefore, a matter of individual judgment.

SECTION VI

Worm Gears

The half-tone, Fig. 109, shows an interesting model. The horizontal shafts, with the hand crank on the near end, carries six gears, all of the same diameter, and all having teeth cut with the same cutter, or the teeth being all of the same normal pitch. The model shows how the worm, so called, is simply a toothed gear as truly as any other. On this shaft the gear at the farther end has 24 teeth, the next 15 teeth, the next 8, then successively 4, 2, and 1. If the one toothed wheel is not a gear, where is the line to be drawn between worm and

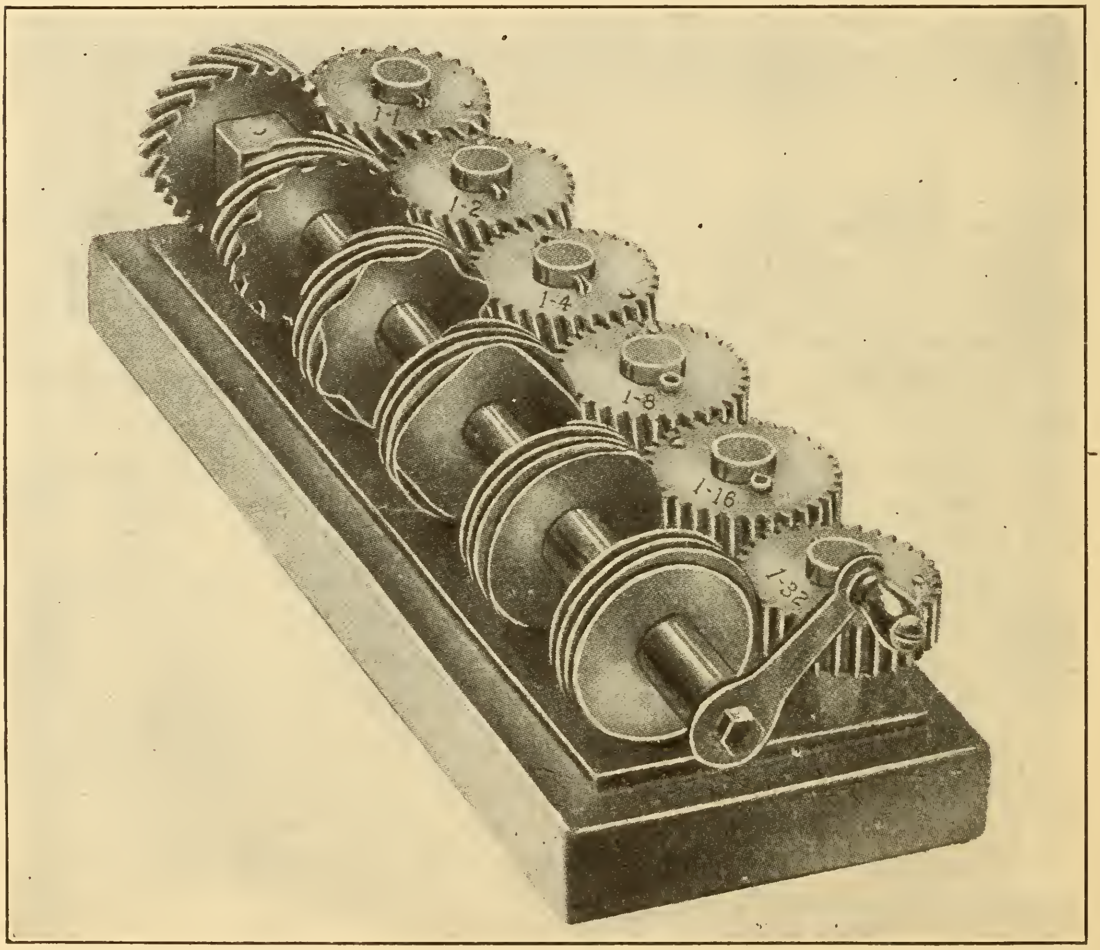


FIG. 109. MODEL OF SPIRAL GEARS OF VARIOUS RATIOS.

gear? The gears on the vertical studs which are driven by those on the horizontal shaft have successively, beginning at the farther end, 24, 30, 32, 32, 32, and 32 teeth. When the shaft is turned 32 times the nearest vertical gear makes one turn, the next two turns and so on successively doubling, the last gear making 32 turns the same as the horizontal shaft.

A study of this cut will give an excellent insight into the design of worm and spiral gears; it also shows the connection between the two types. The first three drivers are evidently worms of single, double, and quadruple thread. They are all spiral gears, however, simply on account of the manner in which they are cut; the driven gears being cut with spiral teeth, not hobbed; also on account of the manner of making the calculations for their cutting. Worm gears are figured from the lineal pitch, sometimes called "axial pitch," of the worm, which corresponds to the circular pitch of the worm gear, the depth and thickness of the teeth being determined from this pitch. Spiral gear calculations ignore this dimension entirely. This, it is supposed, was originally done to avoid odd leads when chasing the worm and hob.

NOTATION FOR WORM GEARS

N = number of teeth in worm wheel.
n = number of threads in worm.
p' = circular pitch (distance from center to center of teeth).
L = lead (advance of worm in one revolution).
D' = pitch diameter of worm wheel.
T = throat diameter of worm wheel.
D = outside diameter of worm wheel.
F = face of worm wheel.
a = distance from center line to point of tooth.
b = length of side.
d' = pitch diameter of worm.
d = outside diameter of worm.
d'' = bottom diameter of worm.
e = radius at throat of worm wheel.
ϕ = angle of sides of face.
B = center distance.
R = number of revolutions of worm to one of wheel.
δ = angle of teeth in wheel with axis (used for gashing teeth).
π = 3.1416.
W = working depth.
W' = whole depth.

f = clearance.
t = thickness of tooth at pitch line.
t^n = normal thickness of tooth at pitch line.
p'^n = normal circular pitch.
s = addendum.
U = width of worm thread at top.
Y = width of worm thread at bottom.

FORMULAS FOR WORM GEARS

$$N = \frac{D' \pi}{p'}.$$

$$D' = N\, p'\, 0.3183.$$

$$T = (N + 2)\, p'\, 0.3183.$$

$$D = T + 2\,(e - e \cos.\, \phi).$$

$$F = \frac{\left(\frac{d}{2} + 0.17\, p'\right) \text{sine}\, \phi}{0.5}, \text{ or } \frac{d + (0.34\, p')}{2}, \text{ when } \phi = 30 \text{ degrees.}$$

$$a = F - (b \text{ sine } \phi).$$

$$b = W' + (0.12\, p').$$

d' = as small as possible. (See discussion.)

$$d = d' + 2\, s.$$

$$d'' = d - 2\, W'.$$

$$e = \frac{d'}{2} - s.$$

$$\phi = 30^\circ \text{ to } 35^\circ, \text{ or sine } \phi = \frac{F}{d + (0.34\, p')}.$$

$$B = \frac{D' + d'}{2}.$$

$$p' = \frac{D'}{0.3183\, N}.$$

$$L = p'^n.$$

$$n = \frac{N}{R}.$$

$$\text{Tang } \delta = \frac{L}{\pi\, d'}.$$

$$t^n = t \cos.\, \delta, \text{ or } t = \frac{t^n}{\cos.\, \delta}.$$

$$U = 0.335\, p', \text{ or } \frac{1.0536}{p}.$$

$$Y = 0.31\, p', \text{ or } \frac{0.9744}{p}.$$

$$p'^{n} = p \cos. \delta, \text{ or } p' = \frac{p'^{n}}{\cos. \delta}.$$

Formulas for tooth parts as given for spur gears apply to worm gears.

DISCUSSION OF FORMULAS

N. D'. p'. The number of teeth, pitch, and pitch diameter of a worm gear are calculated in the same manner as for spur gears.

T. The throat diameter of the wheel is the same as the outside diameter of a spur gear of the same number of teeth and pitch.

D. The extreme outside diameter can be found by this formula, although a measurement of sketch is sufficient when sketch is carefully made, the hob automatically giving this size. It is therefore the practice to simply turn one spot in center of gear face to the throat diameter, leaving plenty of stock on the outside diameter for the hob. The points of teeth are just as well cut off as shown in Fig. 110, as it makes a better appearing gear and one safer to handle. These points are of very little real use.

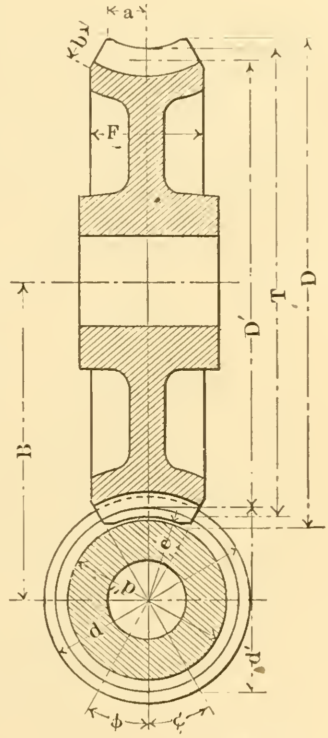


FIG. 110. DIAGRAM FOR WORM GEAR.

F. This formula is for finding the face of a worm-wheel when the angle ϕ is assumed. There can be no real gain in making the face wider than the value given in the second notation of the formula for *F*, which gives the angle ϕ a value of 30 degrees.

d'. The efficiency of a worm drive depends greatly upon the angle of the worm δ. When the lead is fixed the angle is determined from the pitch diameter ($\tan g\ \delta = \frac{L}{\pi d'}$), and in order to make it of an angle that would indicate high efficiency the diameter should be made as small as possible, except where the angle approaches 45 degrees, where either worm or gear may drive. In such a case the conditions should be reversed, and the diameter of worm made large enough to secure the desired angle, unless the wheel is to do the driving, as is sometimes the case.

ϕ. This angle is generally made from 30 to 35 degrees, preferably 30 degrees, but may be found by the formula given when the face is assumed.

This angle may be made 45 degrees, and the face widened to correspond by making the pitch diameter of the wheel equal $D' + 2s$. This will give a full bearing clear to the edge of the tooth, which under ordinary conditions would be cut away, giving the gear the appearance of being spoiled. This, however, is not to be recommended, as the gear will wear more rapidly.

L. Lead is the advance of the worm thread in one revolution of the worm, and is found by multiplying the circular pitch by the number of threads in the worm. Therefore, a 1½-inch pitch worm, double thread, is 3 inches lead.

n. The number of threads necessary to put in a worm of a given ratio is found by dividing the teeth in the wheel by the ratio required. For example: a wheel has 60 teeth and a velocity ratio of 30 to 1; thus, $\frac{60}{30} =$ 2 threads in worm.

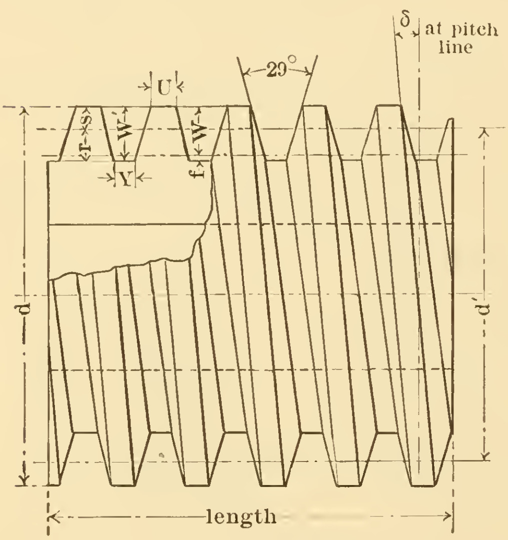


FIG. 111. WORM.

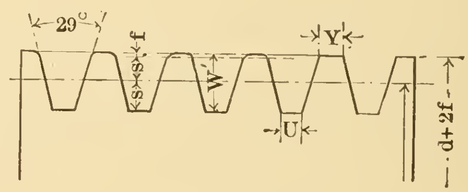


FIG. 112. HOB.

W'. The depth of tooth should be proportioned from the normal pitch (p'^{n}) rather than from the lineal pitch (p'), as is usually the case. It is a mistake to take the depth from the lineal pitch, even for single or double threads; but when quadruple or sextuple threads are used in the worm the error becomes very pronounced, and liable to seriously affect the working of the gears. It is readily seen from Fig. 109 that the normal pitch is the real pitch of the gears. In other words, worm gears might be figured in the same manner as spiral gears. This is absolutely necessary when the angle of worm is increased sufficiently to make the worm gear the driver, as for gears used in cream separators, blowers, etc.

t^{n}. The normal thickness of tooth is found by multiplying the thickness of tooth measured parallel with axis of worm (0.5 lineal pitch) by the cosine of angle of spiral, δ.

The length of worm need be no more than three times the circular pitch, as

there are seldom more than two teeth in contact at once. If, however, the worm is made longer, it can be shifted as it becomes worn, as the worm is always more worn than the wheel. It is common practice to make the length of worm six times the circular pitch.

THE HOB

$d + 2f$. The outside diameter of hob should be made equal to the outside diameter of worm plus twice the clearance. To this should be added $0.03 \times p'$ for wear.

V. For 14½ degrees standard the width of hob thread at top equals 0.31 of the circular pitch. The points of thread should be rounded off a little back to the clearance line, or 0.06 of circular pitch.

U. For 14½ degrees standard the width of thread at bottom equals 0.335 times circular pitch. If a form tool is used to chase the thread, the worm may be cut first, then grind an amount equal to the clearance at the bottom of tooth, f, from the point of tool and use it to chase the hob thread, cutting the same depth as for worm W'.

W'. The depth of thread for hob is made the same as for worm or $0.6866 \times p'$.

LENGTH OF THE HOB

The hob should always be a little longer than the worm, providing, of course, that the entire length of the worm has some contact with the wheel; see Fig. 113; this depends upon the diameter of the wheel engaged. If the hob is shorter than the worm it will not cut the proper shaped tooth; in other words, the generating of the teeth in worm gear will not be complete owing to the shortness of the hob.

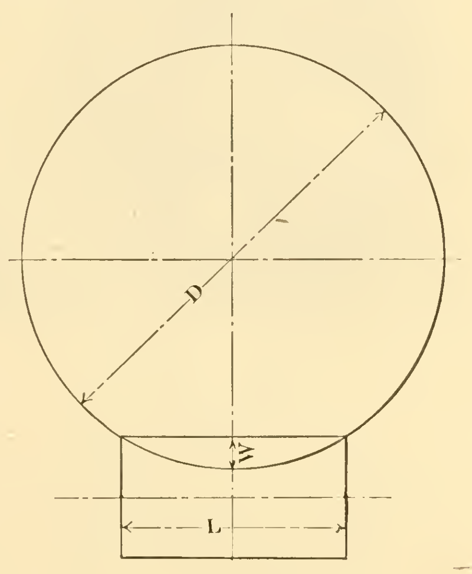


FIG. 113. LENGTH OF THE HOB.

The hob, therefore, should be made long enough to overlap the entire contact of any gear liable to be cut. This length may be found as follows:

$$L = 2\sqrt{(D - W') - W'}.$$

In which W' = whole depth of tooth.

L = length of hob.

D = outside diameter of largest gear to be cut.

NUMBER OF FLUTES

The following by Oscar J. Beal, originally published in AMERICAN MACHINIST, June 22, 1899, throws considerable light on the proper arrangement of the teeth in the hob. This arrangement is of vital importance when the hob is much over quadruple thread.

"In the works of the Brown & Sharpe Manufacturing Company a pair of gears was wanted of the spiral or screw type, and it was thought better to make the large gear, or member, as a worm and the small member as a worm-wheel.

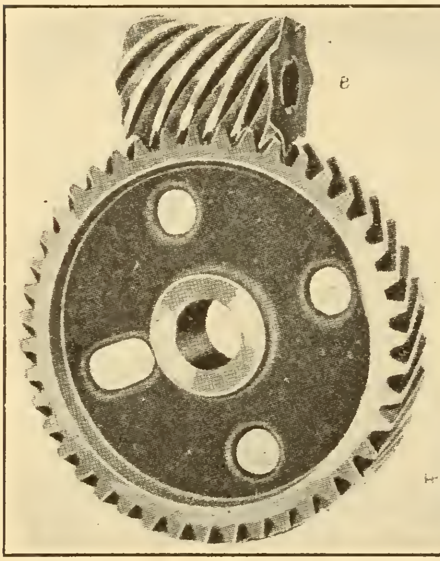


FIG. 114. WORM GEAR *B* AND WORM *A*.

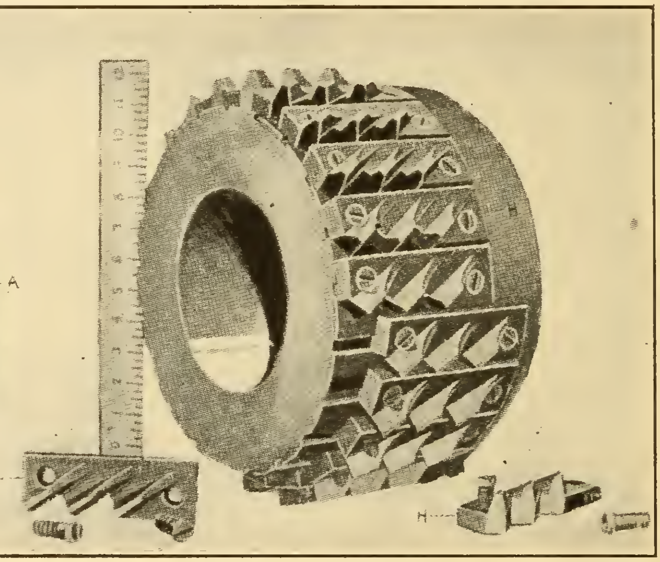


FIG. 118. HOB USED FOR CUTTING WHEEL *B* OF FIG. 114.

Fig. 114 shows the worm and wheel in mesh; *A* is the worm and *B* is the worm-wheel. The large member, or the worm *A*, has 43 threads; the lead of the worm is 60.3 inches, and the thread pitch, or the axial pitch, is 1.4 inch. The small member, or the worm-wheel *B*, has 7 teeth, and the circular pitch of the wheel is, of course, the same as the thread pitch of the worm, 1.4 inch.

Fig. 115 is an axial section of the worm threads. The threads incline 57 degrees from the plane perpendicular to the axis, which is so great that, while the axial thickness of the thread at the pitch line is $\frac{7}{10}$ inch, the actual or the normal thickness is not quite $\frac{4}{10}$ inch. In Fig. 116 the line *C D* shows the inclination of the threads; *C E* is the axial pitch, and *F G* the actual or normal pitch.

In cases where the inclination of the thread is more than 15 degrees, that is, in cases where the normal pitch is less than 0.96 of the axial pitch, it is well to have the depth and the addendum correspond to the normal pitch. Fig. 117

is a normal section of the thread, the depth being the same as a gear tooth of equal pitch, which makes the thread look shallow and thick when seen in the axial section, Fig. 115.

The worm-wheel *B*, Fig. 114, was hobbed, or cut, with the hob shown in Fig. 118.

The worm has more than six times as many threads as the worm-wheel; the pitch diameter of the worm is four times that of the wheel; the wheel is the driver. The hob is made up of a cast-iron body, upon which are fastened lags that are arranged in steps in order to have the lags alike for convenience in manufacturing. Once a large hob was made that did not work, because the cutting edges of the hob teeth did not trim the tops of the worm-wheel teeth narrow enough to clear the backs of the hob teeth, which jammed so hard that the machine could not go. This jamming of the backs of the hob teeth upon the tops of the worm-wheel teeth was owing in part to incorrect spacing of the lags *H H*, Fig. 118, which will be explained.

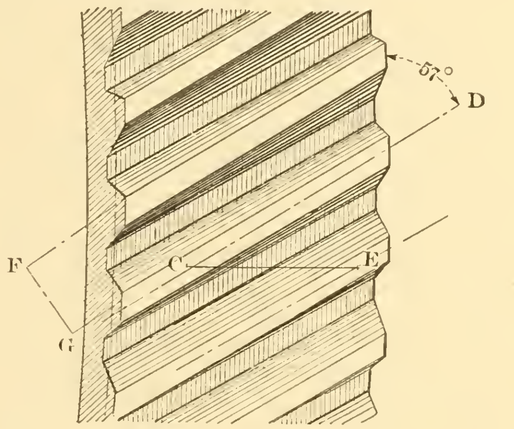


FIG. 116. AXIAL AND NORMAL PITCH.

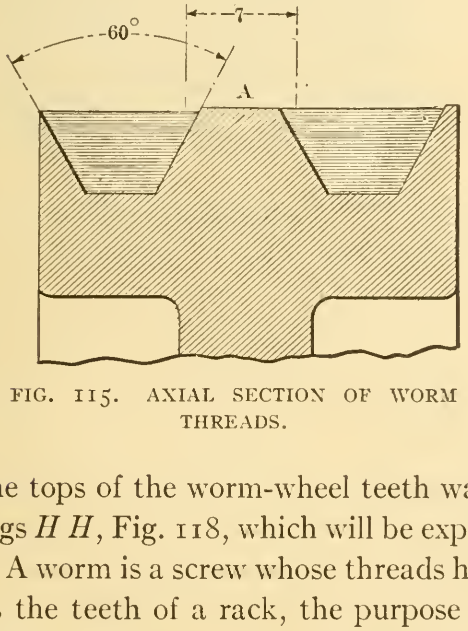


FIG. 115. AXIAL SECTION OF WORM THREADS.

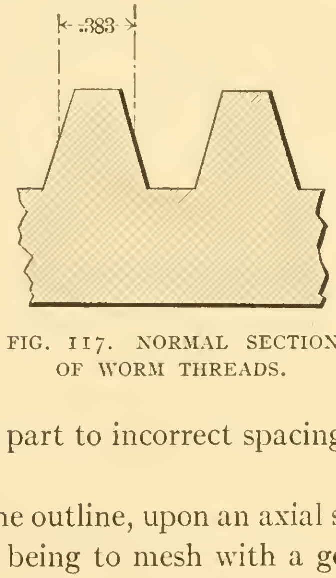


FIG. 117. NORMAL SECTION OF WORM THREADS.

A worm is a screw whose threads have the same outline, upon an axial section, as the teeth of a rack, the purpose of a worm being to mesh with a gear. A worm gear meshes with a worm. The action of a worm meshing with a worm gear is analogous to that of a rack with a spur gear, as stated by Professor Willis in his "Principles of Mechanism." In most worms the outlines of the threads upon the axial section have straight sides, as in Fig. 115, which corresponds to the sides of rack teeth in the involute system of gearing.

Fig. 118 is a hob made up of cast-iron body, into which are fastened lags *H H*. Two of these lags are shown detached. The lags were threaded in axial section like *A*, Fig. 115. The resulting teeth were trimmed and backed off, as in the detached lag on the left. The numbers in the scale are for inches.

I have spoken of the failure of a hob because the backs of the hob teeth jammed upon the tops of the wheel teeth. This interfering action can be explained in several ways; it is analogous to trying to thread a coarse screw in a lathe with a tool that does not lead or incline in the same direction that the thread inclines. A thread tool inclined for a right-hand thread would soon interfere in cutting a left-hand thread. Any grooving tool that has only one cutting edge or face must track in the same groove that it cuts. Sometimes a tool goes wrong and cuts a groove that bends the tool, which is occasionally noticed in cutting off a large piece in a lathe. In cutting a deep narrow groove a thin saw sometimes runs so much to one side that the saw is broken. In the case of the hob the interfering teeth would neither bend nor break, and so the machine had to stop. The teeth of a hob should be so arranged that there will be a cutting edge to take off an interfering point as it comes in the way. A worm-wheel can be cut with a tool that has only one cutting edge by bringing the tool into different positions in relation to the teeth of the wheel. In the American Machinist for May 27, 1897, reference was made to the great number of cutting edges that a hob must have in order to cut a perfect wheel, and a description was given of a machine that cuts worm-wheels with a single tool acting in different positions. Such a machine was patented November 15, 1887, and another July 5, 1898.

In most hobs the cutting edges are straight, and in consequence the sides of the hobbed worm-wheel teeth are made up of straight lines in warped surfaces that meet in angles. These angles are often not noticed in worm-wheels of fine pitch and in wheels having a large number of teeth; but in wheels of coarse pitch and in wheels having few teeth the angles may be quite pronounced. Fig. 119 shows a worm-wheel that has teeth with hobbly sides on account of these angles. This wheel was cut with the hob shown in Fig. 120. The length and the diameter of the blank were great enough to extend beyond the teeth left by the hob that are available to work in connection with the worm. The available part of the teeth occupy about two-thirds the length of the wheel through the mid-part, as between *I* and *J*. Though the teeth are available, yet their sides are so hobbly between *I* and *J* that they will need to have the angles finished off before the wheel can run smoothly with its worm.

Another kind of stepped action of the hob is seen as grooves near *K L*, Fig. 119, which are cut in consequence of the quick travel of the large part *K L*, in proportion to the narrow flats *M M*, Fig. 120, at the tops of the hob teeth. If

the travel of K L had been slow enough or if the flats M M had been wide enough, there would have been no grooves.

The circular pitch and the number of teeth of Fig. 119 are the same as in *B*, Fig. 114.

It is well known that the cutting edges of a hob must act upon the worm-wheel teeth in different positions, and that a tool with a single cutting edge must track in a groove cut with itself; but it was a surprise to learn that a hob of any number of cutting edges can be so made that it will absolutely

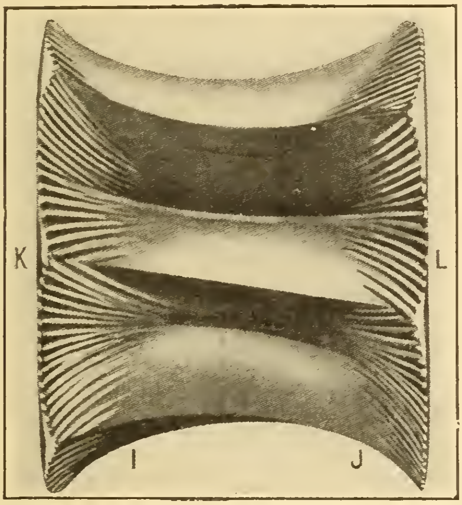


FIG. 119. A WORM GEAR OF FEW TEETH AND COARSE PITCH.

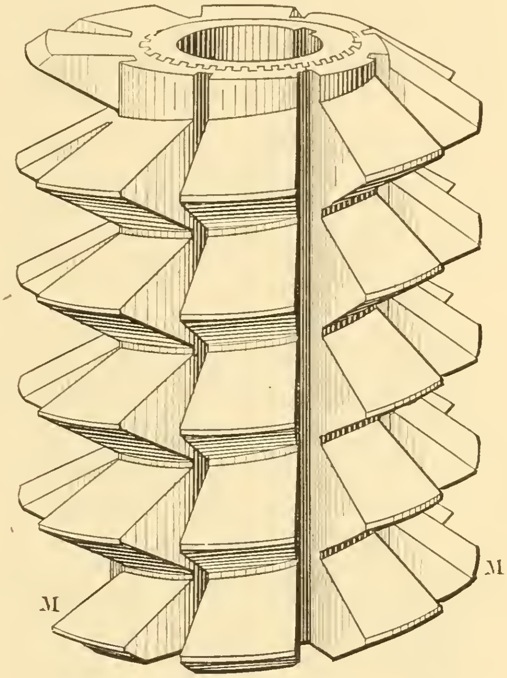


FIG. 120. HOB USED IN CUTTING WORM GEAR OF FIG. 119.

refuse to cut a wheel that has only a few teeth like *B*, Fig. 114. When the workman told me that the hob jammed, I was incredulous, but a glance at the work proved that he was right. I could not believe that my previous experience had been such that I could have known how to make the hob, yet in a few minutes, when the solution came to me, I had the feeling that I must have known it well some time in the long past.

The things that affect this interference might be called variable; there are several of these variables. I am unable to give a rule that will indicate the conditions in which interference would be objectionable; yet, while limits may not be easily defined, an understanding of a few extremes may enable us to keep away outside these limits.

One way of explaining the interference is based upon the fact that, in a gear, any point outside the pitch circle moves through a greater distance, or faster,

than a point in the pitch circle. Fig. 121 shows a single-threaded hob having only one row of cutting edges *O O*, the teeth, or the threads, extending nearly around the hob. Let the teeth in the wheel *P* be shaped as if they had been cut to the full depth with the cutting edges of the hob, and in a low-numbered wheel we shall have tooth faces shaped as shown. Now, place this gear in mesh with the hob at the cutting edges, turn the hob in the direction of the arrow, and we shall soon find that the tooth faces of the wheel will interfere with the hob threads, as shown at *N N*, in consequence of the faces *N N* moving at a different speed from the pitch circle. There would also be interference upon

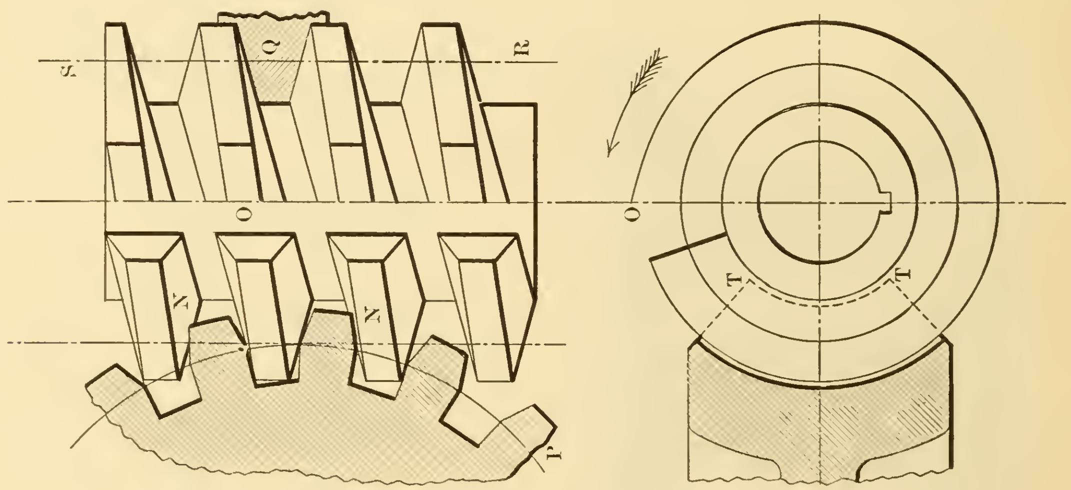


FIG. 121. A SINGLE-THREADED HOB WITH ONE ROW OF CUTTING EDGE.

the flanks of the gear, but it was thought that the cut would be quite as clear if the showing of this flank interference were not attempted. Only a slice section of the wheel is shown at *P;* in the real wheel we should have a still greater interference at the outer part of the teeth *T T*. In moving along a straight path, from *R* to *S*, a close-fitting tooth *Q* would not interfere; but interference would begin as soon as *Q* was moved in a curved path like that of a gear tooth.

From this consideration of Fig. 121 we should conclude that it is impractical to hob a wheel of few teeth with a hob having only one row of cutting edges, like the one shown. Even though we reduce the hob threads back of the cutting edges enough to clear the teeth of the wheel, so that it will be possible to hob the wheel, we shall not shape the teeth so that they will run correctly with the worm.

Another illustration of interference may be seen in Fig. 122. Let a small gear be cut, as shown, with a cutter that is shaped like a gear, as might be done in a Fellows gear shaper. Let every rotative movement of the gear, in order to take

another cut, be through exactly one tooth, a cutter tooth always cutting on the line of centers, as shown. In this way cut to the full depth, moving the gear exactly one tooth at every setting. In our experimental cutting we can let the cutter rotate through one tooth at every movement of the gear, or we can let the cutter remain stationary, so far as rotation is concerned. When we have cut a few spaces to the full depth, we shall find that they are shaped as shown in Fig. 122, the spaces below the pitch line merely fitting a cutter tooth upon the line of centers without any enveloping or shaping of the gear teeth, as there would be in the ordinary working of the Fellows gear shaper. Now let us stop the cutter, leaving its cutting edges just above the side of the gear, and try to rotate the gear

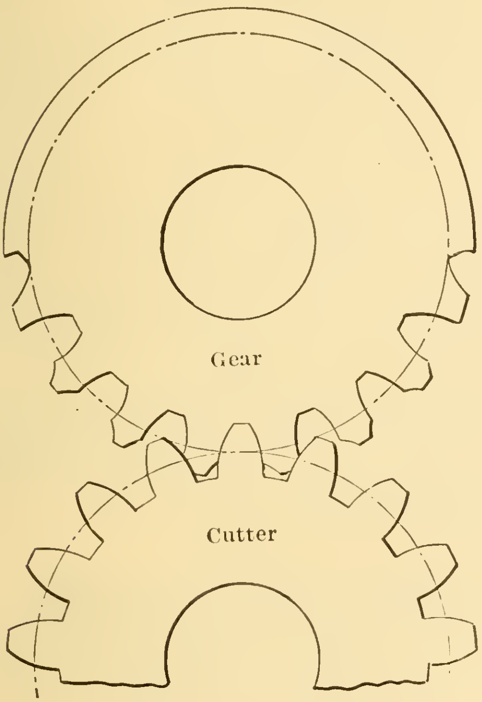


FIG. 122. ABSENCE OF ENVELOPING ACTION IN GEAR CUTTING.

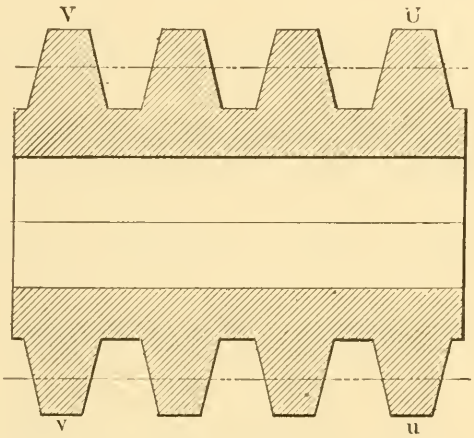


FIG. 123. A DOUBLE-THREAD HOB WITH TWO ROWS OF CUTTING EDGES.

with the cutter in mesh, just as if they were a pair of gears, and we shall at once see that the teeth of the cutter interfere back of the cutting edges, as we should suppose from a mere inspection of Fig. 122.

The same kind of interference that we saw in a single-threaded hob, Fig. 121, will occur in a double-threaded hob that has only two rows of cutting edges if they are evenly spaced and are parallel to the axis. This can be understood from Fig. 123. Any tooth or thread *U* is exactly opposite another tooth *u*, because the thread is double, one thread starting at the end half way around from the other thread. One row of cutting edges *U V* will pass through the spaces cut by the other row *u v* in the same position as regards the worm-wheel teeth, and in consequence the backs of the teeth in both rows will interfere, as in Fig. 121.

A three-threaded hob with three evenly spaced rows of cutting edges will interfere, and so on.

From a careful consideration of the foregoing we arrive at the general principle—*The spacing of a hob must not be equal to the circumferential distance occupied by either one or to any whole number of threads.*

The more teeth there are in a worm-wheel the more teeth it is possible to have in contact with the worm threads at one time, in a worm that is long enough, and in consequence a long hob can possibly cut upon enough teeth at a time; or, what is the same thing, it can cut every tooth in enough positions so that even with only one row of cutting edges it can shape the teeth smooth and without interfering. In practice, however, it is never safe to trust to only one row.

My hob has 43 threads and 21 lags or cutting rows. I had spaced the lags $\frac{2}{43}$ of the circumference apart, which gave just two thread spaces to each lag. Hence, so far as the shape of the worm-wheel teeth is concerned I was not doing any different with the 21 lags (shown in Fig. 118) than I could have done with only one lag.

Another body was made for the hob. The lags were spaced evenly, 21 in the circumference, which gave $2\frac{1}{21}$ thread spaces to each lag. This arrangement afforded twenty-one positions of lags. To accommodate these positions steps were provided, as seen at *H*. The hob was successful."

RELIEVING A SPIRAL FLUTED HOB WITHOUT SPECIAL FIXTURES*

Special fixtures are not necessary to relieve the teeth in a spiral fluted hob. This may be accomplished by indexing for a greater number of flutes than are actually contained in the hob.

Let L = lead of hob.
L_1 = lead of flute milled in hob.
C = pitch circumference.
I = distance gained by spiral flute in one revolution.
C' = circumferential length of each flute.
N = number of flutes to be added.

$$L_1 = \frac{C}{L} C.$$

$$I = \frac{C}{L_1} L.$$

$$N = \frac{I}{C'}.$$

* R. J. Briney.

If N turns out an inconvenient figure it may be changed to the nearest whole or fractional number and the lead of flute (L') changed to suit as follows:

$$L' = \frac{C}{N\,C'}\,L.$$

Example:

What will be the proper index for the relieving attachment for a hob 4 inches pitch diameter and 8-inch lead, number of flute cut in hob 5.

$$C = \pi\,4 = 12.5664.$$

$$L' = \frac{C}{L}\,C = \frac{12.5664}{8} \times 12.5664 = 19.739 \text{ inches.}$$

$$I = \frac{C}{L_1}\,L = \frac{12.5664}{19.739} \times 8 = 5.088 \text{ inches.}$$

$$N = \frac{I}{C'} = \frac{5.088}{\frac{12.5664}{5}} = 2\frac{6}{25}.$$

Substituting 2 for $2\frac{6}{25}$, makes our index $5 + 2 = 7$, instead of 5.

Since the value N is changed from $2\frac{6}{25}$ to 2, we must change the lead of flutes to correspond.

$$L_1 = \frac{C}{N\,C'}\,L = \frac{12.5664}{2 \times \frac{12.5664}{5}} \times 8 = 20 \text{ inches.}$$

REDUCING THE DIAMETER OF WORM GEARS

Increasing the pitch diameter in order to avoid undercut is not good practice, as it tends to shorten the life of a gear, instead of lengthening it. By referring to Fig. 124 it is plain that the pitch of a worm gear at C is greater than at A and, since this pitch the worm can only be made to correspond with the pitch of the gear at one point, generally A, there must necessarily be a great amount of friction, with the necessary loss in efficiency at B and still more at C.

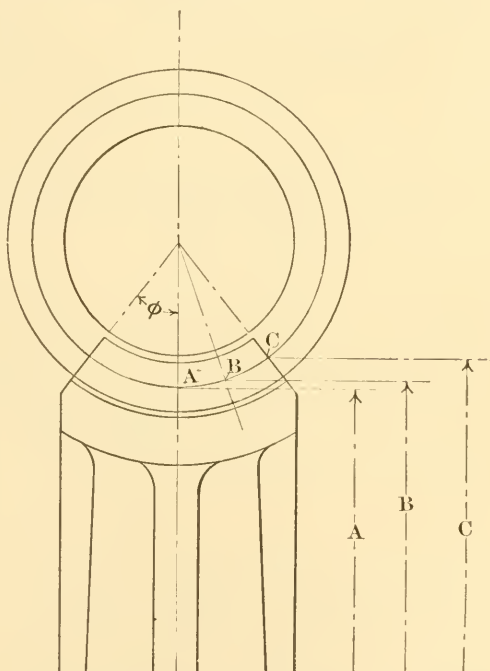


FIG. 124. CORRECTED DIAMETER OF WORM GEAR.

The efficiency and life of worm gears is greatly increased, therefore, by mak-

ing the diameter and, therefore, the pitch of the gear to correspond with the pitch of the worm at point B, or the medium pitch diameter of gear. This will reduce the pitch diameter the following amount, it being assumed that angle of face ϕ, is 30 degrees; or it can readily be found by a careful layout.

Corrected pitch diameter of worm gear $= D' - 2\ (d' - d'\ 0.97)$.

There are in reality as many different pitch diameters between A and C as we would care to take sections, as the pitch is changing constantly. For our illustration, however, but the three main points have been considered.

CUTTING WORM GEARS ON MILLING MACHINE

The worm gear is first gashed out, dropping a cutter approximating the outside diameter of the hob, about two-thirds of the full depth of tooth. The hob is then placed on the cutter spindle and dropped as far as possible into the gashed out gear which is held at right angles, and allowed to turn freely. The hob then completes the gear, driving it around by the action of the cutting edges in contact. Great care must be taken when the cut is first started, or the teeth in hob will lock on some of the sharp corners left by the gashing cutter.

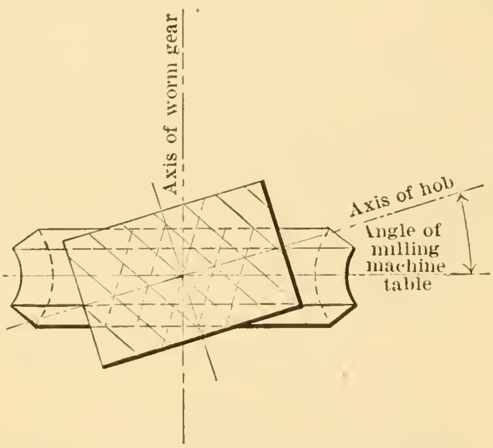


FIG. 125. CUTTING WORM GEAR WITH A HOB OF A DIFFERENT ANGLE FROM THE ENGAGING WORM.

If the only hob on hand is larger or smaller than the worm engaging gear being cut, swing the table to the right or left (depending upon the hand) to an angle equal to the difference in the angle of worm and the angle of thread in hob. Of course, there is a limit to this, depending upon the accuracy desired, but it may be carried further than generally supposed.

In the same manner, a left-hand hob may be used to cut a right-hand wheel, or *vice versa*, the gear being swung around until the angle of its thread corresponds with the angle of the hob, as illustrated in Fig. 125.

STRAIGHT-CUT WORM GEARS

Worm gears with the teeth cut in a straight path, to correspond with the angle of worm thread as shown by Fig. 126, are often employed with excellent results, apparently proving superior to the hobbed worm gear, and in all cases better than such a gear cut with spiral teeth. This type, unfortunately, cannot be used with a worm having a helix angle much greater than 15

degrees, as the cutter naturally does not cut the teeth full depth at the edges of the wheel, the proper dimensions being secured only in the center of the face.

One of its advantages is the fact that side adjustment, which is essential in hobbed worm gearing, is not important, also it is claimed, and not without reason, that the contact is better, the pitch of the wheel corresponding to that of the worm the entire length of the face.

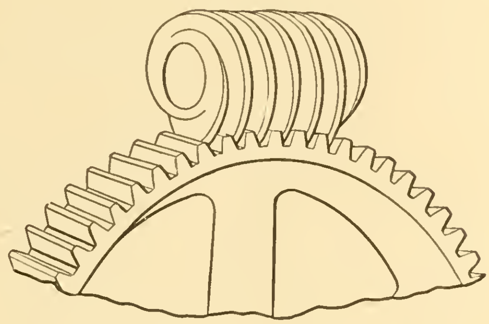


FIG. 126. STRAIGHT-CUT WORM GEAR.

This class of gears is often used for elevator service and is said to operate car with less vibration than the hobbed gear type. This is feasible, as the spacing errors, well known, and unavoidable in hobbed worm gears, may be greatly reduced if care is taken in cutting.

These gears may be cut on a plain milling machine—in fact, the universal machine has no advantage for this work. The table must move on a line parallel to the center line of the cutter; the dividing head and tail stock are

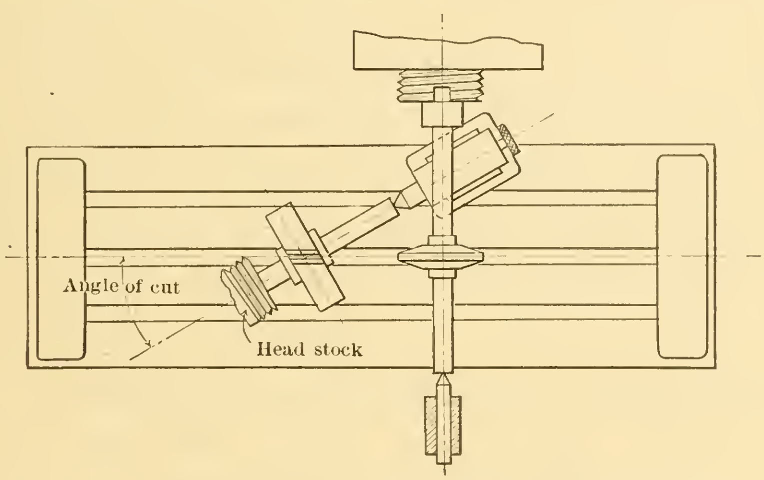


FIG. 127. CUTTING STRAIGHT-CUT WORM GEARS ON MILLING MACHINE.

blocked up and set at the angle of the teeth in the worm gear, as shown in Fig. 127. Gears of this type should be calculated as for spiral gears, that a standard spur gear cutter may be employed to cut the teeth. A slight error is allowable in the pitch to accommodate the nearest even lineal pitch which must be used in chasing the worm, as it is sometimes impossible to gear the lathe for the exact pitch.

CUTTING AN UNUSUALLY ACCURATE WORM GEAR

This worm gear cut had 360 teeth, 5/8-inch pitch, 71.620-inch pitch diameter. The engaging worm was 3 inches outside diameter, single thread.

The cutting and hobbing of this wheel was done in a very unusual manner, made necessary by circumstances, and the necessity of great accuracy, as it was intended to be used to drive a large telescope.

The wheel could not be hobbed upon the largest machine available for that purpose, as the worm-wheel, being of large diameter, would strike the collar of the hob spindle before it could come into contact with the hob; it was also not possible to gear the hobbing machine for a ratio of 360 to 1.

The cutting was done as follows: The wheel was placed upon a hobbing machine geared to cut 180 teeth and a fly cutter about twice the diameter of the worm was used to cut 180 notches in the wheel; the notches produced by this cutter having practically the same angle of spiral as the thread upon the 3-inch diameter worm. The wheel was fed against the cutter slowly by hand until the notches were cut about 1/4 inch deep. The fly cutter was then rotated upon its arbor a half turn so as to cut 180 notches between those first cut, thereby producing 360 notches as a guide for the hobbing which followed.

The hobbing was done in a lathe, the worm-wheel being mounted upon a special plate, planed to fit upon the lathe carriage. A bushing tapered on the outside to fit the taper bore of the worm-wheel was mounted upon the plate with four bolts to hold it down, and four set screws to raise it, so as to permit a little vertical adjustment. The hub of the wheel rested upon three brass blocks resting upon set screws by which the fit of the wheel upon the bushing could be adjusted. The hob was carried upon an arbor upon the lathe centers, and of sufficient length to allow the worm-wheel to clear the headstock.

This worm-wheel was designed with a loose ring occupying one-half the face of the wheel and held to the wheel by 24 cap screws which permit of changing its position 24 times. This ring was rotated one-half turn showing 1/16-inch average error in the original spacing before the beginning of the hobbing.

After hobbing around two or three times, the ring was turned 11-24 of a revolution; and after having changed the position of the ring 14 times and hobbing a little each time, the teeth were cut full depth, and the rotation of the ring showed no error of spacing or coincidence of the teeth in ring and wheel exceeding 0.003 inch.

The grinding was next done with a worm 24 inches long and mounted in place of the hob. No. 180 emery mixed with oil was used, and after grinding and changing the position of the loose ring a number of times, the error in the

spacing of the teeth did not not exceed 0.001 inch, or six seconds of arc approximately.

The hobbing and grinding of this wheel was done by Geo. W. Klages at the shop of the John A. Brashear Company, Ltd., and the wheel will be used to drive the 30-inch refractor of the Allegheny Observatory.

POWER AND EFFICIENCY OF WORM GEARING *

In view of the good results now being obtained with worm gearing, the old prejudice against that form of gearing, on account of its supposed low efficiency and short life, is dying out. These good results are the outcome of the application of principles which are by no means a late discovery, and it is expected that what follows will contain much that to some readers is not new. At the same time it is an undoubted fact that the best practice with worms is understood by but few, relatively speaking, and the corroboration of the theory by examples from practice which follow is believed to be new. No better illustration of the fact that good practice with worm gearing is not yet widely understood could be given than the statement in a recent and excellent work on gearing that "the diameter of the worm is commonly made equal to four or five times the circular pitch," the fact being that such proportions are distinctly bad if the worm is to do hard work.

It should be stated at the beginning that while what follows is not offered as a presentation of all the data necessary for assured success with worms under all conditions, it is hoped to make the general conditions of successful practice plain, and to present the "state of the art" as it exists to-day.

The essential change in practice which has improved the results obtained with worm gearing has been an increase in the pitch angle over what was formerly considered proper. There is no doubt whatever that this change has increased the efficiency of the gear, and, what is of more importance, has reduced the tendency to heat and wear. This is not only a fact, but it is a sound conclusion from theoretical considerations, which might have been predicted under proper examination.

THEORY OF WORM EFFICIENCY

The reason why an increase of pitch, other things being equal, or, in other words, an increase of the angle of the thread, gives these results, will be understood from Fig. 128. If $a\ b$ be the axis of the worm and $c\ d$ a line representing a thread, against which a tooth of the wheel bears, it will be seen that if the tooth bears upon the thread by a pressure P, that pressure may be resolved into two components, one of which, $e\ f$, is perpendicular, while the other, $e\ g$, is

* F. A. Halsey, in the AMERICAN MACHINIST.

parallel to the thread surface. The perpendicular component produces friction between the tooth and the thread. The useful work done during a revolution of the thread is the product of the load P and the lead of the worm, while the work lost in friction is the product of the perpendicular pressure $e\,f$, the coefficient of friction and the distance traversed in a revolution, which is the length of one turn of the thread. Now, if the angle of the thread be doubled, as indicated, the load P remaining the same, the new perpendicular component $f\,h$ of P will be slightly reduced from the old value $e\,f$, while the length of a turn of the thread will be slightly increased. Consequently their product and the lost work of friction per revolution will not be much changed. The useful work per revolution will, however, be doubled, because, the pitch being doubled, the distance traveled by P in one revolution will be doubled. For a given amount of useful work the amount of work lost is therefore reduced by the increase in the thread angle, and, since the tendency to heat and wear is the immediate result of the lost work, it follows that that tendency is reduced. For small angles of thread the change is very rapid, and continues, though in diminishing degree, until the angle reaches a value not far from 45 degrees, when the conditions change and the lost work increases faster than the useful work, an increase of the angle of the thread beyond that point reducing the efficiency.

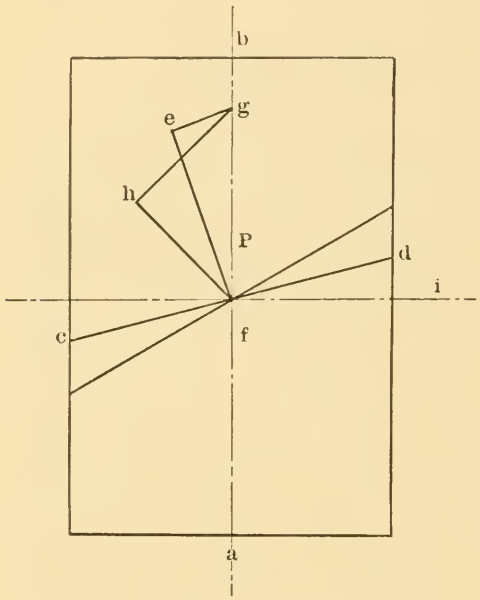


FIG. 128. THE PRINCIPLE OF WORM EFFICIENCY.

This general consideration of the subject shows the principles at the bottom of successful worm design, but a more exact examination is desirable. According to Professor Barr the efficiency of a worm gear, the friction of the step being neglected, is:

$$e = \frac{\tan\alpha\,(1 - f\tan\alpha)}{\tan\alpha + f}$$

in which

e = efficiency,
α = angle of thread, being the angle $d\,f\,i$ of Fig. 128,
f = coefficient of friction.

To study the effect of the step, a convenient assumption is that the mean friction radius of the step is equal to that of the worm. This assumption

would be realized only in cases where the step is a collar bearing outside the worm shaft, and the preceding and following formulas therefore represent extreme cases, one of a frictionless step, which would be approximated by a ball bearing, and the other of a step having about the extreme friction to be met with. Most actual cases would therefore fall between the two. Again, according to Professor Barr, the efficiency of a worm and step on the above assumption is: *

$$e = \frac{\tan \alpha\ (1 - f \tan \alpha)}{\tan \alpha + 2f} \text{ (approximately).}$$

Notation as before.

These formulas give no clear indication of the manner in which the efficiency varies with the angle, and Chart 10 has been constructed to show this to the eye. The scale at the bottom gives the angles of the thread from 0 to 90 degrees, while the vertical scale gives the calculated efficiencies, the values of which have been obtained from the equations and plotted on the diagram. The upper curve is from the first equation, and gives the efficiencies of the worm thread only; while the lower curve, from the second equation, gives the combined efficiency of the worm and step. In the calculations for the diagram it is necessary to assume a value for f, and this has been taken at 0.05, which is probably a fair mean value. The experiments made by Mr. Wilfred Lewis for Wm. Sellers & Co. showed an increase of efficiency with the speed. The present diagram may be considered as confined to a single speed, and at the same time is not to be understood as showing the exact efficiency to be expected from worms, but rather to exhibit to the eye the general law connecting the angle of the thread with the efficiency.

The curves will be seen to rise to a maximum and then to drop. The exact values of the angle of thread to give maximum efficiency may be easily found by the methods of the calculus, the results being:

For worm thread alone the efficiency is at a maximum when

$$\tan \alpha = \sqrt{1 + f^2} - f.$$

Substituting the value of f (0.05) used in calculating the diagram, this becomes $\tan \alpha$ for maximum efficiency = 0.9512, and by referring to a table of natural tangents we find that α for maximum efficiency = $43° \, 34$.

Similarly for the worm and step the result is $\tan \alpha$ for maximum efficiency =

$$\sqrt{2 + f\,4^2} - 2f, \text{ which for } f = 0.05 = 1.318,$$

and a table of tangents tells us again that α for maximum efficiency = $52^2\,49'$.

Of more importance than the angle of maximum efficiency is the general

* In Professor Barr's formulas it is assumed that the worm thread is square in section. Thread profiles in common use affect the results but little.

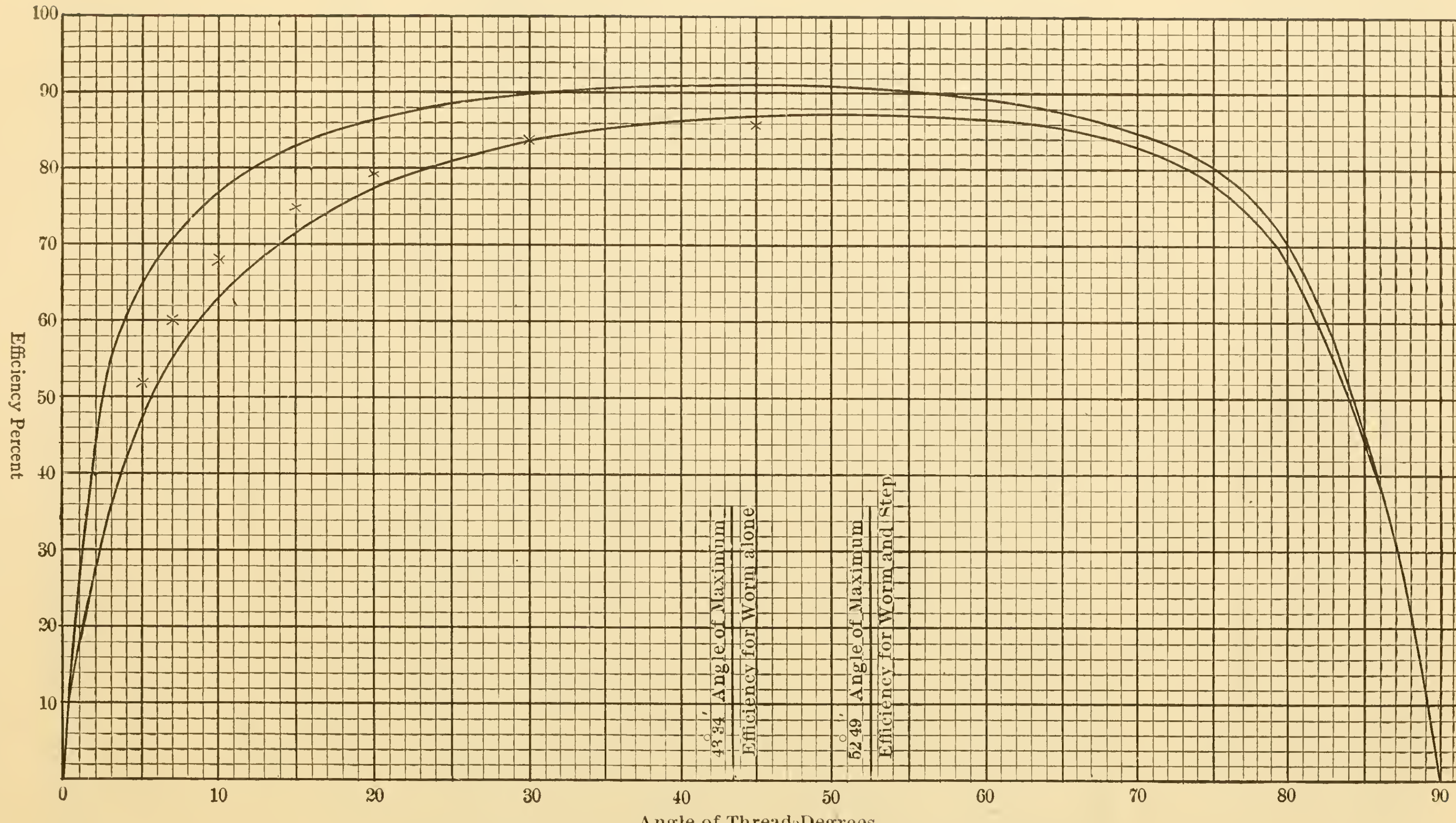


CHART 10. RELATION BETWEEN THREAD ANGLE AND EFFICIENCY.

character of the curves, of which the most pronounced peculiarity is the extreme flatness, showing that for a wide range of angles the efficiency varies but little. Thus, for the upper curve there is scarcely any choice between 30 and 60 degrees of angle, and but little drop at 20 degrees.

At first sight the lower curve might be thought the more useful of the two, as it includes the effect of the step, but a little consideration will show that this is not the case. For most cases in which worms are used the efficiency of the transmission, as such, is of very little account. What the designer concerns himself with is the question of durability and satisfactory working, and the results to be expected in this respect are best shown by the upper curve, in which high efficiency means a durable worm. Throughout this discussion, in fact, the chief significance of efficiency lies in the fact that low efficiency means rapid wear, and vice versa.

EXPERIMENTAL CORROBORATION OF THE THEORY

The experiments of Wm. Sellers & Co., before referred to, go far to confirm the soundness of the above views. From the present standpoint it is unfortunate that those experiments did not cover a wider range of worm-thread angles—those actually used being 5 degrees, 7 degrees, and 10 degrees. Other experiments were, however, made on spiral pinions of higher angles, spiral pinions being understood by Mr. Lewis to mean those pinions having the mating gear a true spur, the pinion shaft being at a suitable angle with the gear shaft to bring the pinion in proper mesh—a construction which is exemplified in the well-known Sellers planer drive. Mr. Lewis gives a formula by which the efficiencies of worms can be calculated from those for spiral pinions, and in the absence of direct experiments on worms of high angles, his results for spiral pinions have been modified by this formula to read for worms. The results for the two forms of gearing differ by less than five per cent. for the extreme case of his experiments. To compare the results obtained by Mr. Lewis with Professor Barr's formula, a speed has been selected from the experiments giving the nearest coefficient of friction to that used in obtaining the curves of Chart 11. The results have been plotted in Chart 11, where they appear as small crosses, and will be seen to have a very satisfactory agreement with the lower curve, with which they should be compared, as the steps of the worms used by Mr. Lewis were of the usual pattern without balls.

The variation of the coefficient of friction with the speed lends an interest to Chart 11, which is a series of curves obtained from the results published by Mr. Lewis in the same manner as the crosses of Chart 10, the curve for 20 feet velocity being in fact the same as that appearing as crosses on Chart 11. The other curves of Chart 11 are obtained from those of Mr. Lewis, and cover a

range of velocities from 3 to 200 feet per minute at the pitch line, as noted at the right. In this diagram the results obtained by Mr. Lewis on worms are plotted direct, but the experiments on spiral pinions have been modified as explained above. Inspection of the curves shows that while there is a progressive increase of efficiency with the speed, there is, nevertheless, not much

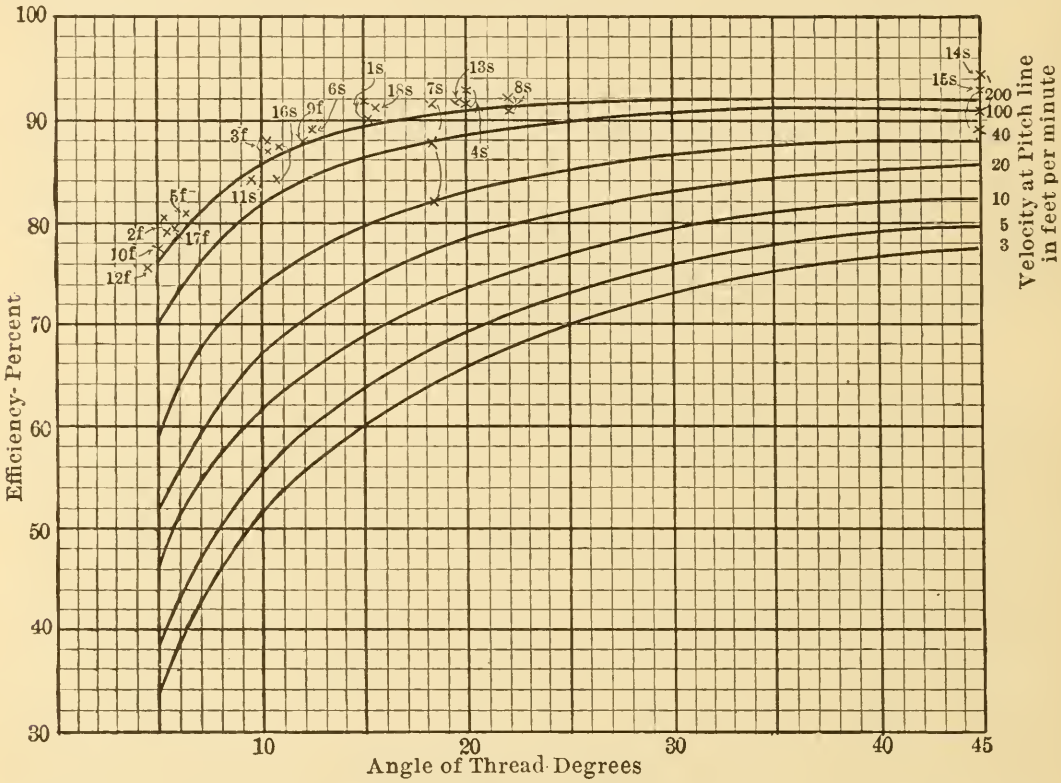


CHART II. RELATION BETWEEN THREAD ANGLE, SPEED AND EFFICIENCY WITH CASES FROM PRACTICE.

probability, or indeed room, for further improvement beyond the speed of 200 feet per minute. It will furthermore be seen that the efficiency drops off much less for low angles of thread at high speeds than at low.

In interpreting this diagram, it should be remembered that the durability of a worm depends upon the amount of power lost in wear, and not upon the percentage so lost. The ability of a given worm to absorb and carry off the heat due to friction is fixed, and does not vary with the speed. That is, a given worm running at 100 revolutions under a given pressure can carry off as much friction heat as the same worm at 200 revolutions, while it, under the same pressure, would transmit but one-half the power in the former case that it would in the latter. In other words, the percentage of lost work might be twice as much at the lower speed as at the higher without increasing the tendency to heat.

The increase of efficiency with the speed is a valuable property of worms, and enables them to do much more work than they otherwise would. Thus the 20 degree worm at 20 feet per minute lost 21½ per cent. of the work in friction. Increasing the speed to 40 feet doubled the work applied, and, had the efficiency remained constant, would have doubled the friction heat to be dissipated. In point of fact, this increase of speed diminished the percentage of loss to 17, and the amount of loss and heat, instead of being doubled, was only increased in the ratio of 160 to 100. It is plain from the diagram, however, that this action does not continue much beyond a velocity of 200 feet per minute, beyond which the amount of loss must be more nearly proportional to the speed, and this doubtless has some connection with the fact observed by Mr. Lewis that 300 feet per minute is the limit of speed when the gears are loaded to their working strength, and that the best conditions are obtained at about 200 feet per minute. It is proper to add, however, that in the cases from practice given later there are three which have been made repeatedly, and which are conspicuously successful, in which the velocity exceeds 600 feet, and one in which it exceeds 800 feet. No doubt, in all such cases, if the pressure on the teeth could be known it would be found to be light.

It will be seen that an increase of speed for any worm under constant pressure leads to an increase of friction work, and the limit is reached when the worm is no longer able to carry off the heat generated fast enough to prevent undue rise in temperature. Furthermore, this limiting speed depends upon the pressure, it being higher for low pressures than for high. A worm having an angle which might be successful at low speed may fail at high speed; but it would seem that any worm which is successful at high speed should also be successful at low, which is in accordance with mechanical instinct.

There are, it will be observed, two methods of increasing the pitch angle. The diameter may be kept constant and the pitch be increased, or the pitch may be kept constant and the diameter be reduced. From a mathematical standpoint, these two methods are identical; that is, at a given pitch-line velocity a worm of a given angle should have the same efficiency, regardless of the diameter; but in a mechanical sense the methods are not identical. The worm of the larger diameter would naturally have a gear of wider face, and the pair, having greater area of tooth surface in contact, would carry a larger load.

EXAMPLES FROM PRACTICE

It is impossible to say who was the first to recognize the significance of the pitch angle as a factor in the satisfactory performance of worm gearing, but it may be mentioned as a matter of interest that the exhibit of the Hewes & Phillips Iron Works at the Newark Industrial Exhibition of 1873 included

several worm-driven planers, in which the worms were double-threaded and had a pitch angle of 15° 15′, a pitch diameter of 3½ inches, a lead of 3 inches, and a speed cutting of 256 and backing of 640 r. p. m., which give pitch-line velocities of 237 and 590 feet. This worm was successful, and was many times repeated; but later on Hewes & Phillips were struck by the high-belt speed idea, and in order to increase the belt speed they changed the worm to 6.16 p. d., 1¾ inches pitch, single thread; speed cutting, 446, and backing 1,110 r. p. m., giving a pitch angle of 5° 15′ and pitch-line velocities of 720 and 1,780 feet. This worm was a failure, and was soon changed to 6.16 p. d., 3½ inches lead, double-thread, speed cutting, 281, and backing 700 r. p. m., giving an angle of 10° 15′ and pitch-line velocities of 452 and 1,130 feet. This worm did better than the last, but not so well as the first. By this time the lesson was learned, and Hewes & Phillips set out to use a worm of 30 degrees pitch angle. Structural considerations, however, prevented the use of so high an angle and they compromised on 20 degrees, the final worm resulting from this experience having a pitch diameter of 2.63 inches, with 3 inches lead, quadruple thread, the speed cutting being 300 and backing 700 r. p. m., giving pitch-line velocities of 205 and 480 feet, and this remained the standard angle as long as these planers were manufactured.

The writer has seen one of these 20-degree worm gears, opened up after twelve years' use, and the wear disclosed was very slight—no shoulder being in existence. As a result of the experience outlined above, this house adopted the standard practice of the worms as small as possible in diameter, and giving the threads in all cases a pitch angle of 20 degrees. The form of tooth used was the epicycloidal, while the materials used were hard cast iron for the gear and case-hardened open-hearth steel for the worms.

These Hewes & Phillips worms are plotted in Chart 11 as crosses 1, 2, 3, 4, of which 1 is the 15° 15′, 2 the 5° 15′, 3 the 10° 15′, and 4 the 20°, the first and last being successes, and the second and third failures.

In plotting these worms, and all others having pitch-line velocities above 200 feet, the crosses are placed near and above the 200 feet curve. It is unfortunate that we have no curves for higher speeds, but Mr. Lewis recommends the use of the 200 feet line for all higher speeds. Leaders connecting different crosses indicate the same worm at different speeds in all cases. The letters *s* and *f* on the diagram mean success or failure in all cases.

Fig. 129 is a drawing of a worm 3 (failure) and Fig. 130 shows worm 4 (success), and no more instructive pair of drawings could be imagined than these. The pitches are not far different, and what difference there is is in favor of the larger worm. The duty is the same, the gears are of about the same diameter, and the revolutions per minute are nearly the same. The essential change is

in the increase of the pitch angle by a reduction of the diameter, and this changed failure to success.

The Newton Machine Tool Works use worm gearing in many of their machines, notably their cold saw cutting-off machines. In the earlier machines of this class the worm had a pitch diameter of 2⅞ inches, with a pitch of 1 inch, single thread, the revolutions per minute being 765. These figures give

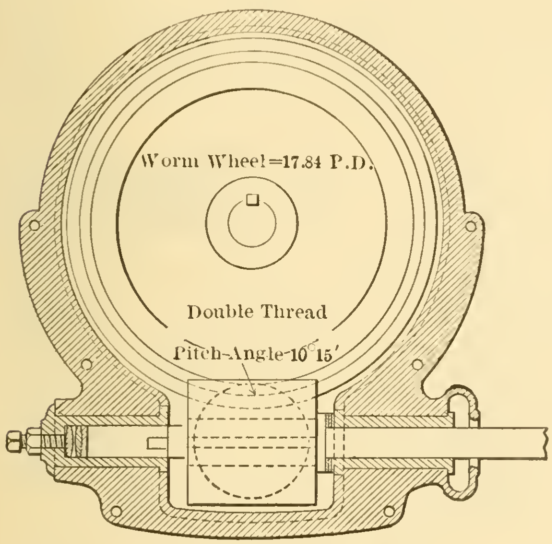


FIG. 129. HEWES & PHILLIPS UNSUCCESSFUL WORM.

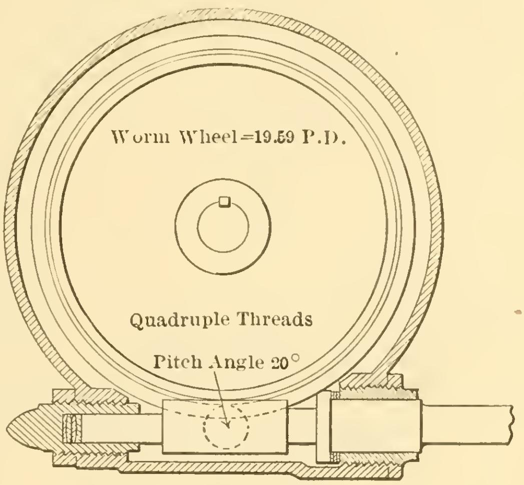


FIG. 130. HEWES & PHILLIPS SUCCESSFUL WORM.

a pitch angle of 6° 20′, and a pitch-line velocity of 572 feet. This machine could be operated, but not with satisfaction on account of the heating and short life of the worm. The worm was then increased in lead by making it double-threaded, giving a pitch angle of 12° 30′, the speed being reduced to 500 revolutions per minute, giving a pitch-line velocity of 375 feet. The change proved to be a great improvement, heavier work than was before possible being done after the change without distress or difficulty, and this worm has since been applied to a large number of machines with entire success. A still later worm used on these machines has a pitch diameter of 3⅞ inches and a lead of 4 inches, triple threads, giving a pitch angle of 18° 15′, and this is found to be a still further improvement. This last worm is used on a wide variety of machines and at a variety of speeds from 40 to 680 r. p. m., giving pitch-line velocities of from 40 to 685 feet, and with uniformly good results. In many cases it is used without an oil cellar, though for comparatively light work. The form of thread used is the involute, and the material is hardened steel for the worm and bronze for the wheel. These Newton worms appear in Chart 11 as 5, 6, 7, of which 5 is nearly a failure, while 6 and 7 are entirely

successful. The second Newton worm—the one appearing in Chart 11 as 6—is shown in Fig. 131.

Another habitual user of worms is John Bertram & Sons, of Dundas, Ontario, Canada, who employ them in all their planers, and use largely a worm of 3.18 inches pitch diameter, 4 inches lead, quadruple threads, the speed cutting being 186 and reversing 744 r. p. m. These figures give a pitch angle of 22 degrees, and pitch-line velocities of 155 and 620 feet. This worm appears in Chart 11 as 8, the vertical position for the higher speed being again uncertain. These worms are highly successful, as the writer knows from repeated obser-

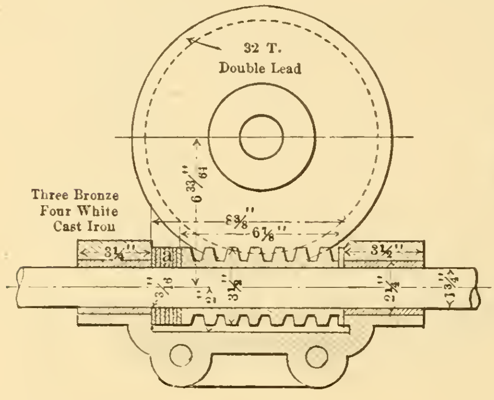


FIG. 131. THE NEWTON WORM AND STEP.

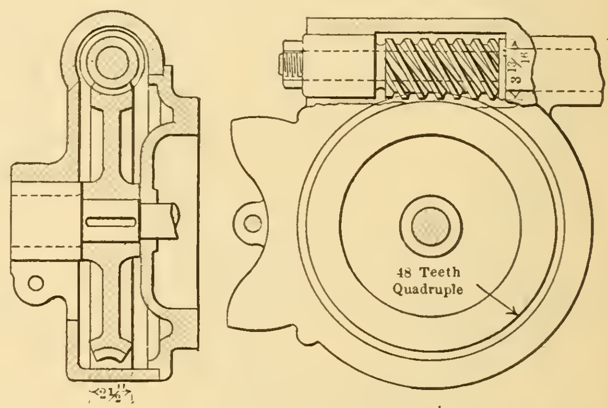


FIG. 132. THE BERTRAM WORM.

vation. Both worm and wheel are of cast iron, the thread being Brown & Sharpe standard. The Bertram worm is shown in Fig. 132. In reading this drawing it should be remembered that the conventional representation of a worm, with the threads shown by straight lines, shows a larger apparent pitch angle than the true one, as shown by a true projection.

Another case of failure was a worm drive applied to a large boring machine, the worm being 12 inches pitch diameter, 8 inches lead, quadruple thread, speed 80 r. p. m. and above, worm of forged steel, wheel of bronze, oil cellar lubrication. These figures give a pitch angle of 12 degrees and a pitch-line velocity of 250 feet. This worm is located on Chart 11 as 9.

Still other cases of change from failure to success are supplied by Mr. Jas. Christie, of the Pencoyd Iron Works. The first of these relates to a boring machine, which was, by the makers, supplied with a worm drive having a worm of 5½ inches pitch diameter, 1½ inches pitch, single thread, steel worm and cast-iron wheel, average speed 150 r. p. m. These figures give a pitch angle of 5 degrees and a pitch-line velocity of 215 feet. This was a failure, but was successfully replaced by a worm of 4⅞ inches pitch diameter, 2½ inches lead, and the same number of revolutions, which figures give a pitch angle of 9°

15′ and a pitch-line speed of 190 feet. These two worms appear as 10 and 11. This successful worm lies in the region of unsuccessful ones, but the influence of the increased lead angle is unmistakable. The fact of its success is probably due to the pressure on the teeth being well below the working strength, or to the speed being moderate, or both.

The second case, of which the data were supplied by Mr. Christie, relates to two heavy milling machines, in which the cutter spindles were driven by worms 6 inches pitch diameter by 1½ inches pitch, single thread. It was found that the cutters could be run much faster than was orginally contemplated, and the worms were consequently speeded up to about 500 r. p. m. In these machines cast-iron worm-wheels were speedily destroyed, while hardened steel worms and bronze wheels would last about a year. Later two more machines were built having steel worms and bronze wheels, the worms being 4½ inches pitch diameter by 5 inches lead, quadruple threads, speed 280 r. p. m. These worms have been in use six years, and are described as being "good as new." The data given for the first worm give a pitch of 4° 30′ and a pitch-line velocity of 785 feet. It appears in Chart 11 as 12. The pitch angle of the second worm is 19° 30′, and its pitch-line velocity 328 feet. It appears in Chart 11 as 13.

Mr. Christie has made many successful changes, of which these are typical, and he now uses worms with great freedom and success. His general conclusion is that good worms begin with those having the pitch about equal to the diameter, giving a pitch angle of 17° 15′.

Another equally striking case of success accompanying an increase of the pitch angle is supplied by Mr. W. P. Hunt, of Moline, Ill., who says:

"In building a special double-spindle lathe I wished to use a worm drive, and having a single thread ¾-inch pitch hob, 2¾-inch outside diameter, I decided to work to that, and made my gear with 26 teeth, giving a speed reduction of 26 to 1. The worm was to run at 460 revolutions per minute, but upon starting the machine I found it impossible to keep the worm and gear cool, and the belts would not pull the cut.

"Accordingly I decided to make a new worm and hob having the same outside diameter as the one first tried, but with double thread and 1-inch pitch, 2-inch lead and a new gear having 48 teeth, giving me a speed reduction of 24 to 1, or less than at first.

"Upon starting the machine with the new worm and gear, not only did it run perfectly cool, but the belts have ample power. We use graphite and oil on the worm, and it is not enclosed."

Mr. Hunt does not give the pitch diameter of his worms, but assuming the threads to have been in accordance with the Acme standard, the pitch diameters

are $2\frac{3}{8}$ and $2\frac{1}{4}$ inches respectively, the thread angles being 5° 44′ and 15° 48′, and the pitch-line speeds 286 and 271 feet per minute. Mr. Hunt's worms are plotted in Chart 11 as 17 and 18.

Three other cases of successful worms under heavy duty are found in milling machines which have been repeated many times. The first two worms would ordinarily be described as spiral gears. The shafts are at right angles.

The first of these, which appears as 14 in the diagram, has a pitch diameter of $2\frac{1}{4}$ inches, a pitch angle of 45°, and a speed varying between 180 and 945 r. p. m., giving pitch-line velocities of 106 to 555 feet per minute. Both gears are of cast iron. The second, 15 in the diagram, is of the same style, and has the same pitch diameter, with speeds varying between 90 and 472 r. p. m., giving pitch-line velocities of 53 to 277 feet per minute. The third, 16 in the diagram, is a true worm, $2\frac{1}{4}$ inches pitch diameter, lead 1.333, triple thread, speed 200 to 1,442 r. p. m., bronze wheel and hardened steel worm. These figures give a pitch angle of 10° 45′, and a pitch-line velocity of 118 to 845 feet per minute. While this worm is entirely successful, it was at first a failure.

LIMITING SPEEDS AND PRESSURES

A very important point connected with worm design, and one on which data are very scarce, is the limiting pressures for various speeds at which cutting begins. The paper by Mr. Lewis contains some information on this subject, and the accompanying table supplied by Mr. Christie, from experiments made by him, supplies the most definite additional data on the subject known to the writer. In all cases the worms were of hardened steel and the worm-wheels of cast iron. Lubrication by an oil bath.

	Single-thread worm 1″ pitch $2\frac{7}{8}$ pitch diameter				Double-thread worm 2″ lead $2\frac{7}{8}$ pitch diameter			Double-thread worm $2\frac{1}{2}$″ lead $4\frac{1}{2}$ pitch diameter		
Revolutions per minute....	128	201	272	425	128	201	272	201	272	425
Velocity at pitch line in feet per minute..............	96	150	205	320	96	150	205	235	319	498
Limiting pressure in pounds	1,700	1,300	1,100	700	1,100	1,100	1,100	1,100	700	400

Limiting Speeds and Pressures of Worm Gearing.

There is real need of a comprehensive series of experiments on this subject. It is obvious enough that a worm, otherwise well designed, might fail from having too high a speed for its load. Were such data at hand it would seem that

with existing knowledge of the influence of the angle of the thread, worm design might be made a matter of comparative certainty. Especially should the behavior of worms at speeds above 200 feet per minute be subjected to further experiment, as it is frequently necessary to use speeds above that figure, and there can be no doubt that higher speeds are entirely feasible if suitable pressures accompany them. The speed as a factor should be kept in mind equally with the pitch angle. A worm may fail because of too high a pitch-line velocity as well as because of too low a pitch angle.

The number of cases cited is too few for certainty in drawing general conclusions, but the testimony is unmistakable in its confirmation of the theory of the influence of the angle of the thread. It will be seen that every case having an angle above 12° 30′ was successful, and every case below 9° unsuccessful, the overlapping of the successful and unsuccessful worms in the intervening region being what is to be expected in the border region between good and bad practice. This band of uncertain results is in fact narrower than we would have any right to expect from a collection of data from miscellaneous sources, and could the inquiry be widened in scope the width of this band would doubtless be increased. As throwing light on these cases, it should be remembered that case 16 is known to have been made successful only by careful attention to the material used, the first worms made having been failures, and that 3, which is near 16 and was a failure, had an excessive speed, while 11, at a lower angle and a success, had a very moderate speed. At a higher speed 11 would probably have failed, and at a lower speed 3 would probably have been a success. It is believed that Chart 11 points out clearly the nature of the worm problem and the conditions of success in its solution.

STEP BEARINGS

The step bearings of worms have been a source of trouble alongside of the worm itself. It is obvious enough that the increase of the thread angle will relieve the step of wear as well as the worm thread. Expedients are possible with the step, however, which are not available with the worm. Ball and roller step bearings have been extensively tried, and while some have been successful with them, the general results are believed to have been unsatisfactory. The troubles from ball bearings arise from the tendency of the balls to break up under heavy loads and to score the pressure plates, while conical rollers, which geometrical considerations call for, have in some instances made trouble from their outward radial pressure cutting out the confining ring.* The multiple

* Mr. C. R. Pratt has had marked success with roller thrust bearings in which the rollers were short cylinders kept in position by a distance plate or cage having suitable openings arranged in spirals. See THE AMERICAN MACHINIST, June 23, 1901.

washer thrust bearing is used by many, and is undoubtedly entirely successful. Many of the readers of this volume have no doubt seen this pattern of bearing without reflecting upon the principle which lies at the bottom of it. When several loose washers are interposed between the shaft collar and the face of the shaft bearing, it is obvious that slipping may occur between any pair of faces, and that this slipping will take place between those surfaces which at the moment offer the least friction. Should these surfaces from any cause increase their resistance the slipping will be at once transfered to another joint, the various surfaces acting as mutual safety valves to one another, any surface which gets into the condition of incipient heating or cutting being at once relieved by another taking up the work. Fig. 131 shows one of these bearings as made by the Newton Machine Tool Works. The Newton Works formerly followed the usual practice and made the washers *a* alternately of hardened steel and bronze, but consider that they have improved on this by substituting white cast iron for the steel. These castings are obtained from the malleable iron founderies, and are in fact unannealed malleable castings. Of course these castings cannot be machined, and they are therefore prepared for use by grinding on a cup-shaved emery wheel. They are dropped into a socket on the end of a shaft, which is revolved by hand, and are thus presented to the face of the cup-shaped wheel. This plan results in the grinding marks crossing the faces in all directions, instead of being circles as in lathe-finished pieces. All mechanics understand the advantage of having the tool marks in a direction different from that of the motion of the parts, which advantage this method of construction secures. Another feature of the Newton bearings consists in making the holes in the washers larger than the shaft on which they are placed. This construction introduces an irregular compound motion of the surfaces upon one another, the advantages of which are well understood. In the Newton washers the holes are $\frac{1}{32}$ inch larger than the shaft, though Mr. Newton considers that this might be increased with probably good results.

Three radial oil grooves are cast in each face of these disks, the use of which is obvious. The Hewes & Phillips step, at the right-hand bearing, will be seen to include three washers, while the left-hand step has lenticular-shaped disks—a construction which has also found favor elsewhere.

INVESTIGATION OF A THREE-THREAD WORM

The following article by C. Bach and E. Roser was translated, in abstract, from *Zeitschrift des Vereines Deutscher Ingenieure* by E. P. Buffet. The experiments were made at the engineering laboratory of the Royal Technical High School, Stuttgart. Originally published in AMERICAN MACHINIST, July 16, 1903.

I. GEARING, APPARATUS AND CONDUCT OF THE EXPERIMENTS

Worm—Left hand.

Material—Steel not hardened.

Pitch diameter of worm—76.6 mm. [3.0167 inches].

Exterior diameter of worm—93.5 mm. [3.6810 inches].

Interior diameter of worm—56 mm. [2.2050 inches].

Profile: Trapesium having an inclination of the flanks to the axis of—75 degrees.

Pitch t—25.4 mm. [1 inch].

Lead 3 t—76.2 mm. [3.015 inches].

Lead or helix angle—$17° 34'$.

Length of worm—148 mm. [5.827 inches].

Worm-wheel—Bronze with teeth.

Number of teeth—30.

Pitch diameter—242.6 mm. [9.511 inches].

Pitch t—25.4 mm. [1 inch].

Breadth of tooth measured on the arc at the roots of the teeth b—78 mm. [3.071 inches].

Breadth of wheel measured parallel to its axis—70 mm. [2.756 inches].

Included angle of face of teeth 2β—91 degrees.

Speed ratio—1 : 10.

Center distance of axes—159.6 mm. [6.283 inches].

ARRANGEMENT FOR THE EXPERIMENTS

The apparatus used in the experiments is shown in Figs. 133 to 137.

Figs. 136 and 137 show the gearing, in a framework which has openings closed on both sides of the worm with plates of glass. The worm shaft, which is of one piece with the worm rests in two bearings A and B with ring lubrication. The axial pressure of the worm is supported by the ball bearing C. In order that the shaft may be rotated in either direction the ball bearing is doubled.

The mechanical work delivered by the line shafting is measured with a tooth-pressure dynamometer, shown in Fig. 135. That given out through the worm-wheel is determined by means of a brake shown in Figs. 133 and 134.

The loss of power through friction (so far as considered) in the dynamometer and its appurtenances up to the movable coupling G, and therefore including the friction in the bearings H and J, is determined when running empty.

The lubricant employed was an extremely viscous steam cylinder oil ("Heinzoline," marked "Extra"). It was placed in the framework so liberally that the lower part of the teeth of the worm-wheel dipped in it as shown in Fig. 137.

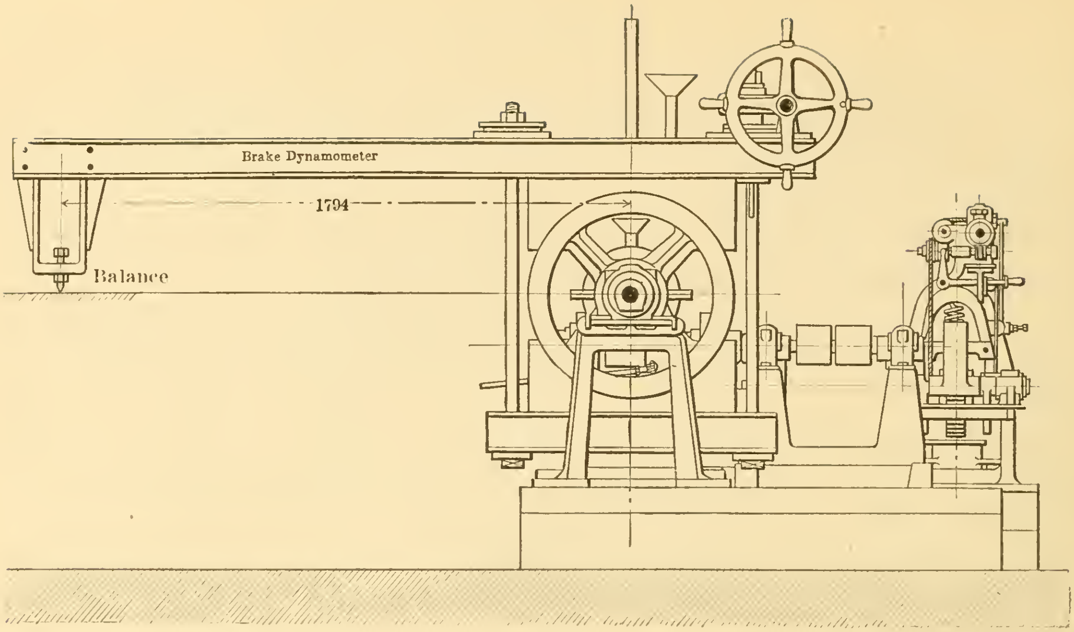


FIG. 133.

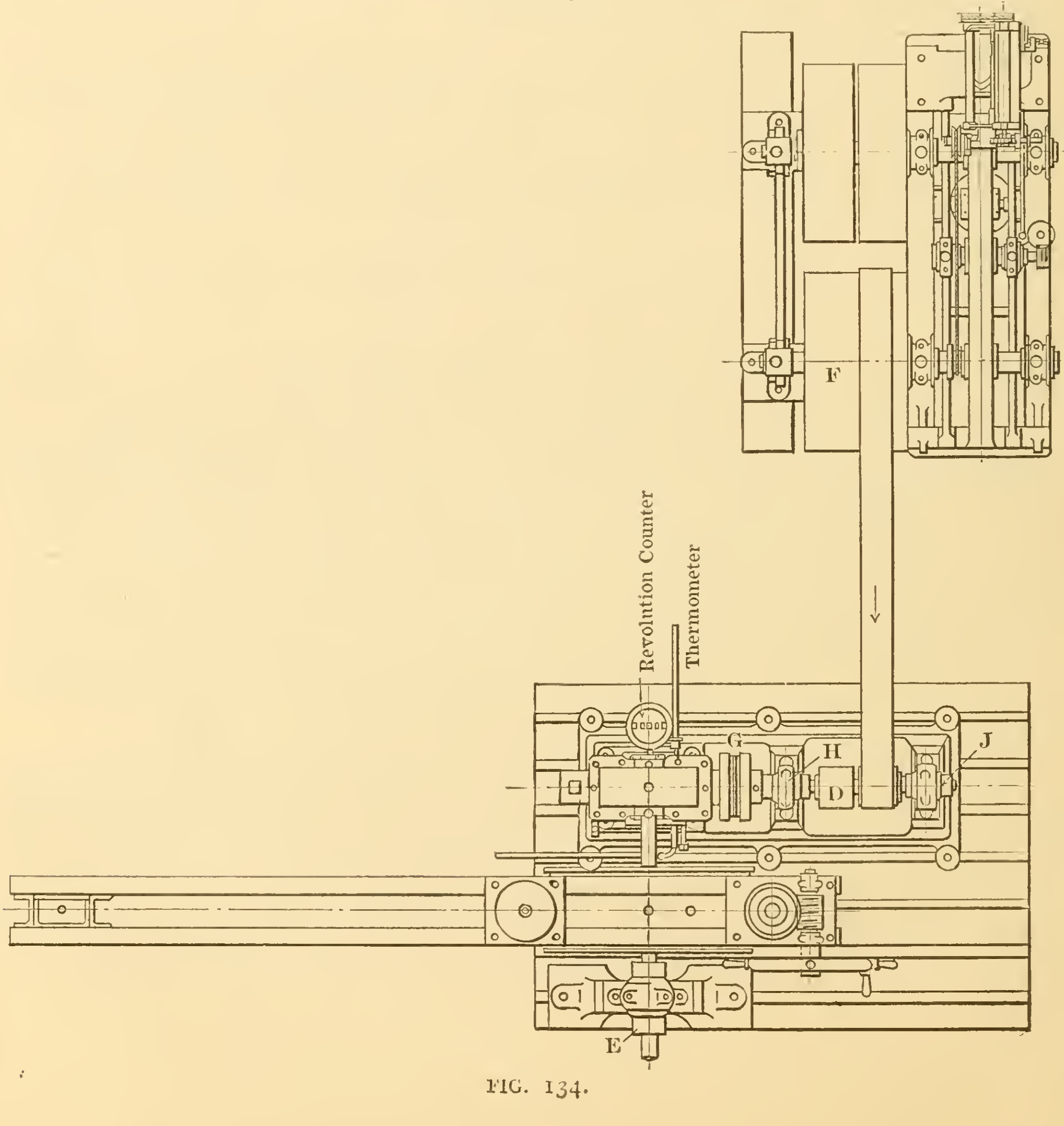


FIG. 134.

When the worm gearing was loaded, diagrams were taken on the tooth-pressure dynamometer, as a rule, every five minutes. As often the oil temperature was observed and readings were taken from the revolution counter on the dynamometer and on the worm wheel shaft.

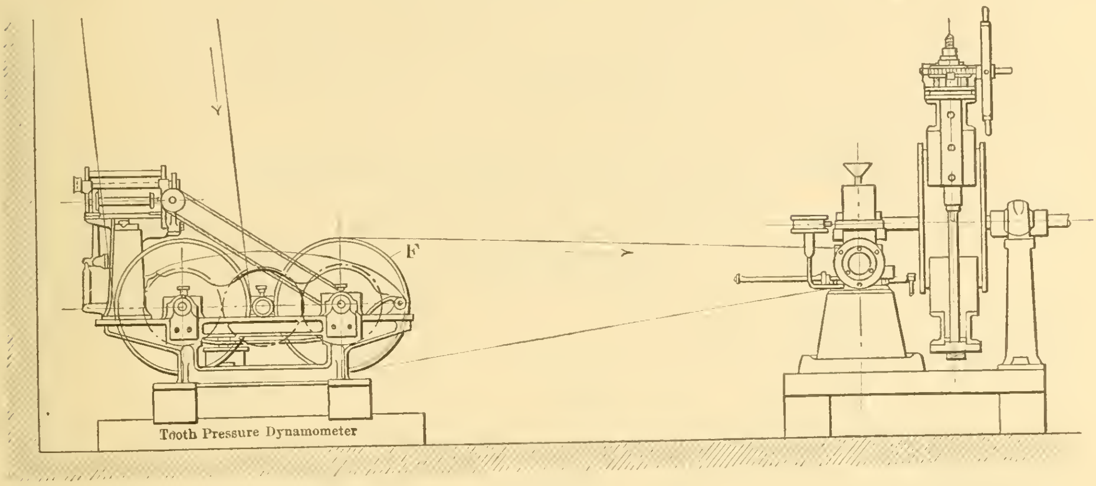


FIG. 135.

Diagrams were taken from the apparatus running free with the worm gearing disconnected (coupling *G* loosened) both before and after each test with a load, commonly to the number of ten or eleven.

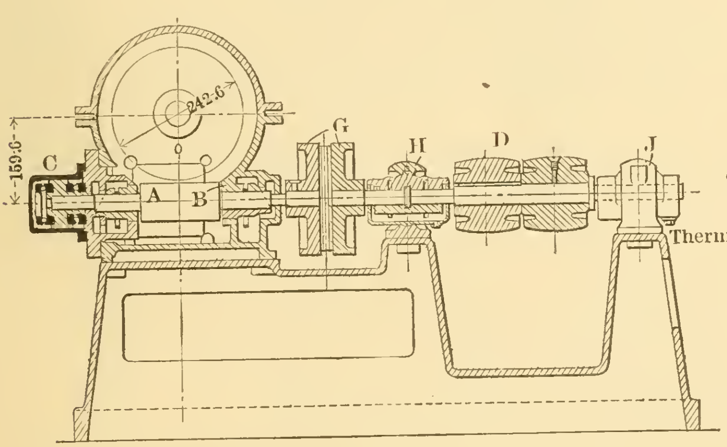


FIG. 136.

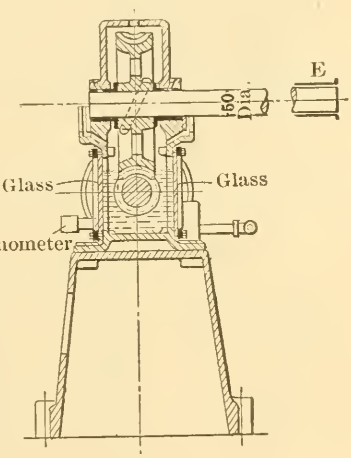


FIG. 137.

Before beginning the experiments proper, the gearing was placed for a considerable time (1 to 2 hours) under conditions similar to those aimed at therein. It was then put at rest and the experiment began after it had cooled.

The result sought to be determined by the investigation was as follows: *To ascertain how high the temperature of the oil rose for different loads and speeds of*

the gearing. Therefore the experiments under loaded gearing were continued until the oil temperature ceased to rise further.

In single cases the test was discontinued when the increase of temperature amounted to only 0.1 degree in 5 minutes.

The length of the experiments was from 55 minutes to 6 hours 55 minutes.

The average sliding velocity of the worm varied within the limits $v = 0.26$ and 8.61 meters per second, corresponding to a speed of the worm of 64 and 2,145.5 turns a minute.

The tooth pressure varied between 111 and 1,257 kilograms.

RESULTS OF EXPERIMENTS *

EXPERIMENT I

Lever arm of brake—1794 mm.

Effective weight on brake Q—7.5 kg.

Pressure on teeth $P = \frac{7.5 \times 1794}{121.3} = 110.9$ kg.

Letting $P = k\ b\ t$ (that is $110.9 = k \times 7.8 \times 2.54$), we have for the value of the coefficient $k = 5.6$.

Therefore $P = 5.6.\ b\ t.$†

Duration of test—100 minutes.

R. P. M. of worm shaft n_s—2,185.2.

Sliding velocity at pitch circle—8.76 meters per second.

At the beginning of the test the temperature of the oil was 22.1 degrees centigrade. The increase as the test progressed is shown in Table 24.

TIME, MINUTES	OIL TEMPERATURE DEGREES CENT.	TIME, MINUTES	OIL TEMPERATURE DEGREES CENT.	TIME, MINUTES	OIL TEMPERATURE DEGREES CENT.
0	22.1	35	58.4	70	61.4
5	42.5	40	59.7	75	61.4
10	47.2	45	60.6	80	61.4
15	50.5	50	61.2	85	61.4
20	53.0	55	61.4	90	61.4
25	55.2	60	61.4	95	61.4
30	56.9	65	61.4	100	61.4

TABLE 24—RISE OF TEMPERATURE DURING THE PROGRESS OF EXPERIMENT I

* In the original text the results of the experiments are given in great detail. Inasmuch as they are summarized in Table 25, the detailed accounts are here considerably abbreviated.—ED.

† In finding the value of k, b and t are obviously taken in centimeters instead of millimeters as given in the table of dimensions. The last equation above thus means that the total load was equal to 56 kilograms per square centimeter of the product of the pitch, by the arc breadth of the wheel at the roots of the teeth.—ED.

The temperature of the surrounding air was 12 degrees Cent. at the beginning of the test; after 40 minutes it had risen to 12.8 degrees; it remained 25 minutes at this point and toward the end of the test sank to 12.5 degrees.

As is evident from the table, the oil temperature after 55 minutes reached a standstill; the difference in temperature between the oil in the frame and the surrounding air amounting to

$$61.4 - 12.8 = 48.6 \text{ degrees},$$

which sufficed to carry off the heat produced by the worm gearing.

In Fig. 138 the rise of temperature is set forth in relation to time. The abscissas represent time and the ordinates oil temperatures. The temperature line belonging to the foregoing experiment is designated "Exp. 1."

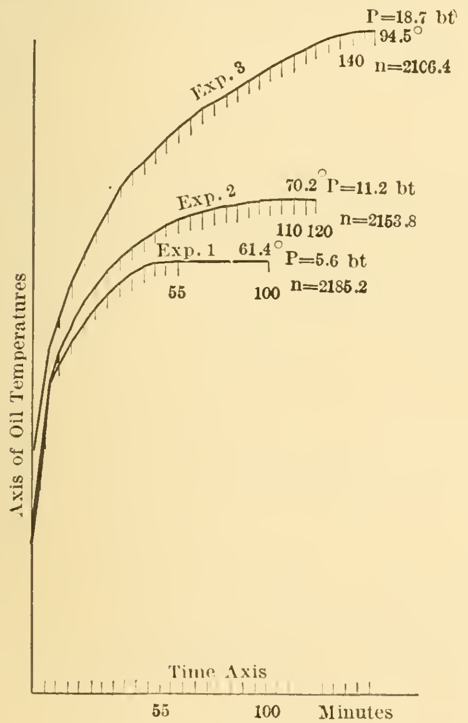


FIG. 138.

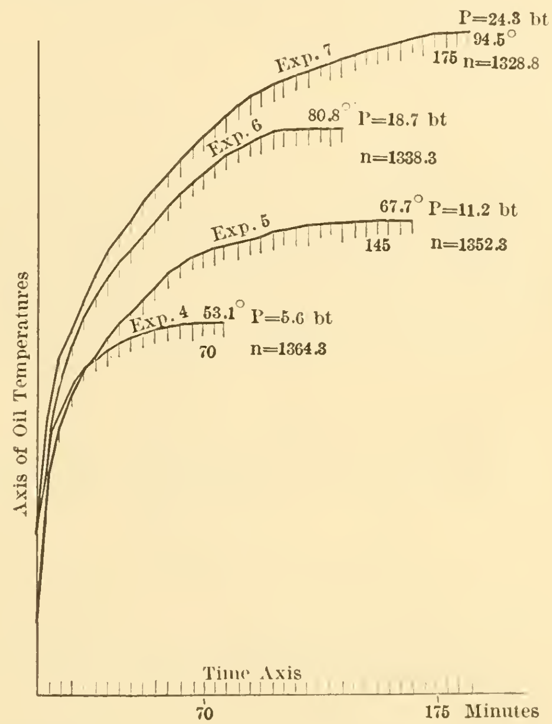


FIG. 139.

RELATION OF TEMPERATURE AND TIME.

The work measured on the brake is calculated as:

$$N = 4.10 \text{ H. P.*}$$

The work absorbed by the worm is found by taking the difference between the dynamometer readings with the worm under load and running free, and is found to have an average value throughout the run of

$$N_1 = 6.11 \text{ H. P.}$$

* All horse-powers are metric, and are 1½ per cent. smaller than English horse-powers.—Ed.

giving for the efficiency:

$$n = \frac{N}{N_1} = \frac{4.10}{6.11} = 0.67.$$

This result is, however, obviously too small, because it does not allow for the increased friction in the worm shaft bearings and dynamometer when under load.

Some supplementary tests, intended to elucidate this point, showed that the value $n = 0.80$ for experiment No. 12 of Table 25 should be increased to 0.84.

A glance at the readings of the dynamometer* shows plainly the influence of the oil temperature upon the consumption of power. Treating the first determination at the end of five minutes in the manner indicated above, gives for the temperature then prevailing: $n = 0.58$, while for the determination after a run of 80 minutes the value becomes $n = 0.72$.

EXPERIMENT 2.

This experiment is distinguished from the preceding chiefly by the fact that the effective load on the brake, and hence the pressure on the teeth, was chosen at double the former value. The speed was as nearly as possible the same and the leading quantities had the following values:

Q—15 kg.
P—221.8 kg.
P—11.2 *bt.*
Duration of experiment—120 min.
n_s—2,153.8.
v—8.63 m.

The results of this test are shown in Fig. 138 by the line "Exp. 2."

The temperature of the surrounding air was at the beginning of the test 10.5 degrees, and finally rose to 13.0 degrees.

The constant condition of oil temperature was attained after 110 minutes, with a difference in temperature between the oil and outer air of

$$70.2 - 13.0 = 57.2 \text{ degrees.}$$

The delivered horse-power was:

$N = 8.09$, and the average absorbed horse-power:
$N_1 = 10.28$, giving $n = 0.79$.

*Not here reproduced.—ED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Experiment No.	Effective load of brake.	Tooth pressure.		Duration of experiment.	Revolutions per minute of worm shaft	Sliding velocity at pitch circle of the worm v.	Temperatures				Temperature difference between oil and surrounding air at the constant condition.	Time from beginning of experiment to entrance of the constant condition.	Work measured on brake N	Work absorbed N_i.	Efficiency $= n = \frac{N}{N_i}$
							Of oil in frame of worm			Of the surrounding air at constant condition.					
		P	k in $P = kbt$				At beginning of exp.	45 min. after beginning of exper.	At constant condition.						
	kg.	kg.		min.	n_s	m.	°C.	°C.	°C.	°C.	°C.	min.	PS	PS	
1	7.5	110.9	5.6	100	2185.2	8.76	22.1	60.6	61.4	12.8	48.6	55	4.10	6.11	0.67
2	15.0	221.8	11.2	120	2153.8	8.63	25.2	65.0	70.2	13.0	57.2	110	8.09	10.28	0.79
3	25.0	369.7	18.7	145.	2106.4	8.45	28.5	76.1	94.5	14.0	80.5	140	13.19	16.52	0.80
4	7.5	110.9	5.6	80	1364.3	5.47	22.4	51.4	53.1	13.4	39.7	70	2.56	3.93	0.65
5	15.0	221.8	11.2	160	1352.3	5.42	8.0	56.0	67.7	13.2	54.5	145	5.08	6.83	0.74
6	25.0	369.7	18.7	130	1338.3	5.37	21.5	65.5	80.8	12.0	68.8	110	8.38	10.49	0.80
7	32.5	480.7	24.3	185	1328.8	5.33	21.2	70.4	94.5	12.5	82.0	175	10.81	13.40	0.81
8	15.0	221.8	11.2	220	695.2	2.79	10.5	36.6	48.9	12.0	36.9	205	2.61	3.29	0.79
9	25.0	369.7	18.7	220	692.4	2.78	13.0	44.6	63.9	10.5	53.4	210	4.33	5.32	0.81
10	32.5	480.7	24.3	205	689.6	2.76	18.5	51.9	77.1	10.5	66.6	195	5.61	6.87	0.82
11	40.0	591.6	29.9	255	686.4	2.75	11.0	62.9	97.6	12.0	85.6	. .	6.87	8.50	0.81
12	15.0	221.8	11.2	195	353.0	1.42	10.0	27.4	40.0	8.5	31.5	. .	1.33	1.66	0.80
13	25.0	369.7	18.7	415	351.2	1.41	13.4	32.4	59.0	11.3	47.7	. .	2.20	2.63	0.84
14	32.5	480.7	24.3	240	350.8	1.41	13.5	46.8	69.2	9.7	59.5	215	2.85	3.56	0.81
15	40.0	591.6	29.9	300	349.5	1.40	15.8	50.1	87.3	12.2	75.1	290	3.50	4.38	0.80
16	25.0	369.7	18.7	240	197.2	0.79	16.5	27.0	39.1	11.2	27.9	225	1.24	1.48	0.84
17	40.9	591.6	29.9	265	195.3	0.78	11.5	37.8	57.7	12.0	45.7	260	1.96	2.36	0.83
18	55.0	813.4	41.1	270	193.5	0.78	11.4	53.5	85.8	12.0	73.8	265	2.67	3.47	0.77
19	70.0	1035.3	52.3	235	193.8	0.78	13.0	60.8	105.4	10.5	94.9	220	3.40	4.52	0.75
20	40.0	591.6	29.9	210	64.4	0.26	9.5	29.5	40.3	12.0	28.3	195	0.65	0.90	0.72
21	70.0	1035.3	52.3	340	64.2	0.26	15.4	36.4	73.5	11.0	62.5	325	1.13	1.59	0.71
22	85.0	1257.1	63.5	310	63.7	0.26	14.2	39.7	89.8	12.2	77.6	295	1.36	2.08	8.65

TABLE 25—SUMMARY OF RESULTS OF AND DEDUCTIONS FROM THE EXPERIMENTS

EXPERIMENT 3

This test differed from the preceding chiefly in the effective load on the brake, which was taken at 25 kg., instead of 7.5 and 15 kg. The leading quantities had the following values:

Q—25 kg.
P—369.7 kg.
P—18.7 *bt.*
Duration of test—145 min.
n_s—2,106.4.
v—8.45 meters.

The temperature of the surrounding air was at the beginning of the test 12.50 and up to the end rose to 14 degrees.

The constant condition of the oil temperature occurred after 140 minutes, with a difference in temperature between the oil and surrounding air of

$$94.5 - 14.0 = 80.5 \text{ degrees.}$$

Fig. 138 exhibits the temperature increase in the curve, "Exp. 3."

The work taken off at the brake is calculated as $N = 13.19$ horse-power, and the average absorbed power as $N_1 = 16.52$ horse-power, giving $n = 0.80$.

EXPERIMENT 4

While in tests 1 to 3 the revolutions of the worm shaft were taken at above 2,100, in tests 4 to 7 it was reduced to a full 1,300. The leading quantities in test 4 were as follows:

Q—7.5 kg.
P—110.9 kg.
P—5.6 *bt.*
Duration of test—80 min.
n_s—1,364.3.
v—5.47 meters.

The temperature of the surrounding air was at the beginning of the test 12.8 degrees, and rose eventually to 13.4 degrees.

The constant condition of oil temperature ensued after 70 minutes, with a difference in temperature between the oil and external air of

$$53.1 - 13.4 = 39.7 \text{ degrees.}$$

Fig. 139 displays the increase of temperature in the curve "Exp. 4."

The work taken off at the brake is calculated as $N = 2.56$ horse-power, and the average absorbed power as $N_1 = 3.93$ horse-power, giving $n = 0.65$.

In like manner the remaining experiments were treated.* The results obtained are collected in Table 25.

The value of e obtained in the last column is always the average found in the respective experiment, as in the above cases Nos. 1 to 4.

A few other experiments were undertaken in which the temperature rose to 120 degrees, and would have gone still higher if it had not been restricted to this degree by pouring on cold oil. Fig. 140 shows the rise in temperature in two of these cases.

The conduct of these experiments, in which the surfaces sliding in contact would naturally have shown a higher temperature than 120 degrees, offered no further particular difficulty. They render it evident *that hight of temperature to which one may go in practice, up to a certain limit, is only an oil question.*

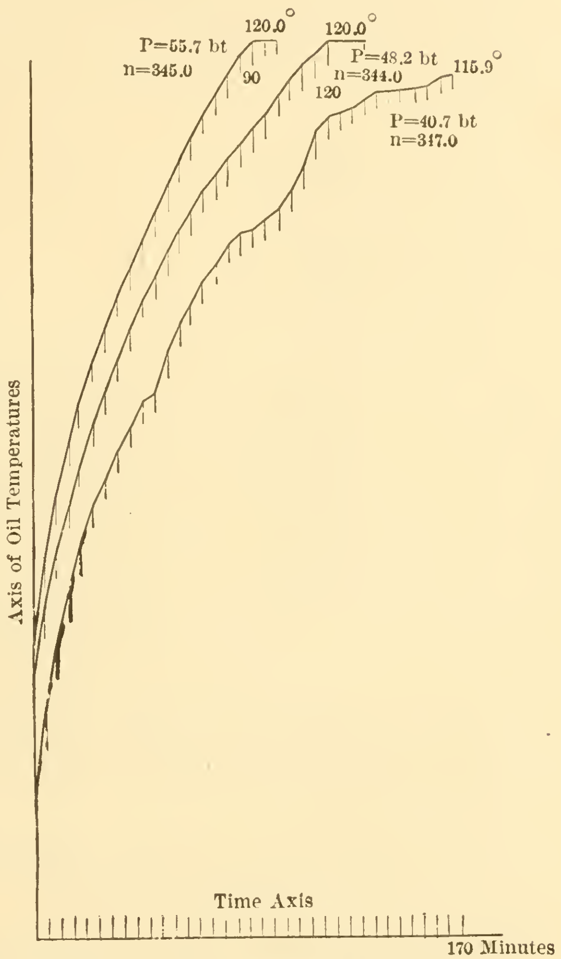


FIG. 140. RELATION OF TEMPERATURE AND TIME.

RESULTS OF THE EXPERIMENTS

1. *Relation between tooth pressure and the difference of oil and surrounding air temperature at the constant condition.*

We shall take up first experiments 1 to 3.

The sliding velocities in these three tests differ little, their average being 8.61 meters per second.

We use for ordinates in Fig. 141 the values of k contained in column 4 of Table 26, and as abscissas the corresponding difference between oil and air

*The original article gives a complete set of temperature curves for all of the experiments. These are all of the general character of Figs. 138 and 139, which we think sufficiently illustrate the whole. The final temperatures and the time required to obtain them are given in Table 26.—Editor.

temperatures at the constant condition; that is, For $k = 5.6$ the difference of temperature $61.4 - 12.8 = 48.6$;*

$= 11.2$ the difference of temperature $70.2 - 13.0 = 57.2$;*

$= 18.7$ the difference of temperature $94.5 - 14.0 = 80.5$.*

Thus we obtain the lower curve, denoted $v = 8.61$ meters.

In like manner we proceed with experiments 4 to 7, which have an average sliding velocity of 5.40 meters, and obtain the second curve.

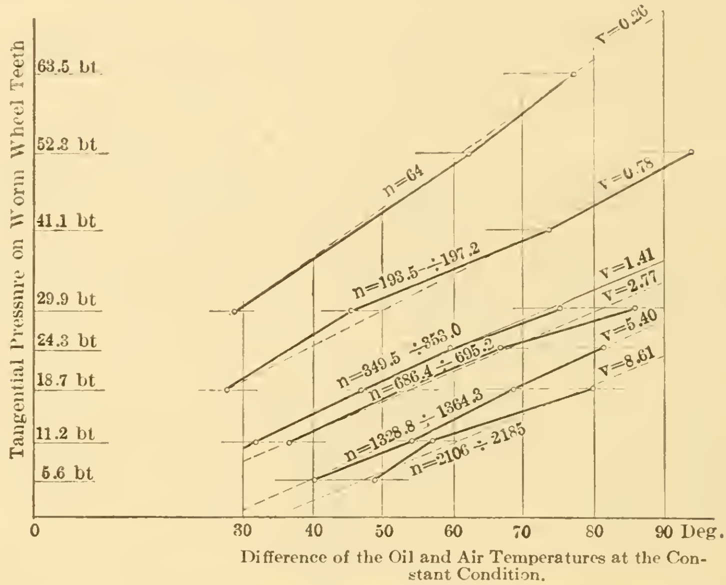


FIG. 141. RELATION OF TEMPERATURE AND PRESSURE.

The experiments 8 to 11, 12 to 15, 16 to 19, 20 to 22 give the remaining curves in Fig. 141.

As is apparent, these curves do not differ much from straight lines, so it may be stated with approximation that:

With equal sliding velocity (equal circumferential speed of the worm) *the difference of oil and air temperature, when the constant condition takes place, is approximately proportionate to the tooth pressure.*

2. *Relation between tooth pressure and sliding velocity for a given difference of the oil and air temperature at the constant condition.*

Consider the broken curves of Fig. 141 replaced by straight lines, and the

* The formation of the difference in temperature is based on the supposition that the outwardly escaping heat would be proportional to this difference, other things being equal. In practice this is not quite true, and probably less so the higher the temperature is. From this it follows—sometimes very notably—also well established, that precisely the same experiment, conducted in one case with the surrounding air temperature at 10° Centigrade for example, and in another at 30° Centigrade does not give the same increase of temperature.

corresponding sliding velocity and tooth pressure for about 70 degrees difference of temperature, determined. Then using the sliding velocities as abscissas and the tooth pressures as ordinates, we get the curve of Fig. 142, denoted 70 degrees. Likewise, for a temperature difference of 50 degrees the curve thus marked is constructed. These curves, which show *that for a*

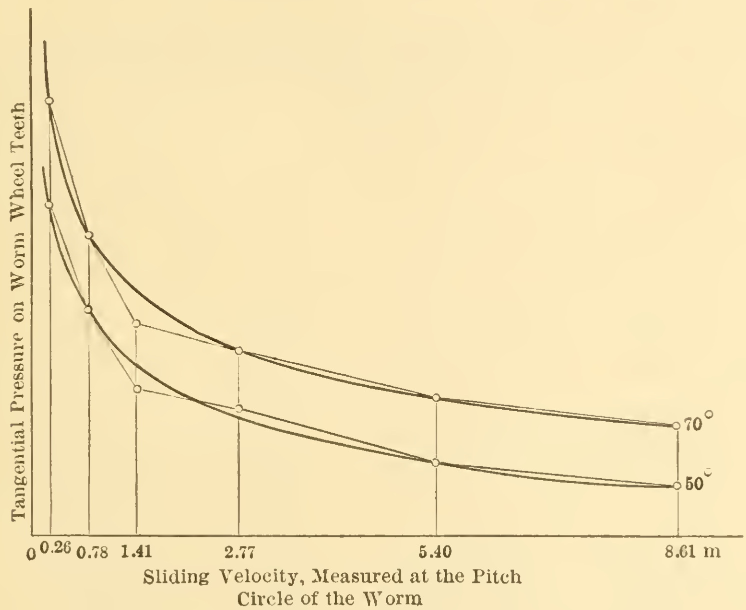


FIG. 142. RELATION OF PRESSURE AND VELOCITY.

constant oil temperature the corresponding tooth pressure decreases with increasing velocity more rapidly at the beginning and afterwards more slowly, bear a distinctly hyperbolic character.*

3. *Relation between tooth pressure, sliding velocity, and difference of temperature.*

Attempts to express this relation by an equation led to an outcome that the formula was not so simple as desirable.

We may compare with the results set forth in Table 25 the formula given by Regierungsbauführer Braun, applicable in this case, viz.:

$$k = a\,(t_o - t_e) + b$$

$$a = \frac{0.0669}{v} \quad 0.4192$$

$$b = \frac{109.1}{v + 2.75} - 24.92$$

* We regard Fig. 142 as giving the most important result of the experiment. It discloses, for the first time we believe, the law connecting the permissible velocity and pressure of worm gearing.—EDITOR.

wherein k denotes the coefficient in $P = k\ b\ t$, representing the tooth pressure in kilograms; v, the sliding velocity in meters per second, measured at the pitch circle of the worm; t_o, the oil temperature; t_e, the air temperature.

For the greater part of the experimental results there is a very good conformity between them and the deductions from the formula, as may be observed in Table 26.

EXPERIMENT NO.	SLIDING VELOCITY v m	a	b	$t_o - t_e$	$k = a\ (t_o - t_e) = b$	k OBSERVED	DIFFERENCE BETWEEN OBSERVATION AND CALCULATION.
1	2	3	4	5	6	7	8
1	8.76	0.4268	− 15.45	48.6	5.30	5.6	+ 0.30
2	8.63	0.4270	− 15.33	57.2	9.09	11.2	+ 2.11
3	8.45	0.4271	− 15.18	80.5	19.20	18.7	− 0.50
4	5.47	0.4314	− 11.65	39.7	5.48	5.6	+ 0.12
5	5.42	0.4315	− 11.57	54.5	11.95	11.2	− 0.75
6	5.37	0.4317	− 11.48	68.8	18.22	18.7	+ 0.48
7	5.33	0.4318	− 11.42	82.0	23.99	24.3	+ 0.31
8	2.79	0.4432	− 5.23	36.9	11.12	11.2	+ 0.08
9	2.78	0.4433	− 5.19	53.4	18.48	18.7	+ 0.22
10	2.76	0.4434	− 5.12	66.6	24.41	24.3	− 0.11
11	2.75	0.4435	− 5.08	85.6	32.88	29.9	− 2.98
12	1.42	0.4663	1.24	31.5	15.93	11.2	− 4.73
13	1.41	0.4666	1.31	47.7	23.57	18.7	− 4.87
14	1.40	0.4670	1.37	59.5	29.16	24.3	− 4.86
15	1.40	0.4670	1.37	75.1	36.44	29.9	− 6.54
16	0.79	0.5039	5.90	27.9	19.96	18.7	− 1.26
17	0.78	0.5050	5.99	45.7	29.07	29.9	+ 0.83
18	0.78	0.5050	5.99	73.8	43.01	41.1	− 1.91
19	0.78	0.5050	5.99	94.9	53.91	52.3	− 1.61
20	0.26	0.6765	11.33	28.3	30.48	29.9	− 0.58
21	0.26	0.6765	11.33	62.5	53.61	52.3	− 1.31
22	0.26	0.6765	11.33	77.6	63.83	63.5	− 0.33

TABLE 26—COMPARISON OF OBSERVED WITH CALCULATED RESULTS.

An important difference between theory and experiment (see column 8 of Table 26) is perceptible among the twenty-two experiments only for No. 2 and Nos. 12 to 15. The causes for these discrepancies are probably to be sought in varying conditions of operation, though such a variation cannot be ascertained.

It is self-evident that the values of a and b depend upon special circumstances

under which the experiments were conducted, and in particular upon the oil employed.

The equation renders it possible with the accuracy resident in the assembled determinations, and under the same suppositions, *to calculate for a given sliding velocity the value of k for a difference of temperature taken at will.*

In order to guard against discrepancies and casualties, considerable pains ought to be exercised in applying the foregoing results of experiments, and care must especially be taken to employ the right kind of oil.

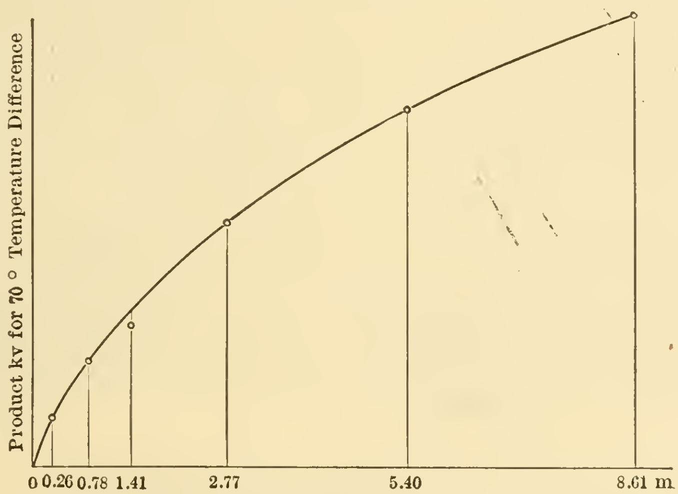


FIG. 143. RELATION OF WORK AND VELOCITY. SLIDING VELOCITY MEASURED AT THE PITCH CIRCLE OF WORM.

In order to construct a diagram showing to what extent the work performed increases with the sliding velocity for a constant difference of temperature fixed at will, the sliding velocities for a temperature difference of 70 degrees are taken as abscissas and the products $k\ v$ as ordinates in Fig. 143.

THE HINDLEY WORM GEAR

The Hindley worm gear is shown in Fig. 144. This type of gear is much used for elevator service, and is supposed to be superior to the ordinary type. The exact nature of the tooth contact is still one of the unsettled questions, so no attempt will be made to go into that subject here. That this contact is peculiar, however, anyone who has attempted to cut this type of gears can testify.

Care must be taken when operating these gears that the bearings do not wear, as the worm will not work without excessive friction in any other than a central position.

Hindley spiral gears are illustrated by Figs. 145 and 146.

A good example of an oblique worm drive is illustrated by Fig. 147. The limiting angles of such a drive are the same as between that of a worm and rack. Fig 147 was used in connection with an article by Wm. H. Ralburn, published in AMERICAN MACHINIST, August 31, 1905.

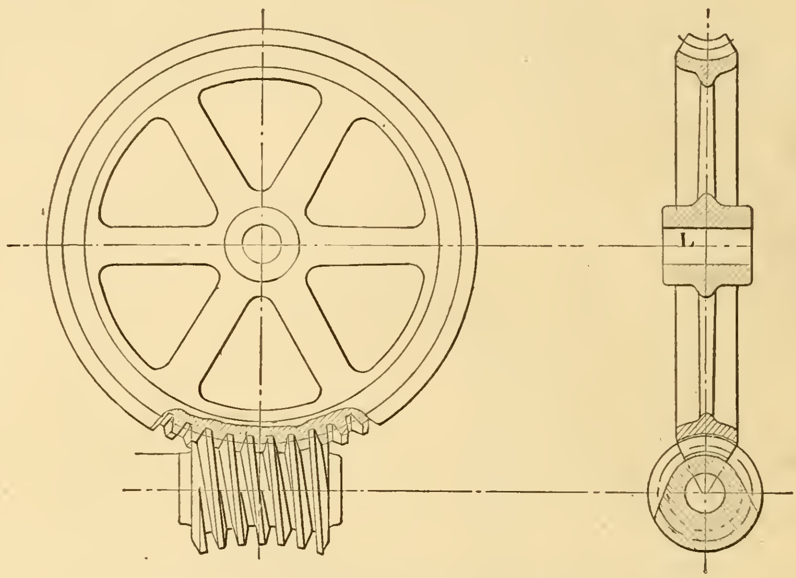


FIG. 144. THE HINDLEY WORM GEAR.

DIAMETRAL PITCH WORMS

"If the proper change gears are provided, it is as easy to cut diametral pitch worm teeth as any. The proper gears can always be easily calculated by the rule that the screw gear is to the stud gear as 22 times the pitch of the lead screw of the lathe is to seven times the diametral pitch of the worm to be cut. For example, it is required to cut a worm of 12 diametral pitch, on a lathe having a leading screw cut six to the inch. We have:

$$\frac{\text{Screw gear}}{\text{Stud gear}} = \frac{22 \times 6}{7 \times 12} = \frac{11}{7};$$

and any change gears in the proportion of 11 and 7 will answer the purpose with an error of $\frac{1}{10000}$ of an inch to the thread of the worm. If 22 and 7 give inconvenient numbers of teeth, the numbers 69 and 22 can be used with sufficient accuracy, and 47 and 15, or even 25 and 8, may do in some cases." *

Care should be taken when using these calculations that the same change gears are used to chase both the hob and the worm, as a slight difference in the lead of one tooth may prove a serious matter in a worm engaging a large

* George B. Grant's Treatise on Gearing, Section 120.

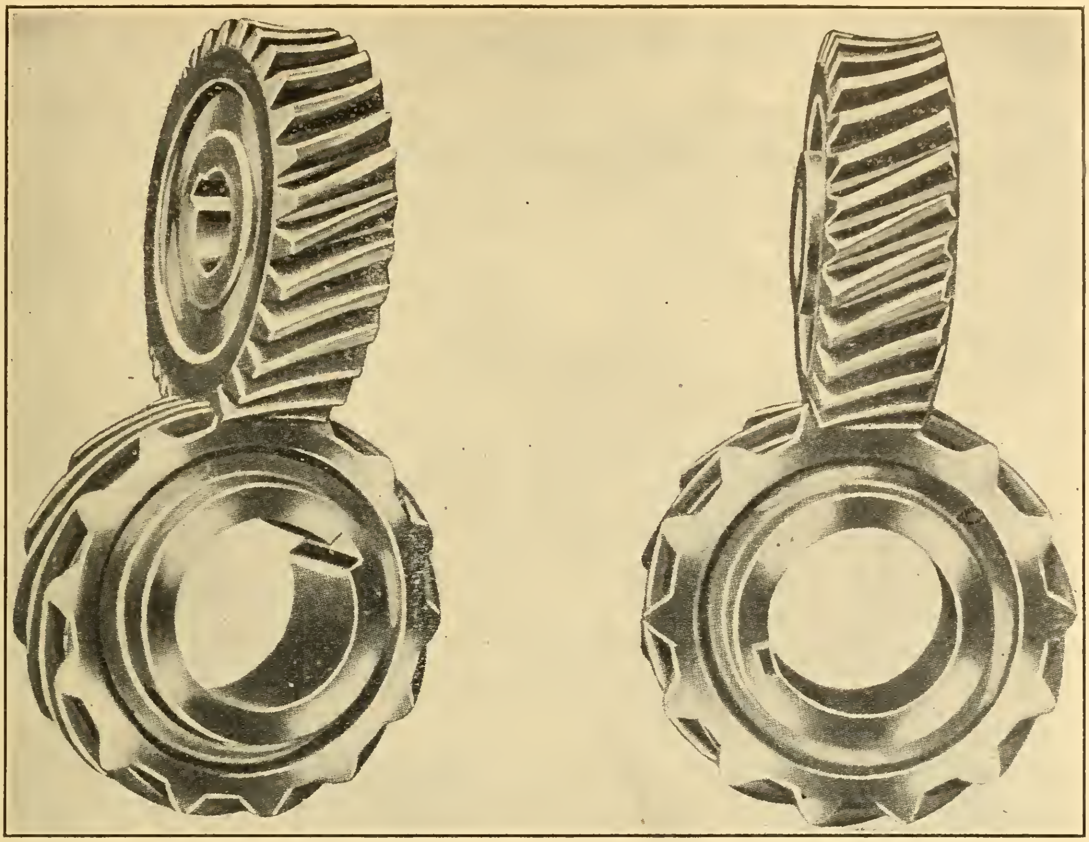


FIG. 145. HINDLEY SPIRAL GEARS. RATIO 2 TO 1.

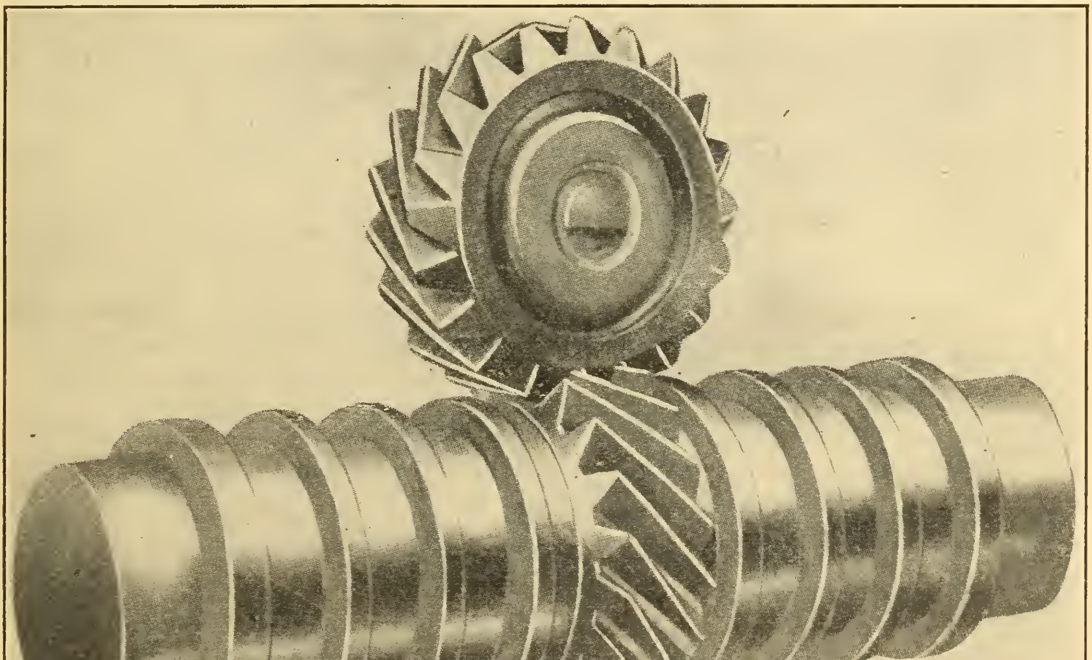


FIG. 146. HINDLEY SPIRAL GEARS. RATIO 1 TO 1.

gear where several teeth will be in contact. This same precaution applies to worms and hobs of a fractional circular pitch.

It should also be remembered that 4 pitch does not mean 4 threads per inch measured on the axis of worm, but 4 threads per inch of diameter of the engaging worm gear. The corresponding circular pitch is 0.7854 inch, not 0.25 inch.

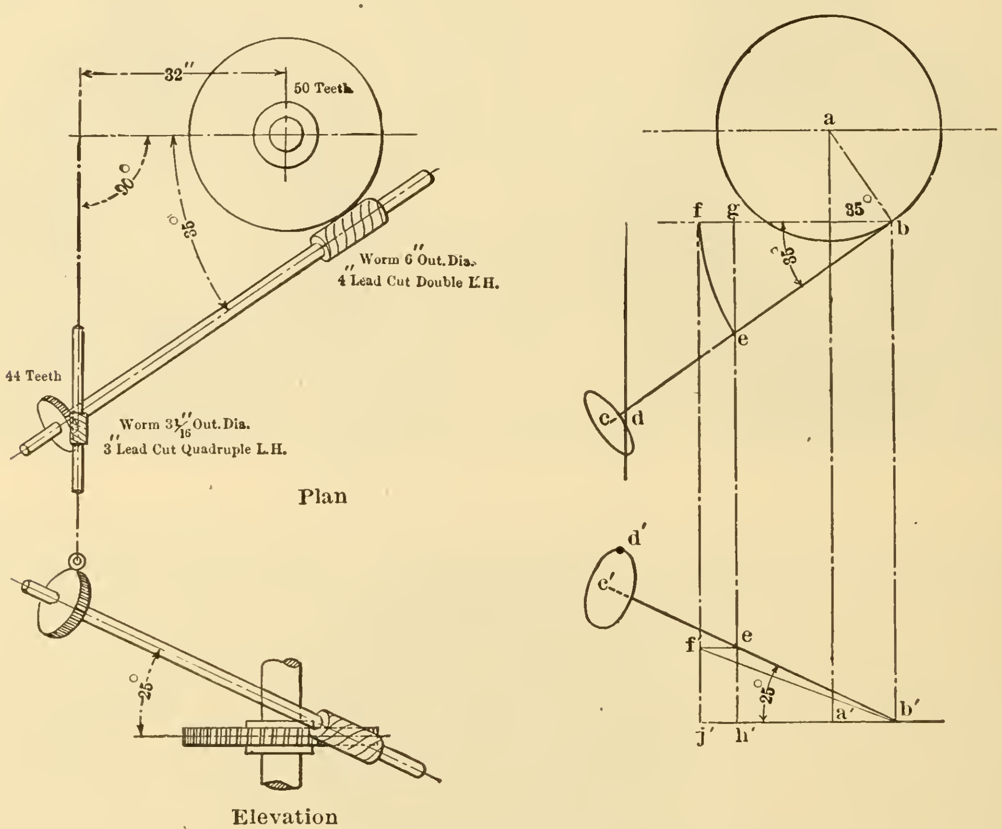


FIG. 147. EXAMPLE OF LAYOUT FOR AN OBLIQUE WORM DRIVE.

SECTION VII

Helical and Herringbone Gears

For connecting parallel shafts where high speed and smoothness of action are essential, helical gears, of the general design as shown in Fig. 148, are generally superior to ordinary spur gears. The smooth operation of this style of gear is due to the spiral action of the teeth, which affords continuous contact of one tooth with its mate, until the next is well engaged, and to the fact that there is always contact at the pitch point; thus transmitting uniform motion without variation in the impulses received by the driven gear.

In the ordinary spur gear the teeth come in contact over their entire length at once, and the whole load comes on the top of the tooth with a leverage equal to nearly its depth, while in helical gears the strain reaches its maximum after the leverage has been materially diminished—an obvious advantage. The objection to these gears is the end thrust caused by the spiral action of the teeth, which, for the same load, makes them less efficient than spur gears. This, however, varies with the angle of spiral. The highest efficiency is obtained by making the angle just great enough so that the end of one tooth will overlap the one adjoining. Therefore it follows that the wider the face the less acute the angle need be and the less the thrust.

In Fig. 149 this angle is shown in such a way that a line drawn from the center of one tooth of a spur and at one edge of the gear will intersect the center of the space between it and the following tooth at the other edge.

If this circular pitch is 1½ inches and the face 10 inches, an angle of 12° 41″ would be sufficient. Nothing would be gained by making the face any wider, except, perhaps, to accommodate the use of a standard cutter, which is often necessary when the speed ratio and center distance are fixed. Another reason for keeping the angle low is the fact that a steep angle, say, 30 degrees, will allow only points of contact across the face, depending of course upon its width, while a small angle will allow more of a line contact, as in spur gears.

Referring to Fig. 150 it will be seen that the thrust on the shaft *B* is entirely avoided and on *A* and *C* it is reduced to a minimum. The drive shown is often used to advantage where the gears are of necessity small and the drive heavy.

In the following formulas the circular pitch system is used throughout, as the introduction of diametral pitch would only tend to be confusing. When a diametral pitch cutter is to be used its circular equivalent should be intro-

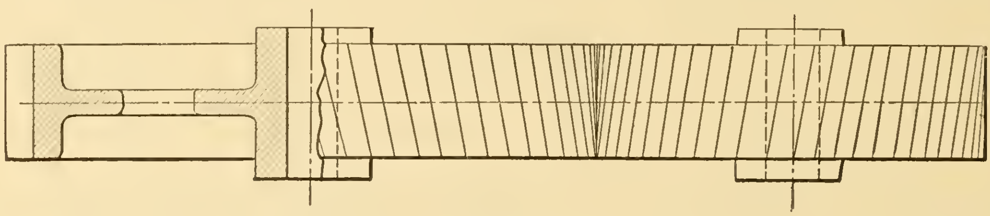


Fig. 148

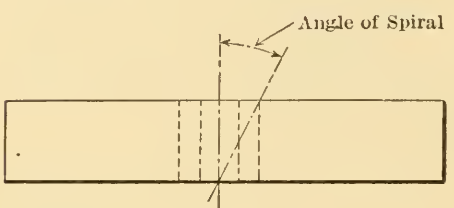


Fig. 149

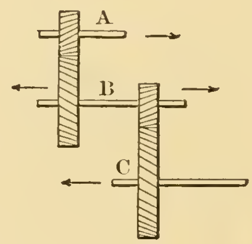


Fig. 150

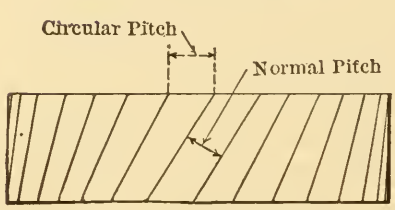


Fig. 151

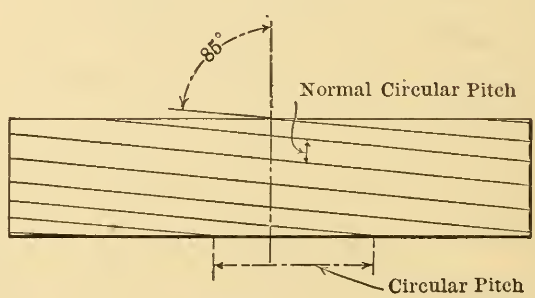


Fig. 152

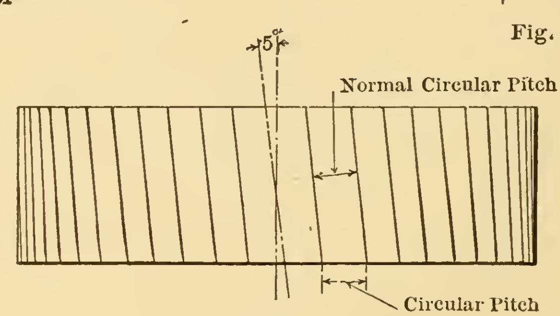


Fig. 153

THE DESIGN OF HELICAL GEARS.

duced into the formulas. The "normal circular pitch" is the shortest distance between two consecutive teeth, measured on the pitch cylinder. The "circular pitch" is the distance between two teeth measured on the pitch line as in spur gears. See Fig. 151.

The following conditions govern the designing of helical gears: The pitch

diameters must be in proportion with the number of teeth, as in the case of spur gears. Therefore, the circular and normal pitch must be the same in both gears of a pair. The angle must be the same in both gears.

Formulas for Helical Gears

D' = Pitch diameter.
D = Outside diameter.
E = Angle of spiral.
p'^n = Normal circular pitch.
p' = Circular pitch.
L = Lead of the spiral.
N = Number of teeth.
π = 3.1416.

$$D' = p' N \, 0.3183. \tag{1}$$

$$D = D' + (p'^n \, 0.6366). \tag{2}$$

$$Cos \; E = \frac{p'^n}{p'}. \tag{3}$$

$$p'^n = p' \cos E. \tag{4}$$

$$p' = \frac{p'^n}{\cos E}. \tag{5}$$

$$\text{The corresponding spur cutter} = \frac{N}{\cos^3 E}. \tag{6}$$

$$L = D' \pi \cos E. \tag{7}$$

1. The pitch diameter is figured from the circular pitch in the same manner as for spur gears.

2. The outside diameter is found by adding to the pitch diameter twice the addendum of the normal pitch.

3. The angle of the spiral is always understood to be taken from the axis of the gear; therefore the gear with the smallest angle is nearest to a spur gear.

The angle is best determined by first selecting the pitch of the cutter to be used (normal circular pitch) and dividing it by the actual circular pitch; the result is the cosine of the angle. If this does not give the required angle the number of teeth must be changed, consequently the circular pitch, and another trial made, unless the diameter can be changed, which will change either the center distance or the speed ratio. When the angle is assured as well as the pitch of the cutter, the diameter necessary to give the proper combination may be found by first determining the circular pitch. For

helical gears, however, a change in the angle to accommodate an even number of teeth will make no difference whatever.

It may be well to state that the angle of the spiral for helical gears must be closely followed, as a slight deviation will cause the teeth to bear only on one end instead of evenly along the face. This also applies to the lead, as an error will result in an incorrectly formed tooth.

4. The normal circular pitch is the pitch of the cutter used and may be found by multiplying the circular pitch by the cosine of the spiral angle. To avoid special cutters the normal pitch should conform as nearly as possible with some standard pitch. The normal pitch is generally assumed, and the angle made to suit. If there must be a variation between the pitch of the cutter used and the normal pitch of the gear it is better to have the cutter pitch finer rather than coarser than the normal.

5. The circular pitch is found from the normal pitch and angle by dividing the normal circular pitch by the cosine of the angle. This must always correspond with an even number of teeth; that is, the circular pitch multiplied by the number of teeth must represent the pitch circumference of the gear in question.

6. After the pitch of the cutter has been settled the next step in importance is the selection of the proper cutter, as upon that point largely depends the success of the gears. The number of teeth for which the cutter should be made is found by dividing the actual number of teeth in the gear by the third power of the cosine of the spiral angle. It will be seen that the greater the angle the nearer we approach the rack tooth, as in a helical gear large in diameter with a small number of teeth, as shown in Fig. 152.

The less the angle the nearer we approach a spur gear, and the nearer the normal and circular pitch correspond, so that in a gear with a very slight angle (say, 5 degrees, as in Fig. 153), the same cutter is used as for a spur gear of the same size. It may be well to note here that for a gear such as is illustrated in Fig. 152, instead of using the regular No. 1 rack cutter made for 135 teeth and over, a straight-side worm-milling cutter be used. In fact this should be done when the value of the formula is found to exceed 250.

7. The spiral lead is the distance traveled by the thread in one complete revolution of the pitch circle and is found by multiplying the pitch circumference by the contangent of the spiral angle. The lead begins at an angle of 90 degrees and increases as the spiral angle decreases until at an angle of 45 degrees it is equal to the pitch circumference of the gear, and, as the angle becomes smaller, the form of the tooth approaches that of the spur gear and the lead becomes longer until it gradually lengthens to infinity for a zero angle.

EXAMPLES IN THE DESIGN OF HELICAL GEARS

When computing a helical gear drive we generally have three conditions to meet, namely: the speed ratio, the center distance, and a suitable angle. This angle should not exceed 20 degrees, but it is sometimes necessary to take the best we can get in order to meet the other two conditions. For example, suppose we desire a pair of gears as follows: Velocity ratio 4 to 1, center distance 12½ inches, and to be cut with a 4 pitch cutter. First, arrange a table, as shown below. Then figure out the dimensions as they are listed.

Gears for center distance, 12½ inches. Kind, helical.

	GEAR	PINION
Pitch diameter	20	5
Pitch circumference	62.832	15.708
Number of teeth	76	19
Circular pitch	0.8267	0.8267
Normal circular pitch	0.7854	0.7854
Angle	18° 11′	18° 11′
Addendum of teeth	0.250	0.250
Whole depth of teeth	0.539	0.539
Thickness of teeth	0.392	0.392
Outside diameter	20.50	5.50
Lead—exact	191.292	47.823
*Lead—approximate.		
Cutter used	No. 2–4 p.	No. 5–4 p.
Hand	R. H.	L. H.

We first set down the assumed pitch diameters and find their circumferences. From the number of teeth find the circular pitch (see fourth place in the table), then assume a normal pitch and determine the angle. The tooth parts that follow are determined from the normal pitch. In case the angle is found too great, either the number of teeth or the center distance must be changed.

On the other hand, suppose the center distance had not been fixed, and a velocity ratio of 4 to 1 was desired. We again arrange a table so that the items will come in rotation as they are assumed or calculated. In this case we assume the number of teeth to be 76 and 19, the spiral angle 15 degrees, and a 4 pitch cutter (0.7854-inch normal pitch).

* It is sometimes necessary to vary the lead a little, and the record should state the variation made.

	GEAR	PINION
Number of teeth	76	19
Normal pitch	0.7854	0.7854
Angle of spiral	15° 0′	15° 0′
Circular pitch	0.8130	0.8130
Pitch diameter	19.668	4.917
Pitch circumference	61.788	15.447
Lead—exact	230.600	57.650
Lead—approximate.		
Addendum	0.250	0.250
Whole depth	0.539	0.539
Thickness	0.392	0.392
Cutter used	No. 2–4 p.	No. 5–4 p.
Outside diameter	20.168	5.417
Hand	R. H.	L. H.

Gear on the worm.
First gear on the stud.
Second gear on the stud.
Gear on the screw.

Example: Suppose two spur gears of 60 and 15 teeth, three pitch, 12½ inches between centers, 4-inch face mounted on parallel shafts. It is desired to replace them with helical gears. The first point is to find a pair with fewer teeth, but of the same ratio, thus preserving the center distance. If, however, the same pitch is inadmissible the number of teeth may be increased.

In the first instance we desire to use a 3 pitch cutter; therefore the number of teeth must be reduced. 56 and 14 are found to be in the same ratio as 60 and 15, and by using the same pitch diameters the circular pitch will be found to be 1.122 inches. According to formula 3 the angle is found to be $21° - 2'$. While this angle might well have been less it is the best that can be had under the circumstances; unless, of course, the centers can be altered. On the other hand, had a finer pitch been decided on—say, a normal pitch of 0.7854 inch (4 pitch cutter)—we increase the number of teeth to 19 and 76, which gives a circular pitch of 0.8267 inch. The normal pitch being 0.7854, the corresponding angle from formula 1 is:

$$\textit{Cosine } E = \frac{0.7854}{0.8269} = 0.95004.$$

The angle whose cosine is $0.95004 = 18° \ 11'$, the angle of the spiral. This latter arrangement would probably be the better of the two.

HERRINGBONE GEARS

A double-helical or herringbone gear may be said to be two helical gears of right and left obliquity placed together on the same shaft so as to form one gear. The object is to eliminate end thrust encountered in helical gears (see Fig. 154). The formulas for herringbone gears are the same as for helical, but the angle of the teeth may be made as acute as desired without fear of axial thrust; 30 degrees is common and even 45 degrees is often used.

This style of gear is extensively used for rotary blowers, air compressors, etc. They are, in fact, indispensable for these and similar purposes, and it is believed the highest possible speed may be thus attained. Just

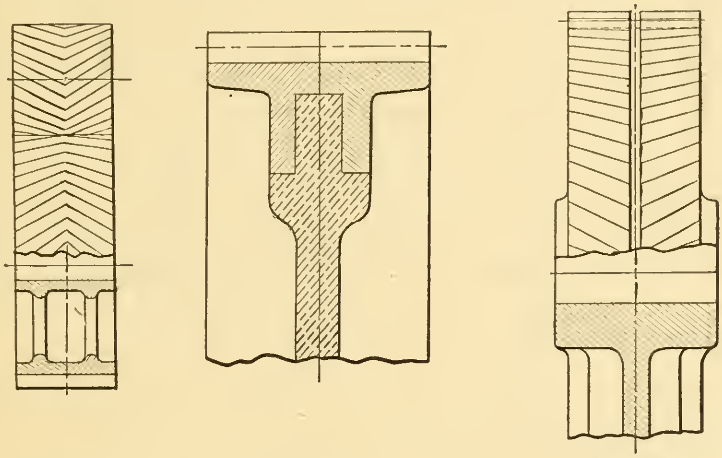


FIG. 154. FIG. 155. FIG. 156.

THE DESIGN OF HERRINGBONE GEARS.

what this speed may be is not definitely known. One authority puts it at one mile per minute. It is likely, however, that by the use of proper material this speed may be safely exceeded. One serious objection to this type of gear is the cost of manufacture. Doubtless but for this fact the use of herringbone gears would become almost universal. However, unless they are very accurately made and great care used in assembling them, they are little better than spur gears.

The most common design for herringbone gears is shown in Fig. 154. This is subject to many modifications in details, such as Fig. 155, where the rims are made separate from the spider to save cost and facilitate replacements. It is no improvement to stagger the teeth except when cutting gears with a central groove. By staggering the teeth this groove may be made narrower than if the teeth are cut directly opposite. A herringbone gear may be made in one piece by grooving between the faces to allow the cutter to run out, as shown in Fig. 156.

When the gear is made in this manner the pinion is sometimes made in two pieces to facilitate the proper engagement of the teeth, as it is extremely difficult to cut such gears interchangeably without special and unusual facilities.

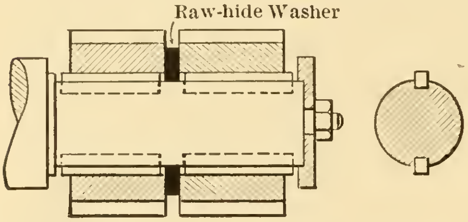


FIG. 157. ARRANGEMENT OF PINION.

When the direction of rotation can be determined, it is not necessary to secure the two halves of the pinion, as the thrust of the teeth may be utilized for this purpose; the pinion being keyseated after the teeth have been adjusted to suit the position of the teeth in the gear, which may be made in one piece.

A still better plan is suggested by "Attic" in the AMERICAN MACHINIST. "Place a washer of leather or other elastic material between the two halves, as per Fig. 157, so that they can slide together as one piece free on the shaft endwise, and also have a slight axial freedom of motion relative to each other, which amounts to exactly the same thing as if they had given in the turning direction. This is obvious, when one considers that the screw line is obtained by the combination of the simple straight-line motion with

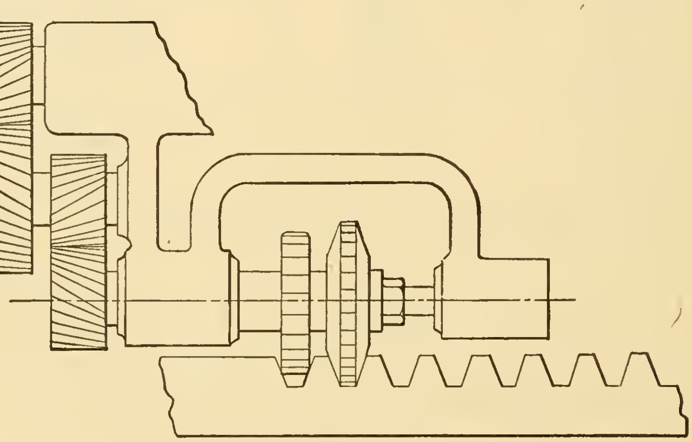


FIG. 158. HELICAL GEARS FOR CUTTER DRIVE.

the circular motion. If a helical gear is moved a fraction of an inch axially, the effect on the teeth is the same as if they had been rotated through a correspondingly small angle. If the two halves of our double-helical pinion are moved axially in opposite directions to each other by means of squeezing the elastic washer more or less hard, we have the same result as if the wheel turned slightly around."

Fig. 158 illustrates the arrangement of gears on a heavy rack cutter. They were made helical after several discouraging experiments with ordinary spur gears. These gears were made of 20-point carbon-steel forgings and case-

hardened. The machine is used in cutting as heavy as one diametral pitch, and on two inch pitch it is used with two cutters, one for blocking and one for finishing, at a heavy feed. They have been in constant use for over two years and show little wear.

A very efficient herringbone gear for light loads may be obtained by employing a short tooth in which the involute curve has been modified so that the teeth will have rolling action only, the portions of the tooth above and below the pitch line that engage in sliding contact being cut away. This would not be possible with spur gears, but as a herringbone gear has continuous contact at the pitch point, the shape of the tooth does not effect the smooth action of the gears if there is no interference.

This is mentioned by Professor McCord in his "Kinematics" and is there designated as "perfection in gearing."

CUTTING HERRINGBONE GEARS

The following is an extract from an article in American Machinist by Percy C. Day:

"The inaccuracies of the usual process of cutting helical and herringbone gears are many, and well known to all gear manufacturers. It is practically impossible to determine the correct shape of the cutter for any particular wheel, because, in the first place, the contour of the tooth as formed only approximates to the cutter form on the normal section. What is required is the correct shape of tooth on the section normal to the wheel axis, which is the same as for straight-toothed gears. It is exceedingly difficult to find the correct form of cutter which, when cutting its own shape in a spiral groove, will leave a tooth of the desired contour on a section in the plane of rotation of the wheel. This difficulty is, moreover, complicated by the fact that a plane disk cutter does not leave its own shape in a spiral groove, so that even the correct normal-tooth section is practically unobtainable.

"Single helical gears can now be cut by a generating process, using a hob as a cutter. The hob is set at an angle in order to make the threads correspond, on the cutting side, with the inclination of the spiral teeth. Although this process is a decided improvement on the first, it has certain inherent defects. In hobbing systems of gear generation the teeth of all wheels are developed from the parent rack. Axial sections of the hob teeth are considered theoretically to correspond with this rack. When the hob is set at an angle other than a right angle in relation to the axis of the wheel blank, the generating section is no longer an axial section, and the tooth form may differ considerably from the involute. . . ."

MILLING PROCESS FOR CUTTING DOUBLE HELICAL GEARS

"In the milling process the teeth are sometimes cut by means of end mills formed to the tooth shape on the normal section. The working principle of the machines usually employed is shown in the diagram, Fig. 159. The end mill *a* is supported by the saddle *b*, which traverses the bed *c*. The mill is driven by the bevel gears *d* from the splined shaft *e* and driving cone *f*. The feed and differential motions are driven from *e* through speed cones or gears *g* and clutch *h*. The traverse of the saddle *b* is actuated by the feed screw *j*. Motion is also transmitted from *j* through change wheels *k*, reversing gears *l*, dividing change wheels *m*, worm *o*, dividing wheel *w*, and work spindle *p* to the blank *q*.

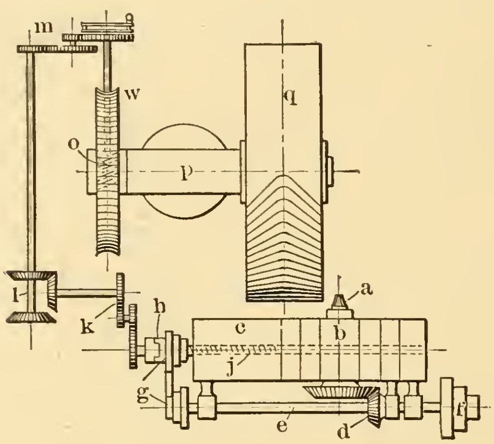


FIG. 159. DIAGRAM OF MILLING PROCESS OF CUTTING DOUBLE HELICAL GEARS.

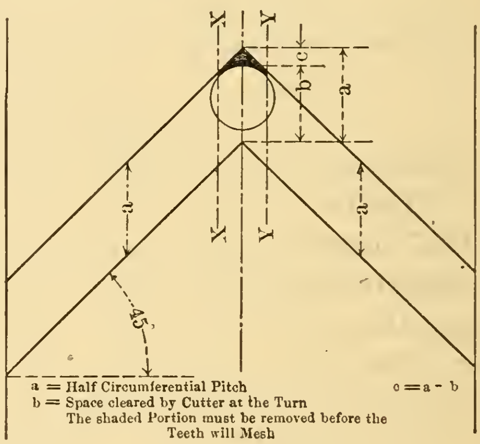


FIG. 160. DIAGRAM OF SPACE NOT CUT BY MILLING PROCESS OF CUTTING DOUBLE HELICAL GEARS.

"While the end mill traverses from one edge of the blank to its center, the blank is rotated through an angle which gives the requisite spiral form to the tooth. The saddle then operates a stop which is in connection with the reversing gear *l*, and the rotation of the blank is reversed until the end of the cut. A quick-return mechanism, not shown in the diagram, comes into action at the end of the cut, and the mill is returned to the starting position. The dividing mechanism *m* is then operated by hand, and the cutting process is repeated on another tooth.

"The end-milling process can be readily adapted for cutting double helical bevel gears.

"The disadvantages of the process are principally of a practical nature. End mills are small tools, and are liable to rapid wear. Since the teeth are cut singly, any wear on the mill causes a change of tooth shape and thickness. The reversal of the angular motion of the blank while cutting proceeds allows

the inevitable blacklash in the mechanism to take effect in a manner which is not conducive to accurate work. The cutter must be formed to the normal tooth section, and has not the circumferential shape of the teeth which it cuts. The width of the tooth space at the apex corresponds to the normal instead of to the circumferential pitch, hence the space must be cleared out by hand or in a separate operation (see Fig. 160).

"The tendency to wear is greater when the end mills are small, and wheels on this system are generally made of coarser pitch than is really necessary or even desirable from the user's point of view, in order to minimize the manufacturing difficulties by the use of large mills.

HOBBING PROCESS FOR CUTTING DOUBLE HELICAL GEARS

"The latest method of producing double helical wheels is an adaptation of the hobbing process. A double-hobbing machine of special design is employed, so that the right- and left-hand teeth are cut simultaneously. The working principle of the machine is shown in Fig. 161.

"The blank *a* is mounted on the vertical mandrel *c*, and is driven by the faceplate *b* from the work spindle *d* and the dividing worm wheel and worm *e*. The hobs *f f* are mounted in vertical slides *g g*, which move up and down on the standards *h h*. The standards are mounted to slide on the bed of the machine *j* and are provided with micrometer feed screws for adjusting the depth of cut.

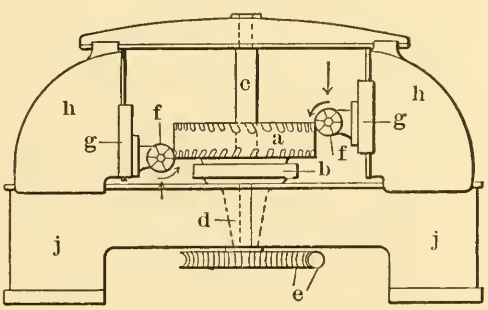


FIG. 161. DIAGRAM OF HOBBING PROCESS FOR CUTTING DOUBLE HELICAL GEARS.

"The hobs are rotated in the direction shown by the curved arrows, and the dividing wheel *e* is driven at the correct relative speed by means of change wheels. The hob threads are right- and left-hand, and the slides *g g* are fed down and up simultaneously.

"In connection with the feed mechanism there is a second train of change wheels which operates a differential gear for correcting the speed of the blank, to give the required spiral lead to the teeth. The process is entirely automatic from the first setting to the completion of a cut all round the wheel.

"The teeth of wheels cut on this system are of the staggered or herringbone type, a tooth on one side of the rim being coincident with a space on the other side. This arrangement allows the cutters to clear at the center of the rim, and, incidentally, has the effect of halving the pitch without reducing the strength of the teeth.

"This method of producing double helical wheels has the great advantage that a complete interchangeable system for any pitch is produced from a single pair of hobs. The teeth of all wheels are cut at the same angle, usually 23 degrees. The hobs are constructed with a definite relationship between lead and pitch diameter, so as to give the threads the same angle as the wheels to be cut. As a consequence there is no need to incline the hob axes, which are set at right angles to the axis of the wheel blank. . . .

ADVANTAGES OF THE DOUBLE HELICAL SYSTEM

"The adoption of the double helical principle in gearing, if properly applied, reduces noise to a minimum and practically eliminates vibration without any necessity for departure from sound mechanical principles. In this type of gear, pinions may be chosen of sufficient hardness to wear evenly with the wheels, and soft materials do not enter the proposition. This is due to the absolute continuity of engagement which is characteristic of double helical gears when accurately cut and correctly designed to suit the working conditions. The effect of vibration is not by any means confined to the gears themselves, but acts injuriously on the shafts and machinery connected therewith. Many failures of haulage and other gear-driven shafts have been directly traced to this cause.

"Consider a pair of wheels transmitting 100 horse-power with an efficiency of 96 per cent. If we assume only one-tenth of the lost energy to be dissipated in vibration which is absorbed in the wheel shaft, the result is somewhat surprising. Under such conditions the shaft is called upon to absorb energy at the rate of nearly eight million foot pounds during each working day of 10 hours duration. The result is finally expressed in crystallization of the shaft material. . . .

"Another interesting application of machine-cut, double helical gears is the reduction of speed from high-power steam turbines. No other type of gear can be used for this class of work, because absolute smoothness of action is essential. The essence of this problem is to avoid excessive velocity by keeping the pinion diameter small, but at the same time it is undesirable to reduce the number of teeth below a certain point because absolute continuity of engagement must be maintained. The result of these conditions is that the gears must be of extremely fine pitch and great relative width. For example, a set of gears recently constructed for a 500-horse-power steam turbine, to reduce from 3,000 to 300 revolutions per minute, were 4 diametral pitch with face width 10 inches and pinion of 19 teeth.

"There is probably no field of application for double helical gears which offers such substantial advantages as for driving machine tools. In most

modern machine shops there is a tendency to dispense with shafting as far as possible, and to drive the tools individually from separate motors. This method allows a more economical distribution of machines over the available floor space, and leaves the space overhead clear for rapid handling. On the other hand, motor-driven tools require far more gearing than when the drive is effected by belts, and it has been found difficult to obtain uniformity of motion under the new conditions. If machine-cut, double helical gears are used for this purpose, the quality of the work turned out is much improved and, by reason of reduced vibration, higher speeds and coarser feeds can be employed.

' The diagram, Fig. 162, shows the relative normal tooth pressures, pitch line sections and stresses for angles of 23, 45, and 60 degrees.

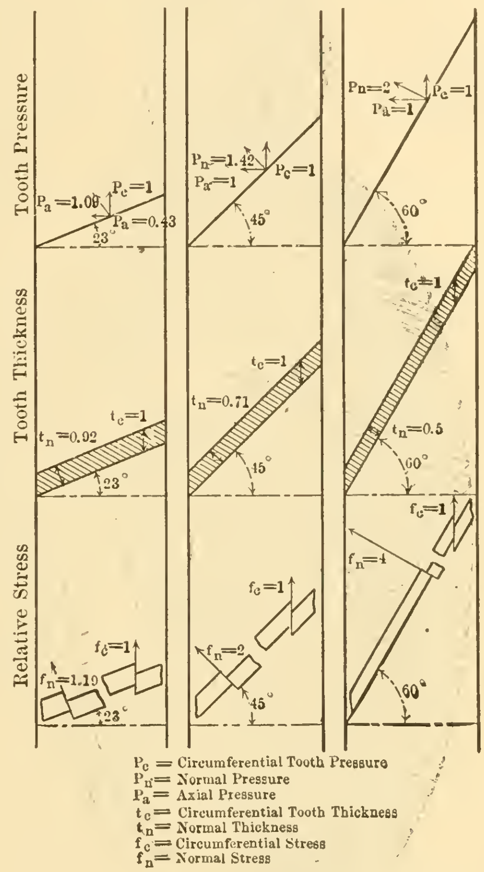


FIG. 162. COMPARISON OF TOOTH PRESSURE, THICKNESS, AND RELATIVE STRESS FOR THE TOOTH ANGLES OF DOUBLE HELICAL GEARS.

"One of the greatest advantages of machine-cut, double helical wheels is to be found in their adaptability for high ratios of reduction. The number of teeth which can be used with success in the smallest pinion lies far below the practical limit for straight spurs, and pinions of four or five teeth are by no means uncommon for special purposes. Since, however, the pitch can be made very fine, it is rarely necessary to reduce the number of teeth so far, and most high-ratio gears are made with pinions of 11 to 20 teeth. Pinions for high ratios are generally cut solid on their shafts, in order that the diameters may be kept low to bring the wheels within reasonable proportions.

"Single wheels and pinions will transmit heavy powers with ratios between 10 and 20 to 1, so that they can be used in place of worm gears or double trains of ordinary spurs. As against worm gears the gain lies in the direction of increased efficiency and life. A set of double helical gears with 20 to 1 ratio has an efficiency of about 95 per cent. against a maximum of about 80 per cent. for a worm gear of equal ratio.

AN INTERESTING APPLICATION OF DOUBLE HELICAL GEARING

"An interesting example of this difference came under my notice a short time ago. A worm gear of first-class manufacture and modern design had been in use for some 2½ years, driving a deep-well pump from a 50-horse-power motor with reduction 480 to 22 revolutions per minute. This gear was replaced by a double train of machine-cut, double helical wheels, the ratios being 480 to 60, and 60 to 22. The records of power consumption and pump duty were regularly kept, and after the new gear had been running for a year the figures showed a net saving of over 17 per cent. in its favor as against the average for the whole life of the worm gear. It was also shown that the efficiency of the double helical gear had actually improved after a year's daily work, while the worm gear had steadily deteriorated in this respect from the day it was started.

"Fig. 163 shows a set of double helical gears that are representatives of their design and construction.

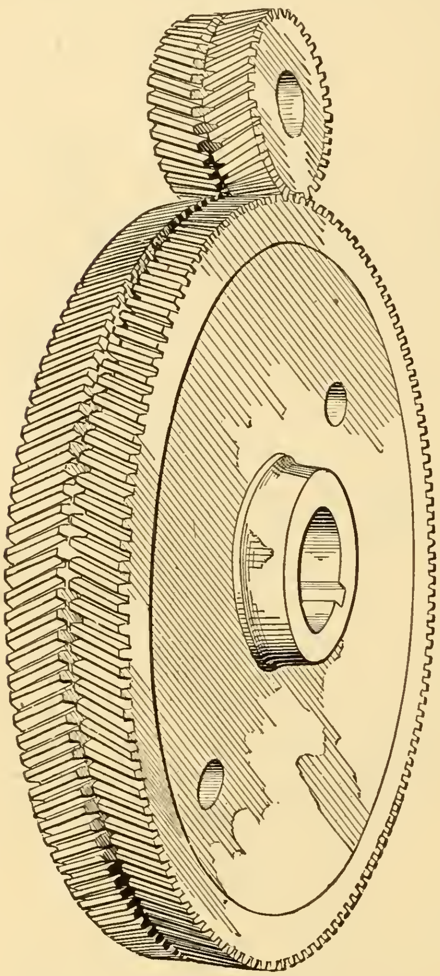


FIG. 163. TYPICAL HIGH-RATIO GEARS WITH STAGGERED TEETH.

IMPORTANT POINTS IN APPLYING DOUBLE HELICAL GEARS

"In conclusion it is desirable to add a word of caution to those who are about to adopt this class of gear for the first time. It must not be forgotten that there are three fundamental points of difference between machine-cut, double helical wheels and ordinary spur gearing:

"(a) The pitch is finer.
"(b) The face width is greater.
"(c) The tooth pressures are generally higher.

"To insure satisfactory working it is necessary that the shafts shall be parallel, true, and rigidly supported. The center distance must also be adjusted with great care on account of the fine pitch and small clearances allowed.

Motor pinions of high-ratio gears should be mounted on extended shafts with an outer bearing. Anything in the nature of an overhung drive should be avoided wherever possible.

"To avoid undue wear from magnetically controlled end-thrust in motors, the pinions should be mounted on two parallel feathers set at 180 degrees and carefully bedded to the keyways (see Fig. 164). The pinions should be a good tight fit on the motor shafts, but there should be just sufficient freedom to allow them to move along under the influence of continued side pressure, so that the motor armature can reach a neutral position where the pressure ceases. It is unnecessary to allow the pinions to slide freely on the shafts, and if this is done there may be trouble from excessive wear of the keys and keybeds."

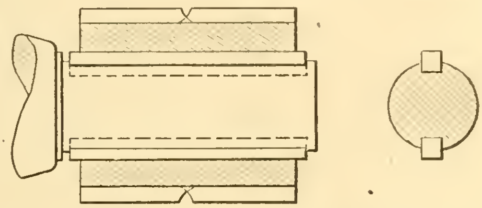


FIG. 164. AN IMPROVED METHOD OF KEYING FOR GEARS.

DETERMINING LEAD AND ANGLE FROM SAMPLE

To produce a herringbone gear to operate with a sample, the calculations for which are unknown, is generally a matter of cutting and trying until a satisfactory gear is produced, as for herringbone or helical gears the angle and lead must be exceptionally accurate, the teeth having contact their entire length, and a slight error is noticeable. There is more or less leeway for spiral gears, but the method as described can be applied to them as well.

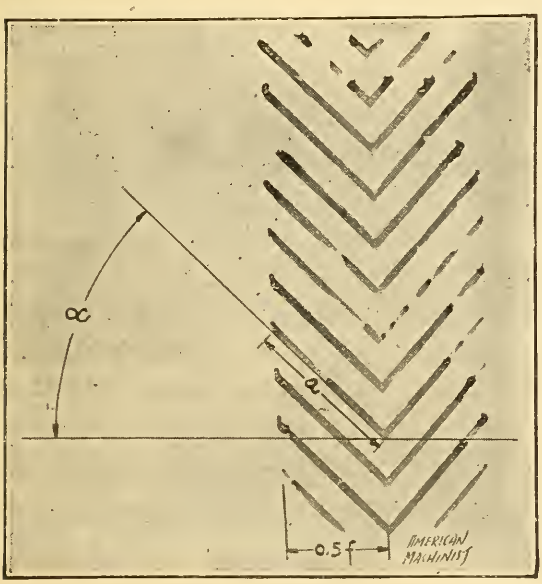


FIG. 165. IMPRESSION MADE BY ROLLING SAMPLE HERRINGBONE GEAR.

Cover the points of the teeth in sample with an application of lampblack, or anything that will make a clear impression on a piece of clean white paper. Roll the gear thus treated on the surface of the paper, being careful not to allow it to slip, until a sharp impression of the points of the teeth is made, as illustrated in Fig. 165. This will represent a development of the teeth at the outside circumference.

The angle of the teeth at the outside circumference may then be measured

with a protractor by extending the lines of the tooth as developed on the paper.

f = Face of herringbone gear.
a = Length of the tooth from center of face.
β_1 = Angle of spiral at outside diameter.
β = Angle of spiral at pitch diameter.
L = Lead of spiral.
C_1 = Outside circumference.
C = Pitch circumference.

$$\cos \beta_1 = \frac{0.5 f}{a}.$$

For helical gears this formula would be:

$$\cos \beta_1 = \frac{f}{a}.$$

The next step is to find the lead:

$$L = C_1 \cos \beta_1.$$

As the lead is necessarily the same at the outside diameter as it is at the pitch diameter of a helical or spiral gear when cut with a rotary cutter, the angle of spiral at the pitch line may be found by formula 4.

$$Tan\ \beta = \frac{C}{L}.$$

The fact that the lead is the same at all points when cutting a spiral, helical, or herringbone gear cutter, using a single rotary cutter, makes the solution of this problem a simple matter. Fig. 166 is self-explanatory.

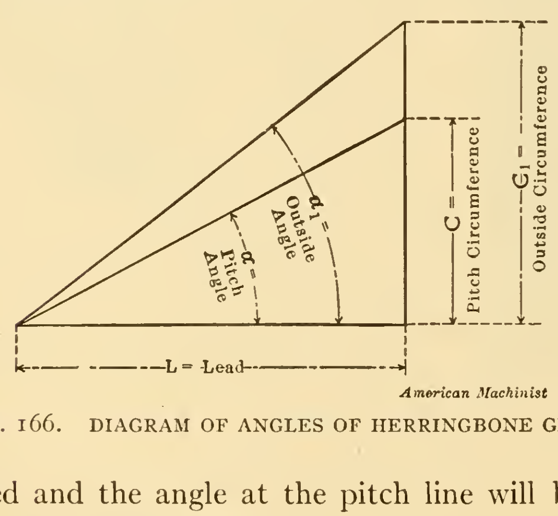


FIG. 166. DIAGRAM OF ANGLES OF HERRINGBONE GEAR.

This being the case, it is apparent that such a cutter cannot reproduce its own shape in the gear blank, as to do this the angle and lead must be proportional to all parts of the tooth. When the teeth are generated this condition is fulfilled and the angle at the pitch line will be proportional to the pitch and outside circumferences, or:

$$\beta = \frac{\beta_1 C}{C_1}.$$

SECTION VIII

Spiral Gears

Before going into the matter of calculations it may be well to direct the readers to a careful consideration of the accompanying perspective sketches originally published in American Machinist, October 11, 1906, by H. B.

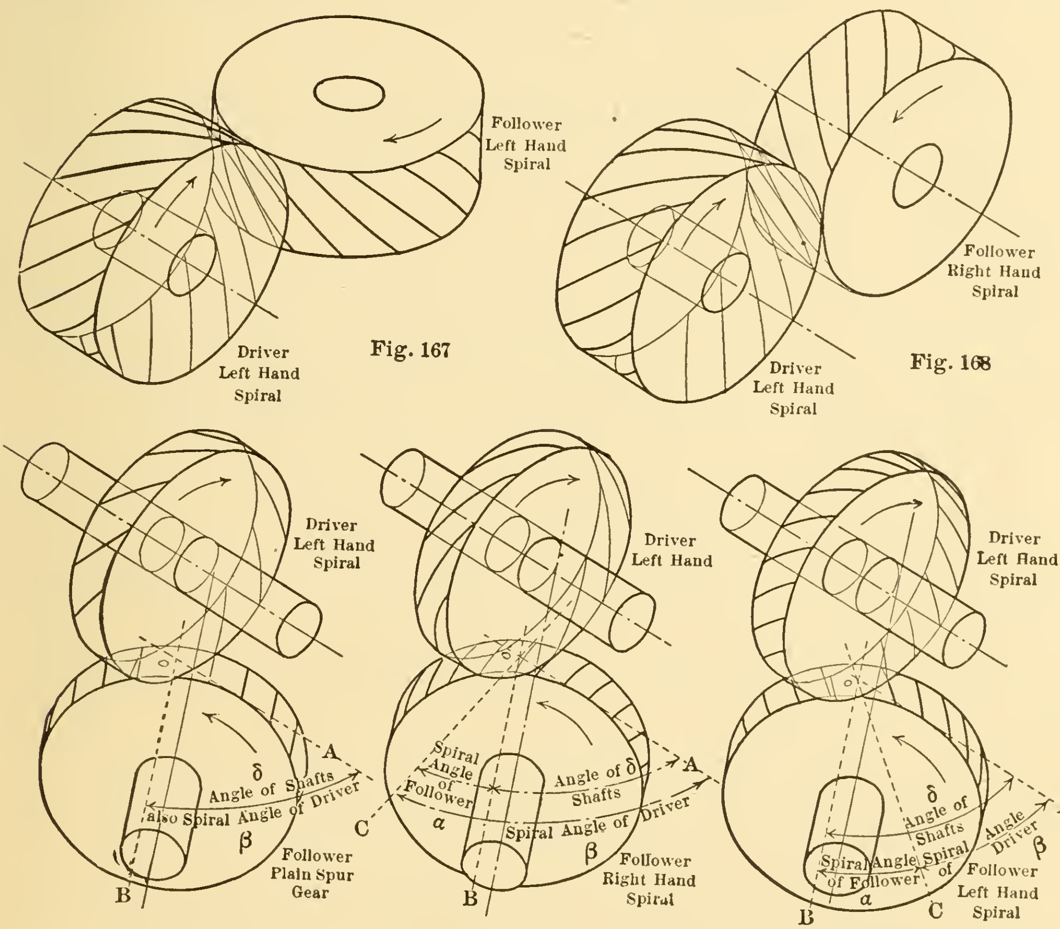


Fig. 167 Fig. 168 Fig. 169 Fig. 170 Fig. 171

SPIRAL-GEAR DIAGRAMS.

McCabe. "In Figs. 167 to 171, inclusive, the driving gear of each pair is shown as if transparent, the teeth being represented by lines. Fig. 167 shows a pair of gears on shafts at right angles and Fig. 168 a pair on parallel

shafts. Note that in Fig. 167 both spirals are left hand, while in Fig. 168 one is left and the other right hand; that is, in the first case they are the same hand and in the second case they are opposite hands. (The word hand as here used has the same significance as in the case of threads.) It is evident that these two are fixed conditions for shafts respectively at 90 degrees and parallel.

"Now when the shafts are at any angle between 90 degrees and 0 degrees either of these conditions may exist; that is, the spirals may be both the same hand or they may be opposite hands. This may be made plain by observing carefully Figs. 169, 170, and 171, in which the shafts are at an acute angle, all conditions in the three views being exactly alike except that the teeth are at different spiral angles in each. Note that in Fig. 169 the spiral angle of the driver is the same as the angle of the shafts which makes the follower a plain spur gear. Also note that the spiral of the driver is left hand. Now letting the spiral of the driver remain left hand, but increasing its angle a little we have the condition of Fig. 170. By decreasing it a little we have the condition in Fig. 171, making in the first case the spirals opposite hands and in the second case the spirals the same hand.

The lines $O\ A$ and $O\ B$ in these figures are drawn parallel to the shafts and the line $O\ C$ is drawn tangent to the spiral of the teeth and makes with $O\ A$ and $O\ B$ respectively the spiral angles of the driver and of the follower. Note that in Fig. 171 the angle of the shafts $A\ O\ B$ equals the spiral angles $A\ O\ C + B\ O\ C$, and in Fig. 170 the same angle $A\ O\ B$ equals $A\ O\ C - B\ O\ C$.

RELATION OF SHAFT AND SPIRAL ANGLES

"The following general rules are now evident:

"1. When the spirals are the same hand the angle of the shafts is the sum of the spiral angles.

"2. When the spirals are opposite hands the angle of the shafts is the difference of the spiral angles.

"3. When the spiral angle of one gear is the same as the angle of the shaft the spiral angle of the other will be zero, making it a plain spur gear.

"4. When the shafts are at right angles the spirals must both be the same hand.

"5. When the shafts are parallel the spirals must be opposite hands. (Helical gears.)

"6. When the shafts are at any acute angle the spirals may be either the same hand or opposite hands."

The following is an extract from an article on spiral gearing originally published in American Machinist by F. A. Halsey:

" Spiral gears are not to blame for the undoubted fact that they are somewhat troublesome to lay out, the difficulties of the problem being due to the limitations of workshop facilities and not to the geometrical nature of the gears themselves. It is easy to understand and explain the action of an existing pair of spiral gears. More than this, it is easy to lay out a pair of such gears which shall exactly meet all the conditions of the case except one; they cannot, except through rare good luck, be made with the appliances at hand. To be more specific, the circumference cannot usually be divided into an exact whole number of teeth by any stock cutter, and the real problem becomes the

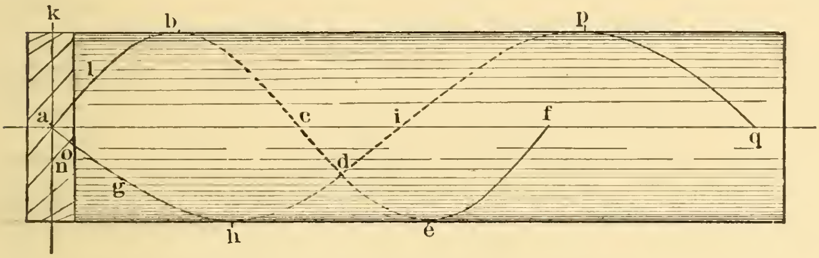


FIG. 172.

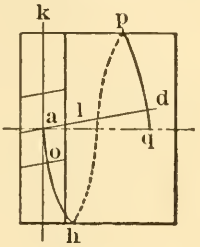


FIG. 173.

readjusting of the diameters of the gears and the angle of the teeth, so that stock cutters shall make an exact whole number of teeth.

" With spur gears it is only necessary to multiply the (circular) pitch of the cutter by the number of teeth to be cut to obtain the circumference of the gears. With spiral gears this operation gives the length of a portion of a spiral, or, more properly, helix, wound upon the pitch surface. We do not know the angle of this helix, the diameter of the pitch cylinder upon which it is wrapped, or even what part of a complete turn the known portion comprises. The length is known for each gear and nothing more, and it becomes a matter of trial to find the diameters of the gears and the helix angle to suit this portion of the helix and at the same time to fill the required center distance.

" Fig. 172 is a conventional representation of the pitch surface of a spiral gear, the surface being extended beyond the limits of the gear in order that the two helixes with which we are concerned may be shown. The first of these, *a b c d e f*, is the tooth helix and the second, *a g h d i p*, is the normal helix. The tooth helix is of importance because it defines the angle of the teeth. Given the diameter of the pitch surface, the helix may be defined by the angle *k a l* or by the length *a f*, in which it makes a complete turn—that is, by its pitch. For the determination of the speed ratio of a pair of gears the former method is the more convenient, but the tables supplied with universal milling machines which are used in setting up the machine employ the latter method.

" In all spiral gear problems we have two pitches to deal with—the pitch of the

tooth helix and the pitch of the teeth. The latter may be measured in several ways. First is the value *a n* measured on the circumference or the *circular pitch*, which is analogous to the pitch of spur gears; second is the value *a o* measured on the normal helix or the *normal pitch*, for which the cutters must be selected; third is the value *a r* measured parallel with the axis or the *axial pitch*. Since the cutters must be selected with reference to the normal pitch, the length of the normal helix is naturally of importance in connection with the number of teeth in the gear. The normal pitch multiplied by the number of teeth must naturally equal the length *a g h d* of this helix measured between its intersections *a* and *d* with the helix of a single tooth. Note that the length of the normal helix to be considered is the length *a g h d* between its intersections with the tooth, and not the length *a g h i p q* of a complete turn around the cylinder. That this is true may be seen by reference to Fig. 173, in which the angle *k a l* is nearly a right angle. It is apparent from this illustration that the length of the normal helix from *a* to *d* takes in all the teeth, and that *a o*, multiplied by the number of teeth, must equal *a h p d* and not *a h p q*. This length *a h p d* is always less than *a h p q*, and usually much less. Fig. 174, *A*, is a development of Fig. 173 on a reduced scale, *a d* being the developed length of the normal helix. Fig. 174, *B*, and Fig. 174, *C*, show how with the same

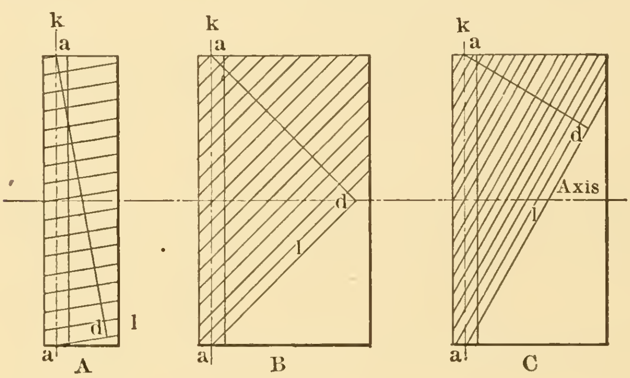


FIG. 174.

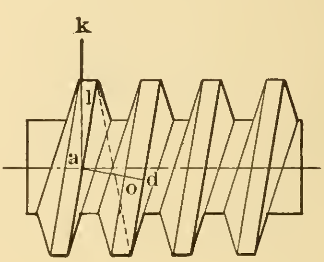


FIG. 175.

circumferential pitch and the same number of teeth but a reduced value of the angle *k a l*, the length of the normal helix which cuts all the teeth grows shorter until it may make but a small part of a complete turn around the cylinder. It is clear that in all cases the line *a d* cuts all the teeth precisely as does the circumference *a a*, which goes completely around the cylinder. It is also clear that if the normal pitch is decided upon at the start, a diameter of cylinder and a helix angle must be found such that the normal pitch, multiplied by the number of teeth, shall equal the length of the normal helix between two intersections with the tooth helix.

"It is natural to ask, Why not employ the circumferential pitch and so deal directly with the circumference instead of the normal helix? Because we do not know what it is. The normal pitch is determined by the cutter used, while the circumferential pitch depends also upon the helix angle, and until this angle is known the circumferential pitch is not known.

"In the extreme case of a spiral gear in which the helix angle is so small that the gear becomes a single thread worm, as in Fig. 175, points *o* and *d* coincide and the length of the helix between *a* and *d* becomes the normal pitch. It is, however, true as before that the normal pitch, multiplied by the number of teeth, which is now one, is still equal to the length of the normal helix between two intersections with the tooth helix.

"A glance at Fig. 174 will show that in gears of the same diameter the length of the normal helix * grows shorter as the angle *k a l* grows less, and hence that it and its gear will contain successively fewer and fewer teeth of the same normal pitch. That is to say, the number of teeth in a gear varies with the helix angle as well as with the diameter, and *the number of teeth in two gears of the same normal pitch is not necessarily proportional to the diameters.* In fact, it is never so proportional, except when the angle *k a l* is equal to 45 degrees. *The diametral pitch of the cutters and the diameter of the gear thus do not determine the number of teeth.*

"The two facts thus developed are fundamental and will bear restating:

"First, *the number of teeth is equal to the length of the normal helix divided by the normal pitch.*

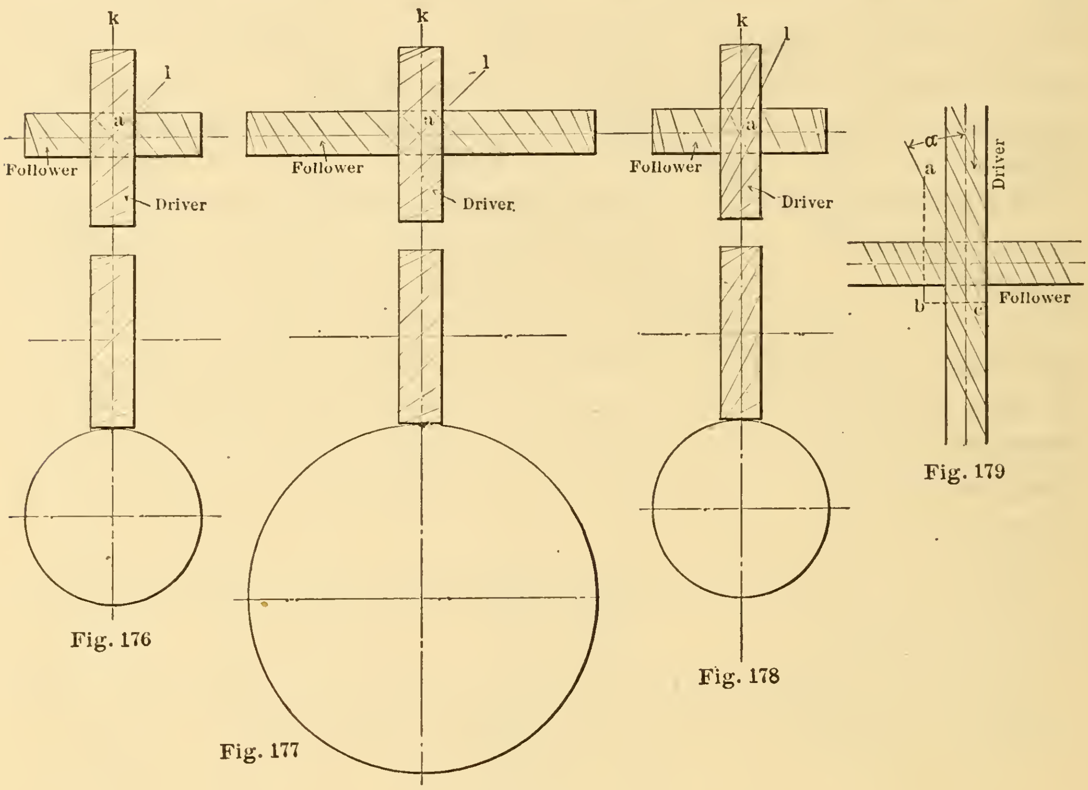
"Second, *the numbers of teeth in a pair of gears are not proportional to the diameters, except when the angle of the tooth helix is 45 degrees.*

THE SPEED RATIO

"Fig. 176 illustrates the simplest possible case of a pair of spiral gears. The gears are of equal size and the tooth helix has an angle of 45 degrees. Such a pair of gears will obviously run at the same speed—that is, have a speed ratio of 1—and as obviously both will have the same number of teeth. Now, unlike spur gears, there are two ways in which the speed ratio of such a pair of spiral gears may be varied. First, the diameters of the gears may be as in Fig. 177; and second, the angle of the helix may be changed, the diameters changed, as with spur gears, the angle of the tooth helix remaining unchanged, of the gears remaining unchanged, as in Fig. 178. These methods act in very different ways. The first method is analogous to the procedure with spur gears. As with spur gears, the circumferential or pitch-line speed of the two

* "Length of normal helix" is to be understood as meaning the length of that helix between two intersections with the same tooth helix.

gears remains, as before the change, equal, but the length of the circumference of the two gears is unequal and the largest one thus has a less number of revolutions than the smaller one. The second method is entirely unlike anything seen in connection with spur gears. By it the pitch-line speeds of the two gears are made unequal, and hence, while their diameters are equal, the lower one revolves the more slowly. This points out another



THE SPEED RATIO.

fundamental difference between spiral and spur gears: With spiral gears, unless the helix angle is 45 degrees, *the pitch-line speeds of two mating gears are not the same.*

"The two methods of changing the speed ratio shown in Figs. 177 and 178 may be combined. That is, part of the desired change in speed may be obtained by changing the diameters of the gears and the remainder by changing the angle of the helix. Given the speed ratio and the diameter of one of the gears, we may assume a helix angle and find a diameter for the second gear to go with it which shall give the desired speed ratio, and, having done this, a second angle may be assumed and a second diameter be found. There are thus an indefinite number of combinations of angles and diameters which will give the required speed ratio. Note, however, that with the diameter of one

gear fixed, every change in the diameter of the other changes the distance between centers, that not every angle of helix can be obtained by the gears which are furnished with universal milling machines, and that if ready-made cutters are to be used, the lengths of both normal helices must be exact multiple of the normal pitch of the teeth.

"The limitation of the helix angle is not, however, as serious as is usually supposed. The tables for spirals which have heretofore been supplied with universal milling machines give but a few of the spirals which can be obtained with the change gears which are regularly supplied with the machines. For universal milling machines, about two thousand spirals can be cut with these gears.

"Geometrically speaking, there is a wide range of choice in the helix angle. As regards the desirability of different angles from the standpoint of durability, the conditions are essentially the same as in worm gearing. Reference to Charts 10 and 11 under worm gears will show that the most favorable angle for durability is at about 45 degrees. There is, however, but a trifling increase in wear down to 30 degrees, no serious increase down to 20 degrees, and no destructive increase down to about 12 degrees. As the angle of worm is the complement of the angle of the driving spiral gear, the angle selected from Charts 10 and 11, for worm gears, should be the angle of the follower α, which is measured from the axis. Where gears are to transmit considerable power the best results should attend the use of angles between 30 and 45 degrees, while angles as low as 20 degrees may be used without hesitation, and as low as 12 degrees if the gears are to run in an oil bath or do light work only. The angle may also be increased above 45 degrees by similar amounts and with similar results.

"Fig. 179 is a development of the gears of Fig. 178, the angle α of Fig. 179 being equal to $k\ a\ l$ of Fig. 178, but in reversed position, because in Fig. 178 the upper side of the driver is seen, while in Fig. 179 the direction of the teeth is that of the lower side of the driver."

NOTATION FOR SPIRAL GEARS

The angle as given for spiral gears is from the axis, which is the opposite or complement of the angle for a worm, therefore the angle governing the efficiency of spiral gears should be determined from tables on worm gears as the angle of the follower (α).

The greatest angle must always be the driver, except where the angle is 45 degrees, when either gear may drive.

All of the tooth parts are derived from the normal pitch. The pitch diam-

eters are derived from the circular pitch, which is never the same in both gears of a pair, except where the angle of both gears is 45 degrees.

As the diameter of the spiral gear is no indication of its speed ratio, the terms gear and pinion are liable to be confusing, therefore follower and driver are used.

N_2 = number of teeth in follower.
N_1 = number of teeth in driver.
d_2 = pitch diameter of follower.
d_1 = pitch diameter of driver.
α = angle of follower.
β = angle of driver.
p_2' = circular pitch of follower.
p_1' = circular pitch of driver.
p'^n = normal circular pitch (the same in both gears of a pair).
P = normal diametral pitch (the same in both gears of a pair).
L_2 = lead of follower (length of tooth helix).
L_1 = lead of driver (length of tooth helix).
D_2 = outside diameter of follower.
D_1 = outside diameter of driver.
s^n = addendum of normal pitch.
r_2 = revolutions of follower.
r_1 = revolutions of driver.
δ = angle of shafts.
C = center distance.

EXAMPLES

Specifications for a pair of spiral gears are sometimes given in this manner:

Required a pair of spiral gears; ratio 3 to 1, to operate on 5-inch centers. The outside diameter of the driven gear must not exceed 7 inches; to be in the neighborhood of 6 diametral pitch.

As the most efficient spiral angle is in the neighborhood of 45 degrees, the follower should be made as large as possible, as to obtain this angle the diameter of both gears must be in proportion to their number of teeth, as for spur gears. As the pitch mentioned in connection with spiral gears is always the normal pitch, to obtain a trial pitch diameter for the follower twice the addendum of the normal pitch subtracted from the outside diameter will give the pitch diameter, according to formula 18:

$$d_2 = D_2 - \frac{2}{P} = 7 - \frac{2}{6} = 6\tfrac{2}{3} \text{ inches.}$$

and,

$$d_1 = 5 \times 2 - 6\tfrac{2}{3} = 3\tfrac{1}{3} \text{ inches.}$$

	DRIVER		FOLLOWER		REMARKS
	TO FIND	FORMULA	TO FIND	FORMULA	
1	β	$Tan\ \beta = \frac{d_1\ r_1}{d_2\ r_2}$	α	$90^\circ - \beta$	Axes at right angles only.
2	β	$Tan\ \beta = \frac{p'_1}{p'_2}$	α	$90^\circ - \beta$	Axes at right angles only.
3	β	$Cos\ \beta = \frac{p'^n}{p'_1}$	α	$\delta - \beta$	
4	β	$Tan\ \beta = \frac{d_1\ \pi}{L_1}$	α	$\delta - \beta$	
5	$p_1{}^n$	$\frac{d_1\ \pi}{N_1} cos\ \beta$	p'^n	$\frac{d_2\ \pi}{N_2} cos\ \alpha$	Same in both gears.
6	p'^n	$p'_1\ cos\ \beta$	p'^n	$p'_2\ cos\ \alpha$	Same in both gears.
7	p'_1	$\frac{p'^n}{cos\ \beta}$	p'_2	$\frac{p'^n}{cos\ \alpha}$	
8	p'_1	$\frac{d_1\ \pi}{N_1}$	p'_2	$\frac{d_2\ \pi}{N_2}$	
9	L_1	$p'_2\ N_1$	L_2	$p'_1\ N_2$	Axes at right angles only.
10	L_1	$d_1\ \pi\ tan\ \alpha$	L_2	$d_2\ \pi\ tan\ \beta$	
11	N_1	$d_1\ P\ cos\ \beta$	N_2	$d_2\ P\ cos\ \alpha$	
12	N_1	$\frac{d_1\ \pi}{p'_1}$	N_2	$\frac{d_2\ \pi}{p'_2}$	
13	d_1	$\frac{2\ C}{\left(\frac{r_1}{r_2}\ tan\ \alpha\right) + 1}$	d_2	$\frac{2\ C}{\left(\frac{r_1}{r_2}\ tan\ \beta\right) + 1}$	Axes at right angles only.
14	d_1	$\frac{2C}{\left(\frac{r_1}{r_2}\ \frac{cos\ \beta}{cos\ \alpha}\right) + 1}$	d_2	$2\ C - d_1$	
15	d_1	$N_1\ p'_1\ 0.3183$	d_2	$N_2\ p'_2\ 0.3183$	
16	d_1	$\frac{N_1}{P\ cos\ \beta}$	d_2	$\frac{N_2}{P\ cos\ \alpha}$	
17	D_1	$d_1 + 2\ s^n$	D_2	$d_2 + 2\ s^n$	
18	D_1	$d_1 + \frac{2}{P}$	D_2	$d_2 + \frac{2}{P}$	$14\frac{1}{2}^\circ$ standard only.
19	Cutter See chart 14	$\frac{N_1}{cos^3\ \beta}$	Cutter	$\frac{N_2}{cos^3\ \alpha}$	
20		$C = \frac{N_1}{2\ P\ cos\ \beta} + \frac{N_2}{2\ P\ cos\ \alpha}$			

Formulas for Spiral Gears.

The next step is to find the angle of driver by formula 1.

$$Tan\ \beta = \frac{d_1 r_1}{d_2 r_2} = \frac{3\frac{1}{3} \times 3}{6\frac{2}{3} \times 1} = 1.5, \text{ or } 56^\circ\ 19'.$$

The angle of driver = 90 56° 19′ = 33° 41′.

Find the provisional number of teeth by formula 11.

$$N_1 = d_1\ P\ cos\ \beta = 3\tfrac{1}{3} \times 6 \times 0.5546 = 11.094.$$
$$N_2 = d_2\ P\ cos\ \alpha = 6\tfrac{2}{3} \times 6 \times 0.8321 = 33.282.$$

Naturally the number of teeth must be whole numbers, so it will be necessary to change either the center distance, or to make numerous calculations and shift the diameters. Practically, however, it is possible to have quite an error in the normal pitch; the normal pitch, or the pitch of the cutter, preferably being under size rather than over. The teeth are thus cut enough deeper than standard to secure the proper thickness of tooth at the pitch line.

This difference may be 0.02 of the circular pitch in some cases.

If the cutter is heavier than the normal pitch it will be impossible to secure enough clearance at the bottom of the tooth as the proper thickness of tooth will be reached before getting the depth of tooth required.

If the center distance can be changed the pitch diameters may be shifted by the method explained on page 66 of Mr. F. A. Halsey's book—"Worm and Spiral Gearing," as follows:

$$\frac{\text{final diameter}}{\text{provisional diameter}} = \frac{11}{11.094}$$

or

$$\text{final diameter} = \text{provisional diameter} \times \frac{11}{11.094}.$$

That is:

$$\text{final } d_1 = 3\tfrac{1}{3} \times \frac{11}{11.094} = 3.361;$$

and,

$$\text{final } d_2 = 6\tfrac{2}{3} \times \frac{33}{33.282} = 6.723;$$

and

$$d_1 \times d_2 = 3.361 + 6.723 = 10.084 = \text{twice the corrected center distance.}$$

In the present example the normal circular pitch for the nearest even number of teeth, 11 and 33, by formula 5 would be:

$$p_1^n = \frac{d_1\ \pi}{N_1}\ cos\ \alpha = \frac{3\frac{1}{3} \times 3.1416}{11} \times 0.5546 = 0.5280.$$

As the pitch of the cutter is 0.5236, this error will not prevent a first-class job being turned out if proper precautions are taken, and no change will be required in the center distance.

These points being settled, the remaining calculations are simple. Before making any calculations, the requirements should be put in the form of a table to avoid confusion, as follows:

DIMENSIONS	DRIVER	FOLLOWER
Pitch diameters	3⅓	6⅔
Revolutions	3	1
Angles	56° 19′	33° 41′
Number of teeth	11	33
Circular pitch	0.9520″	0.6345″
Normal pitch	0.5280″	0.5280″
Cutter used	No. 2–6*p*	No. 3–6*p*
Lead, exact	6.9795″	31.4160″
Lead, approximate	6.9670″	31.5000″
Addendum	0.1680″	0.1680″
Outside diameter	3.6690″	7.0030″
Whole depth of tooth	0.3630″	0.3630″
Thickness of tooth	0.2640″	0.2640″
Gear on worm	86	72
First gear on stud	48	40
Second gear on stud	28	56
Gear on screw	72	32

An error of 0.5 inch in a lead of 50 inches would not ordinarily be prohibitive, but the angle must be changed to suit any alteration of the lead or the cutter will drag. If too much alteration is made in the lead and angle, the teeth must be cut a little deeper than standard to allow the gears to assemble on the proper shaft angles.

The amount of adjustment that can be made depends, of course, upon the accuracy required, and should be done by some one accustomed to the work. This is not possible when cutting helical or herringbone gears, as the tooth has contact the entire length of face and a slight error is noticeable. The accuracy of the final calculations may be checked by the angles, obtained from the circular pitch by formula 3.

Another way of presenting this problem is as follows:

Required, a pair of spiral gears; ratio 4 to 1; about 8 diametral pitch (0.3927-inch circular pitch). Angle of spiral for driver, β, to be about 55 degrees. $\alpha = 90 - \beta = 35$ degrees.

Find the diameter of driver by formula 13.

$$d_1 = \frac{2\,C}{\left(\frac{r_1}{r_2}\,tan\;\alpha\right) + 1} = \frac{2 \times 6}{\left(\frac{4}{1} \times 0.7002\right) + 1} = 3.1572 \text{ inches.}$$

The diameter of follower $d_2 = 2\,C - d_1 = 12 - 3.1572 = 8.8428$ inches. The remaining dimensions are found as in the first example.

Still another example:

The ratio of a spiral gear drive is 4 to 1. The diameter of the driver cannot be less than 8 inches, on account of the size of the shaft. The distance between centers to be 5½ inches. No pitch mentioned.

Assumed diameter of driver $d_1 = 8$ inches.

Diameter of follower $d_2 = (5\tfrac{1}{2} \times 2) - 8 = 2$ inches.

According to formula 1:

$$Tan\ \beta = \frac{d_1\,r_1}{d_2\,r_2} = \frac{8 \times 4}{2 \times 1} = 16.0, \text{ or } 86^\circ\ 25'.$$

Try 10 diametral pitch:

According to formula 11:

$$N_1 = d_1\ P\ cos\ \beta = 8 \times 10 \times 0.0625 = 5 \text{ teeth;}$$
$$N_2 = d_2\ P\ cos\ \alpha = 2 \times 10 \times 19.96 = 0.9980, \text{ say } 20 \text{ teeth;}$$

which just happens to come out even.

If the center distance is not specified, the best plan is to assume number of teeth, angles, and pitch of cutter, P or p'^n and find the corresponding center distance by formula 20.

Example:

What center distance will be required for a pair of spiral gears 11 and 33 teeth, 6 diametral pitch, the angle of the 11 tooth drive being 56° 19′ and the angle of the follower 33° 41′.

According to formula 20:

$$C = \frac{N_1}{2\ P\ cos\ \beta} + \frac{N_2}{2\ P\ cos\ \alpha} = \frac{11}{2 \times 6 \times 0.5546} + \frac{33}{2 \times 6 \times 0.8321} = 1.6529$$
$$+ 3.3049 = 4.9578 \text{ inches center distance;}$$

1.6529 being the pitch radius of the pinion, and 3.3049 the pitch radius of the gear.

When the center distance is approximate, this is the simplest solution of the problem, the speed ratio being used in place of a trial number of teeth, and the number of teeth made to suit the desired center distance.

A CHART FOR LAYING OUT SPIRAL GEARS

Chart 13 with the following explanation of its deviation and use will be an aid in solving spiral gear problems once the provisional numbers of teeth are obtained. This diagram and explanation were originally published in AMERICAN MACHINIST, February 27, 1902, by J. N. Le Conte.

" The provisional numbers of teeth will not in general be whole numbers,

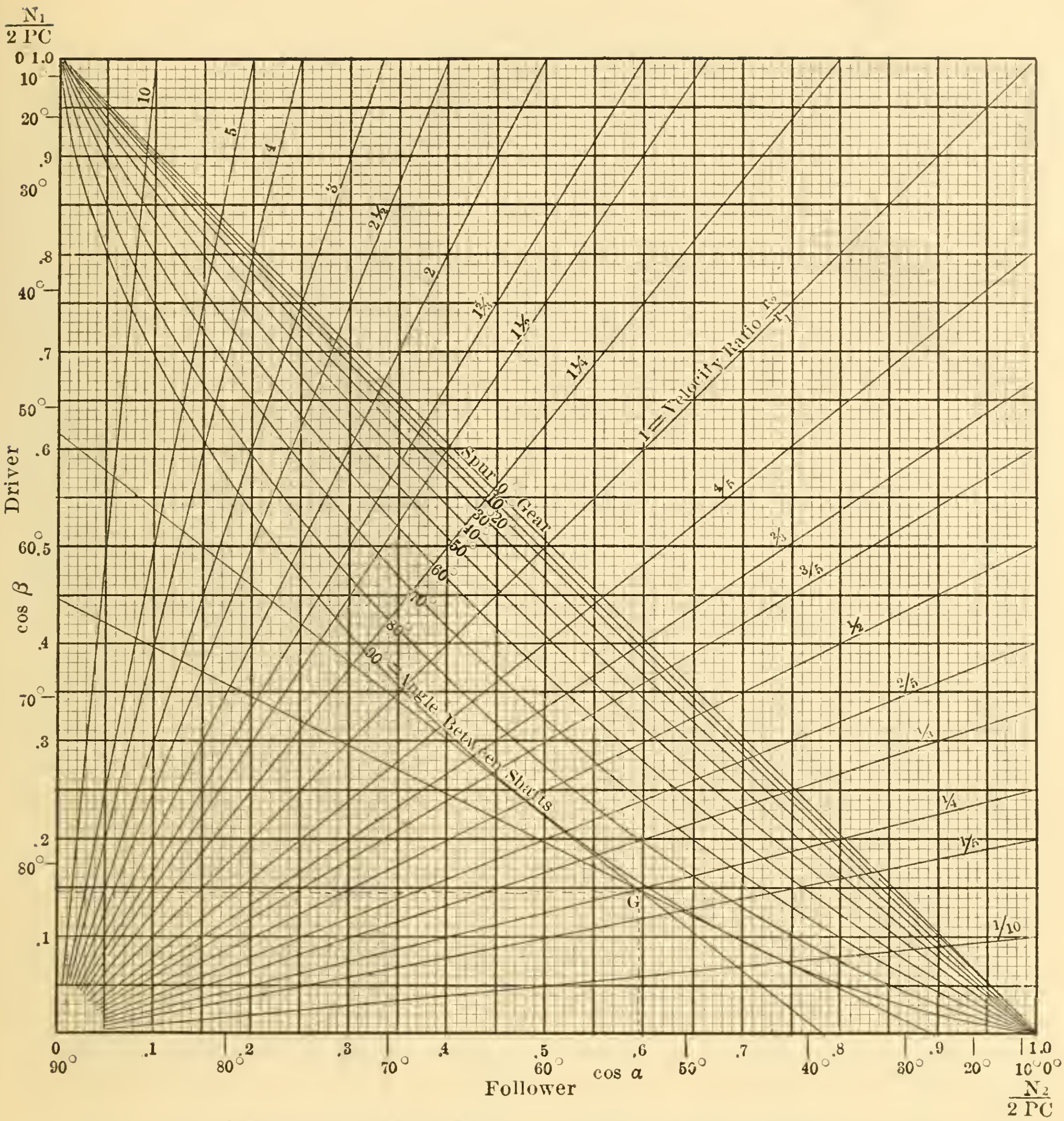


CHART 13. DIAGRAM FOR LAYING OUT SPIRAL GEARS.

but we must choose the nearest whole numbers to the ones obtained, and recalculate the angles and radii to fit the new case, as has been previously shown. The direct solution of this depends upon the solution of the equation:

$$\frac{N_2}{\cos \alpha} + \frac{N_1}{\cos (\delta - \alpha)} = 2\,P\,C$$

"In which N_1 and N_2 are the nearest whole numbers of teeth to the calculated ones, and δ is the shaft angle, or $\delta = \alpha + \beta$. As is well known, this equation cannot be solved by any simple means, for it is of the fourth degree, and, though possible of solution, such solution is not practical. Furthermore,

there are four real values of α which will satisfy it. Graphic methods of solution, or continued approximations, must then be resorted to. Chart 13 gives a method by which the angle can be read off directly. Having obtained the nearest whole number of teeth on the gears, find on the diagram the point G whose co-ordinates are $\frac{N_1}{2\,P\,C}$ and $\frac{N_2}{2\,P\,C}$ on the inner scales. Through this point draw a line or merely lay a straight-edge tangent to the curve representing the shaft angle. The outer scales on the bottom and left will give roughly the angles α and β respectively, and the inner scales the values of cos α and cos β quite accurately. The radial lines of velocity ratio will facilitate the location of the desired point, for if the ratio be one of those given, the point must lie on its line. It will also be noticed that for shafts crossing at 90° the position of the line gives the desired angles at its two extremities.

"It is interesting to note that two lines can be drawn through a given point tangent to the curves, as shown. As a matter of fact, four such lines could be drawn provided the whole of the curves were laid in, but that portion shown is the only portion giving positive angles, *i. e.*, angles within the angle δ. But there will be two separative positive values of the angle α, which, with a given velocity ratio, number of teeth and shaft distance, will work correctly together, giving of course different values of the radii. Which of these is the one required can always be told as lying nearest to the first approximation of the angle. If the point G lies on one of the curves, the two positions coincide (a limiting case), and if it lies on the concave side the solution is impossible within the angle δ.

"As an example of the use of the diagram, take the oft-quoted case of Mr. De Leeuw. Here angular velocity

$$y_1 = \frac{\gamma_2}{\gamma_1} = \tfrac{1}{4},\ N_1 = 8,\ N_2 = 32,\ C = 4.468'',\ \delta = 90^\circ, \text{ and } P = 6.$$

Then

$$\frac{N_1}{2\,P\,C} = 0.149,\ \frac{N}{2\,P\,C} = 0.596.$$

"These co-ordinates give the point G on the diagram. A line through G drawn tangent to the lower part of the 90° curve gives quite accurately cos $\alpha = 0.894$, and cos $\beta = 0.447$, or:

$$\alpha = 26^\circ\ 35' \text{ and } \beta = 63^\circ\ 25',$$

agreeing quite closely with the result derived analytically. If the second line be drawn through G tangent to the upper portion of the curve, it gives: cos $\alpha = 0.787$ and cos $\beta = 0.617$, or:

$$\alpha = 38^\circ\ 61' \text{ and } \beta = 51^\circ\ 54'.$$

These fulfill the requirements."

SPIRAL GEAR TABLE

While it is better in every case to understand the principles involved before using a table, as this tends to prevent errors, they can be used with good results by simply following the directions carefully. The subject of spiral gears is

	To obtain the circular pitch for one tooth divide by the required diametral pitch.	To obtain the pitch diameter, divide by the required diametral pitch and multiply the quotient by the required number of teeth.	To obtain the lead of spiral, divide by the required diametral pitch and multiply the quotient by required number of teeth.		To obtain the pitch diameter, divide by the required diametral pitch and multiply the quotient by the required number of teeth.	To obtain the circular pitch for one tooth divide by the required diametral pitch.	
ANGLE OF SPIRAL DEGREES	CIRCULAR PITCH	ONE TOOTH OR ADDENDUM	LEAD OF SPIRALS		ONE TOOTH OR ADDENDUM	CIRCULAR PITCH	ANGLE OF SPIRAL DEGREES
Small Wheel.	Small Wheel.	Small Wheel.	Small Wheel.	Large Wheel.	Large Wheel.	Large Wheel.	Large Wheel.
1	3.1419	1.0001	180.05	3.1420	57.298	180.01	89
2	3.1435	1.0006	90.020	3.1435	28.653	90.016	88
3	3.1457	1.0013	60.032	3.1458	19.107	60.026	87
4	3.1491	1.0024	45.038	3.1492	14.335	45.035	86
5	3.1535	1.0038	37.077	3.1527	11.473	36.044	85
6	3.1589	1.0055	30.056	3.1589	9.5667	30.055	84
7	3.1652	1.0075	25.728	3.1651	8.2055	25.778	83
8	3.1724	1.0098	22.573	3.1724	7.1852	22.573	82
9	3.1806	1.0124	20.082	3.1807	6.3924	20.082	81
10	3.1900	1.0154	18.092	3.1901	5.7587	18.092	80
11	3.2003	1.0187	16.464	3.2003	5.2408	16.464	79
12	3.2145	1.0232	15.076	3.2105	4.8097	15.104	78
13	3.2242	1.0263	13.966	3.2294	4.4454	13.988	77
14	3.2377	1.0306	12.986	3.2378	4.1335	12.986	76
15	3.2522	1.0352	12.138	3.2524	3.8637	12.138	75
16	3.2679	1.0402	11.393	3.2678	3.6279	11.397	74
17	3.2848	1.0456	10.417	3.2821	3.4203	10.745	73
18	3.3116	1.0514	10.192	3.3032	3.2360	10.166	72
19	3.3225	1.0576	9.6494	3.3225	3.0715	9.6494	71
20	3.3430	1.0641	9.1848	3.3433	2.9238	9.1854	70
21	3.3650	1.0711	8.7662	3.3652	2.7904	8.7663	69
22	3.3882	1.0785	8.3862	3.3833	2.6694	8.3862	68
23	3.4127	1.0863	8.0399	3.4129	2.5593	8.0403	67
24	3.4451	1.0946	7.7379	3.4391	2.4585	7.7242	66
25	3.4661	1.1033	7.4332	3.4663	2.3662	7.4336	65
26	3.4953	1.1126	7.1664	3.4952	2.2811	7.1663	64
27	3.5258	1.1223	6.9198	3.5257	2.2026	6.9197	63
28	3.5579	1.1325	6.6912	3.5575	2.1300	6.6916	62
29	3.5918	1.1433	6.4799	3.5919	2.0626	6.4799	61
30	3.6276	1.1547	6.2778	3.6277	2.0000	6.2832	60
31	3.6650	1.1666	6.0979	3.6652	1.9416	6.0997	59
32	3.7043	1.1791	5.9282	3.7044	1.8870	5.9282	58
33	3.7457	1.1923	5.7710	3.7459	1.8360	5.7680	57
34	3.7894	1.2062	5.6181	3.7826	1.7882	5.6178	56
35	3.8349	1.2207	5.4754	3.8351	1.7434	5.4770	55
36	3.8830	1.2360	5.3431	3.8834	1.7013	5.3448	54
37	3.9336	1.2521	5.2201	3.9261	1.6616	5.2200	53
38	3.9867	1.2690	5.1028	3.9921	1.6242	5.1026	52
39	4.0482	1.2867	4.9866	4.0416	1.5890	4.9920	51
40	4.1010	1.3054	4.8873	4.1012	1.5557	4.8874	50
41	4.1626	1.3250	4.7885	4.1540	1.5242	4.7884	49
42	4.2273	1.3456	4.6949	4.2272	1.4944	4.6948	48
43	4.2956	1.3673	4.6065	4.2956	1.4662	4.6062	47
44	4.3671	1.3901	4.5223	4.3675	1.4395	4.5225	46
45	4.4428	1.4142	4.4428	4.4428	1.4142	4.4428	45

Table 27—Spiral Gear Table Shaft Angles 90 Degrees

For one Diametral Pitch.

so much more complicated than other gears that many will prefer to depend entirely on tables.

This table gives the circular pitch and addendum or diametral pitch and lead of spirals for one diametral pitch and with teeth having angles from 1 to 89 degrees to 45 and 45 degrees. For other pitches divide the addendum given and the spiral number by the required pitch, and multiply the results by the required number of teeth. This will give the pitch diameter and lead of spiral for each gear. For the outside diameter add twice the addendum of the normal pitch, as in spur gearing.

Suppose we want a pair of spiral gears with 10 and 80 degree angles, 8 diametral pitch cutter, with 16 teeth in the small gear, having 10-degree angle and 10 teeth in the large gear with its 80-degree angle.

Find the 10-degree angle of spiral and in the third column find 1.0154. Divide by pitch, 8, which is 0.1269. Multiply this by the number of teeth; $0.1269 \times 16 = 2.030 =$ pitch diameter. Add two addendums or $\frac{2}{4} = 0.25$ inch. Outside diameter $= 2.030 + 0.25 = 2.28$ inches.

The lead of spiral for ten degrees, for small gear, is 18.092. Divide by pitch $= \frac{18.092}{8} = 2.2615$. Multiply by number of teeth, $2.2615 \times 16 = 36.18$, or lead of spiral, which means that the tooth helix makes one turn in 36.18 inches.

For the other gear with its 80-degree angle, find the addendum, 5.7587. Divide by pitch, $8 = 0.7198$. Multiply by number of teeth, $10 = 7.198$. Add two addendums, or 0.25, gives 7.448 as outside diameter.

The lead of spiral is 3.1901. Dividing by pitch, 8, $= 0.3988$. Multiply by number of teeth $= 3.988$ the lead of spiral.

When racks are to mesh with spiral gears, divide the number in the circular pitch columns for the given angle by the required diametral pitch to find the corresponding circular pitch.

If a rack is required to mesh with 40-degree spiral gear of 8 pitch, look for circular pitch opposite 40 and find 4.101. Dividing by 8 gives 0.512 as the circular pitch for this angle. The greater the angle the greater the circular or linear pitch, as can be seen by trying an 80-degree angle. Here the circular pitch is 2.261 inches.

PITCH OF CUTTER	CORRESPONDING CIRCULAR PITCH	CORRESPONDING DIAMETRAL PITCH	ADDENDUM
P	p'	p	s^n
2	2.2214	1.4142	.70710
2¼	1.9745	1.5909	.62853
2½	1.7771	1.7677	.56568
2¾	1.6156	1.9445	.51426
3	1.4809	2.1213	.47140
3½	1.2694	2.4748	.40406
4	1.1107	2.8284	.35355
5	.8885	3.5355	.28284
6	.7404	4.2426	.23570
7	.6347	4.9497	.20203
8	.5553	5.6568	.17677
9	.4936	6.3638	.15713
10	.4443	7.0710	.14142
12	.3702	8.4853	.11785
14	.3173	9.8994	.10101
16	.2776	11.3137	08838
18	.2468	12.7279	.07856
20	.2221	14.1421	.07071
22	.2019	15.5563	.06428
24	.1851	16.9705	.05892
26	.1708	18.3847	.05439
28	.1586	19.7990	.05050
30	.1481	21.2116	.04714
32	.1388	22.6274	.04419
36	.1234	25.4558	.03928
40	.1111	28.2842	.03535
48	.0925	33.9411	.02988

TABLE 28—SPIRAL GEARS OF 45 DEGREES

For determining the pitch diameters of spiral gears when the pitch of cutter is assumed and angle of spiral is 45 degrees: Multiply the addendum of the normal pitch found in fourth column by number of teeth.

DIRECTION OF ROTATION AND THRUST OF SPIRAL GEARS

The use of spiral gears generally causes some study on the part of the designer to determine the proper direction of the teeth, having given the direction of rotation of the two shafts which are to be connected. Another point about spiral gears which also causes some study after the direction of the teeth has been determined is the direction of the axial thrust, that is, the direction in which the spiral gears tend to move along their axes when transmitting motion. The proper direction of thrusts is very important to locate correctly the ball-thrust bearings of other suitable anti-friction devices.

It sometimes occurs that a newly designed machine, when started for the first time, has a shaft which is driven by spiral gears running in the opposite direction to that which was intended, or that the anti-friction washers have

been located on the wrong side of the helical gear. To obviate this and reduce the chance for mistakes in directions of rotation and strains in spiral gears, four diagrams, 180 to 183, are arranged to readily illustrate every pos-

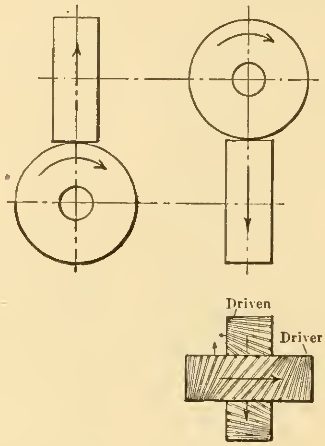


FIG. 180.

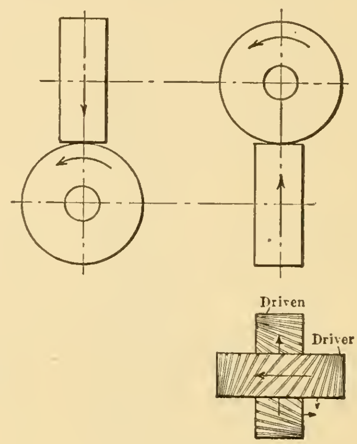


FIG. 181.

RIGHT-HAND SPIRAL GEARS.

sible combination; giving the direction of the teeth, their rotation, and the direction of the lateral strains when they are transmitting motion in the directions indicated. These diagrams eliminate the necessity of consulting

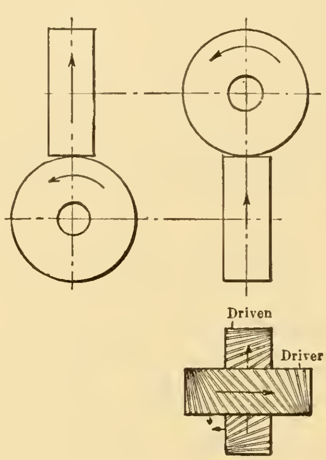


FIG. 182.

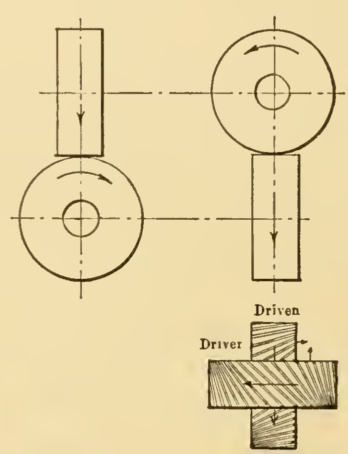


FIG. 183.

LEFT-HAND SPIRAL GEARS.

a gear model, nor is it necessary to go through a series of hand manipulations describing the rotations in the air.

In the diagrams, Figs. 180 and 181 represent a pair of right-hand helical gears with the direction of rotation of the drivers reversed.

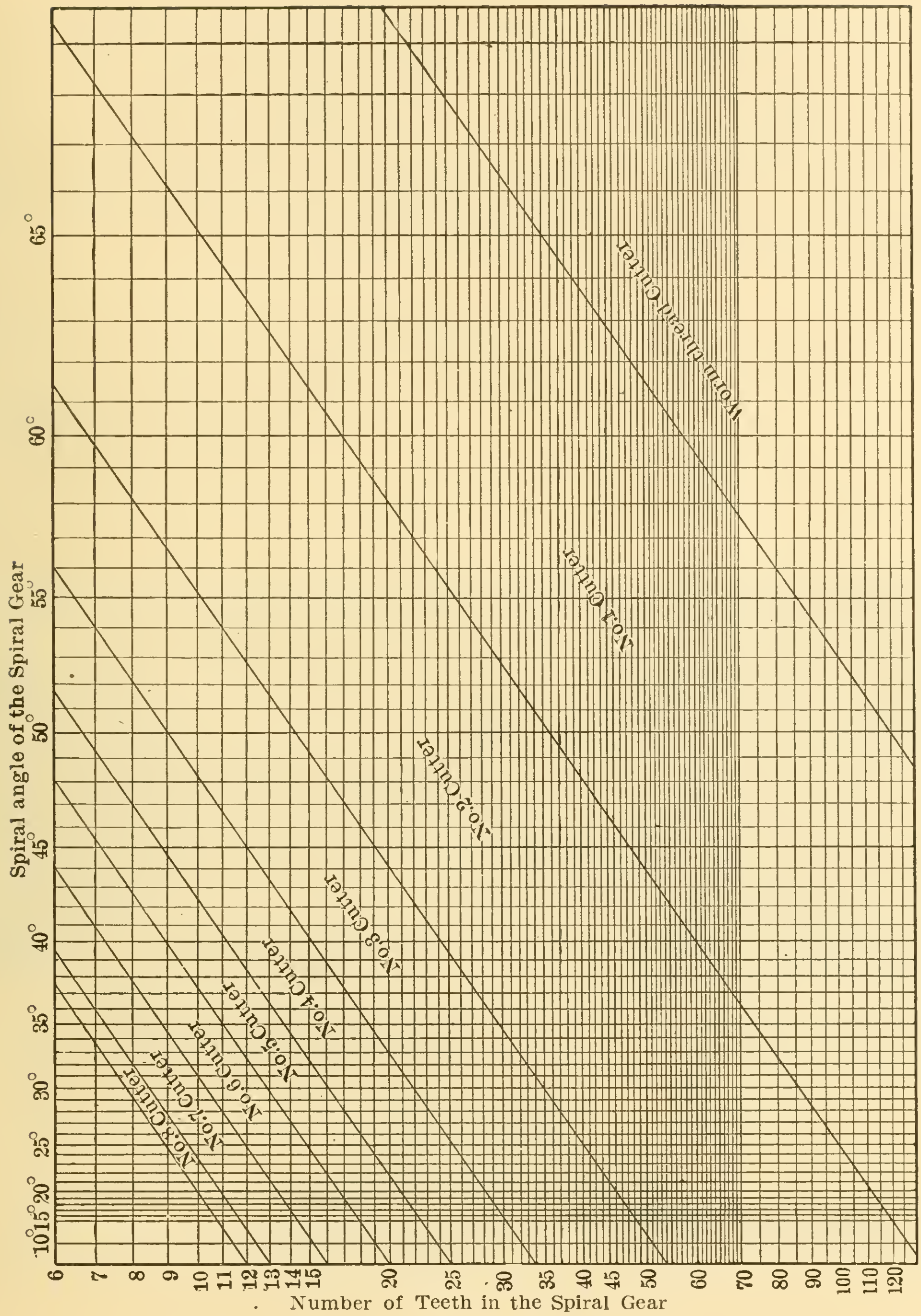


CHART 14. CUTTERS FOR SPIRAL GEARS.

The diagrams Figs. 182 and 183 each show a pair of left-hand helical gears, also with the directions of their drivers reversed. It should be noted that reversing the direction of rotation of the drivers reverses the directions of their axial thrusts. Also, if the driven gears are made the drivers and rotating in the same direction, as shown, the lateral strains are also reversed;

that is, if in Fig. 180 the driven gear is made the driver and rotates as indicated, the gear marked the driver, which is now the driven gear, will rotate as shown, but the axial thrust of each gear would be as in Fig. 181. If the driven gear of Fig. 181 is made the driver, the lateral strains are as shown in Fig. 180. This is also true of the left-hand combinations shown in Figs. 182 and 183, originally published in the American Machinist by William F. Zimmerman.

SECTION IX

Skew Bevel Gears

When the axes are at an angle, and not in the same plane, and the distance between the shafts is not great enough to allow the use of spiral or worm gears, the skew bevel gear (illustrated by Fig. 184) is the only possible solution for a single pair of gears.

It is not the purpose to treat this subject theoretically, but to show practical means of producing such gears. The skew bevel gear is supposed to be

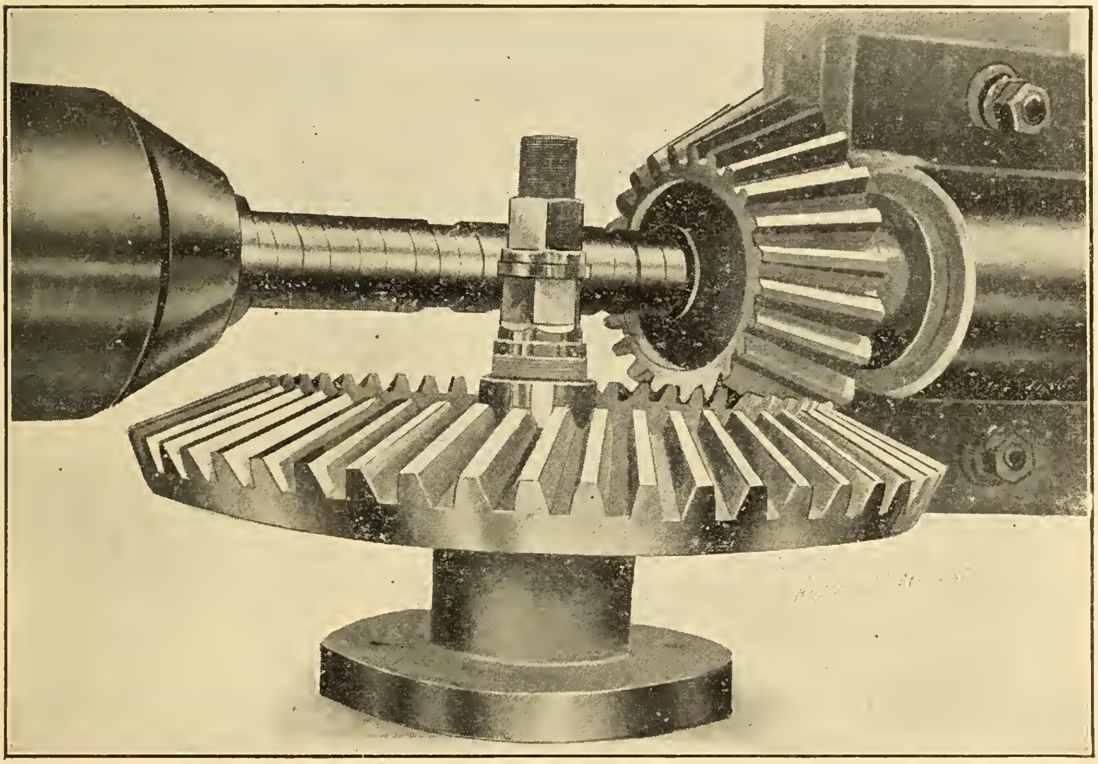


FIG. 184. SKEW BEVEL GEARS.

a combination of the bevel and the spiral gear, or a bevel gear with spiral teeth.

J. A. Utts describes a process of cutting skew bevel gears based on this assumption, as follows:

"The axes of the gears are at right angles, but in planes a distance of 2.25 inches from each other (see Fig. 186). A little study of this sketch will show that the line *A B* corresponds to the line of contact of the pitch cones of a pair of bevel gears, if continued to the point of intersection; also that the line *A B* must be at 45 degrees angle as that of a pair of miter gears; also by referring again to Fig. 185, it will be seen that the ends of the teeth have the same appearance as have the teeth on a common bevel gear, or, in other words, the teeth must not be canted, but at every point throughout the length of the

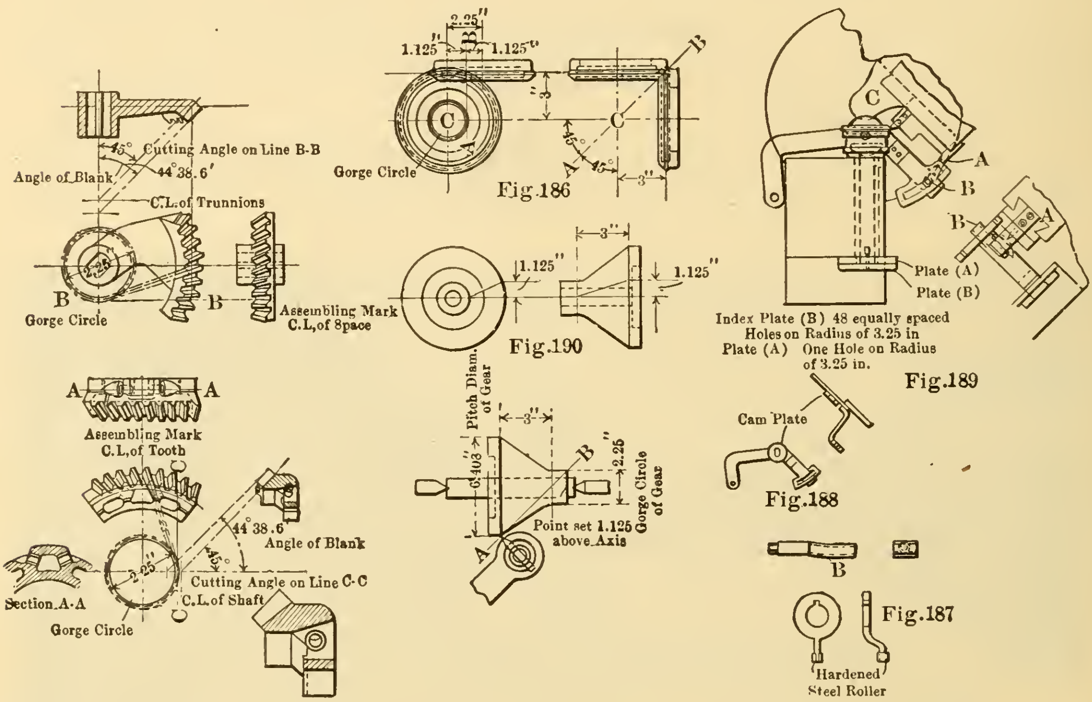


Fig. 185. Details of bevel gears.

SPECIAL ATTACHMENT DESIGNED TO PLANE SKEW BEVEL GEARS.

tooth, a line, drawn through the vertical center of the tooth, must pass through the axis of gear."

"Any mechanic who has watched a gear shaper at work will know that if the blank can be rotated while the tool is cutting, a spiral tooth will be produced; but in this case, the spiral not being constant, the rotating of the blank must be in the proper ratio at every point along the path of the tool. As the job was done on a Gleason gear shaper, it is now fully presented to the reader.

"The rotating is accomplished by an arm (secured to the spindle) actuated by a cam slot (Fig. 187); the cam piece is secured to the lever (Fig. 188), which is moved in unison with the tool by the arm *A* (Fig. 189) secured to the tool head. Note the piece *B* (Fig. 189) set upright in the arm; this is a

close working fit in the arm and the circular surfaces (see *B*, Fig. 187) are concentric with the common center of the machine, as is also the circular slot in the end of the lever. These circular surfaces are necessary to allow universal movement of the tool-head arm, and at the same time impart as little motion to the lever as possible."

"The master form (see Fig. 190) was finished by setting the turning tool 1.125 inches above the center and feeding with the compound rest at 45 degrees; this produced the curve surface shown. On taking the finishing cut it was necessary to revolve the work in opposite directions in order that a cutting edge would be presented. The line *A B* was then drawn with the point of a scratcher set in same position as the tool and drawn across the form with the compound rest. This line corresponds with the compound rest. This line corresponds to the line of contact on the pitch cones of a pair of miter gears.

"The spindle is in two parts; the outer or quill has a plate with index pin keyed to one end and the rotating arm (Fig. 187) keyed fast to the other end. The inner spindle has an index plate keyed to one end and arbors for holding blanks at the other end; the two parts are rotated as one when the index pin is inserted, and the inner can be revolved within the outer for spacing the blank when the index pin is withdrawn. The arbors are so made that both gears are given the proper 'placing distance' without moving the spindle head, thereby maintaining the rotating arm in the proper location in relation to the cam slot."

"To generate the cam slot a short temporary arm was substituted for the arm (Fig. 187). The master form was held on the arbor as a gear blank would be for machining, a scratcher was held in the tool holder with the point in the same location as the cutting point of a tool; the machine was then fed up until the point just touched the form; the form was then revolved on the arbor until the line *A B* was brought to coincide with the scratcher point and then clamped fast to the arbor. The cam piece (see Fig. 188) was secured to the lever; a circle was drawn on it by a hardened cylinder of the same outside diameter as the roller on the arm (Fig. 187) but cut away at an angle to its axis so as to make a sharp point at its circumference; as the cylinder was free to slide and turn on the arm, it was a simple matter to scribe a circle on the cam piece, concentric with the arm. The tool head was then moved forward a fraction of an inch, the spindle was revolved until the line *A B* again coincided with the scratcher point, when another circle was scribed on the cam piece. This was repeated until a series of circles were scribed, completing the laying out of the cam slot. The cam piece was then taken off; the slot was roughed out and then again secured in place; the cylinder was replaced by a shell reamer and the operation of laying out was repeated, except

that the reamer was used to cut down through the slot at every stop, thus making a series of finished points in the cam slot. The slot was then finished with a file; it was a simple operation, as it consisted simply of taking off the small points left between the reamer cuts. It will be seen that the cam slots thus generated compensates or absorbs all false or inaccurate motions of the lever.

"Now with the cam piece and rotating arm in place and a blank on the arbor, the cutting of the teeth was proceeded with, the same as if cutting a common bevel gear (see assembled view, Fig. 189).

"A detail worthy of note is the floating piece that forms one side of the cam slot, and another, not shown, that forms one side of the circular slot in the lever. These pieces are free to slide on guide pins and are held against roller in the cam slot and against the piece *B* (Fig. 189) in the lever by springs, the object being to provide a constant takeup in case of wear and also keep the working side of all parts in close contact when the tool was on the cutting stroke.

"It was found that the common 14½-degree involute tooth would not clear, but by making the angle 20 degrees the teeth cleared nicely. No fitting with a file was done; the teeth were machined to a finish.

"These gears are required to work together smoothly without any lost motion and also to be interchangeable. These requirements were fulfilled. Of course, there are some slight inaccuracies in the teeth, but as the best authority I have thus far been able to find says that the correct shape of tooth on this form of gear can only be approximated in practice, I believe I have produced a practically perfect gear of its kind."

There is a much simpler method used to cut skew bevel gears on a Gleason planing machine.

The gears are turned up according to bevel gears of the same number of teeth, pitch and ratio, no alteration being made in the diameters or angles due to the fact that the apex points of the two gears do not intersect.

The teeth are cut diagonally across the face in a straight path, the gear being held stationary while each tooth is cut. This diagonal cut is accomplished by dropping the former a proportional space of the distance between the shaft centers. Thus, when cutting miter gears with 2 inches between the shafts, the former would be dropped 1 inch in each case. For a gear ratio of two to one and the same shaft centers the former would be dropped $\frac{2}{2+1}$ $= 0.666$ inch for the pinion and $0.666 \times 2 = 1.333$ inch drop for the gear. It is not at all necessary that this drop in the former be so divided; it may all be put in either gear of a pair. But if it is evenly divided in proportion to

the number of teeth or ratio of each gear the contact will be better and less trouble will be experienced in cutting the teeth.

Dropping the former produces a left-hand tooth, and raising the former produces a right-hand tooth. Lowering the former when cutting the gear and raising the former for the pinion will produce a left-hand gear and a right-hand pinion which will run together, the only difference being that the pinion

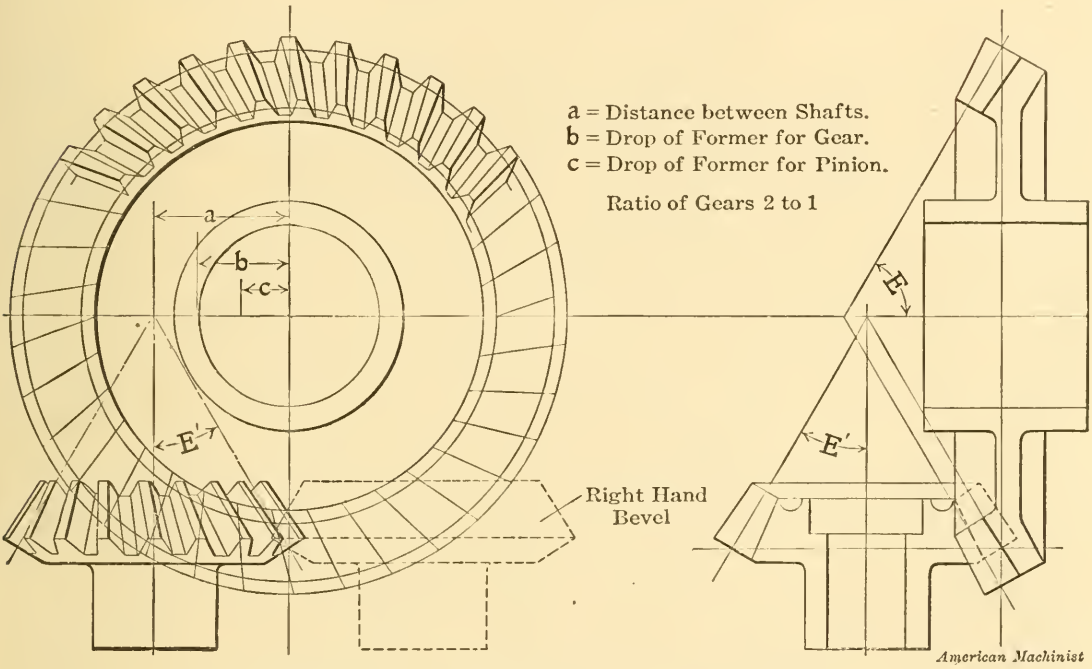


FIG. 191. LAYOUT FOR SKEW BEVEL GEARS.

will turn the gear in the opposite direction, being located in the right-hand side instead of the left-hand side, as shown by dotted lines in Fig. 191.

The former must be inclined to the same angle as that used for cutting the teeth or the teeth will lean. The gears illustrated in half tone (Fig. 184) were 1½-inch circular pitch, 4½-inch face, the larger being approximately 36 inches pitch diameter. The distance between the shaft centers was $3\frac{9}{16}$ inches. These gears operated with the usual amount of backlash, there being even contact the entire length of the tooth. These gears were cut by the R. D. Nuttall Company of Pittsburg. Many smaller gears have been cut in the same manner, all of which have been entirely satisfactory.

By this method the skew bevel gear, which has long been avoided as being next to impossible of solution, is made a commercial possibility.

SECTION X

Intermittent Gears

Intermittent gears are designed to allow the driven gear or follower one or more periods of rest during each revolution of the driver. This may be accomplished in a rough manner by cutting out a number of teeth in the follower as illustrated in Fig. 192, but the cut and try method must be employed to obtain a definite ratio. This type of intermittent gear is seldom used, there being nothing but the spring *B* to keep the follower from moving during a period of rest, and the first tooth of the driver enters contact in a very uncertain manner, it sometimes being necessary to shorten the first tooth in the driver to prevent it from striking the top of the first tooth in the follower.

The proper design of intermittent gears is not as difficult as it first appears. The pitch and outside diameters are found as for an ordinary spur gear, the pitch desired must correspond to an even number of teeth. The blank space on the driving gear is milled to the pitch line, and the stops in the follower are cut by a cutter of a diameter corresponding to the pitch diameter of the driver. If no such cutter is at hand, use the nearest to that size to rough out the stops and finish them with a fly cutter which can be set to any desired radius.

It is well not to have the gears too near the same size; the driver should be the smallest in order to secure all the contact possible in the stops.

The simplest form of intermittent gear is shown by Fig. 193, the follower being moved but a short distance for each revolution of the driver. It will be noticed that a small amount of fitting will always be required at the point *a* to allow the point of the stop to clear.

A more complicated drive is shown in Fig. 194, the follower being moved one-sixth of a revolution for each revolution of the driver. Each of these gears is turned up as for a spur gear of 30 teeth 5 diametral pitch. The cutting operation would be as follows: Index for 30 teeth; cut the first three teeth, then index for two teeth without cutting, and so on around the blank. The six stops are then milled, with a cutter 6 inches in diameter, to a depth of 0.2 inch, or the addendum of the gear, which completes the follower. Four teeth are then cut in the driver, and the remainder of the blank milled to the pitch line. A little filing and the gears are complete.

A still simpler method of cutting these gears, and one that avoids the necessity of first laying them out, is as follows: Drop a cutter equaling the pitch diameter of the driver into the blank of the follower, to the depth of the addendum at the points stops are desired. Then cut the first tooth at a

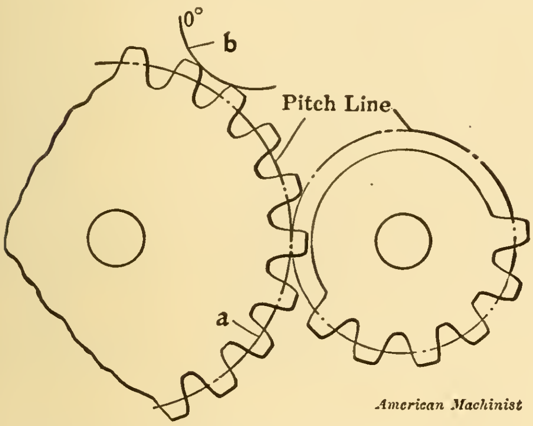


FIG. 192. POOR DESIGN OF INTERMITTENT GEARS.

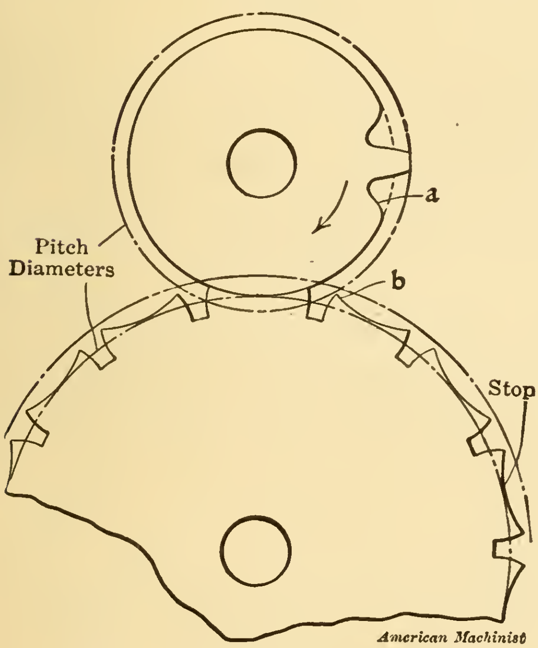


FIG. 193.
INTERMITTENT GEARS WITH TWELVE STOPS.

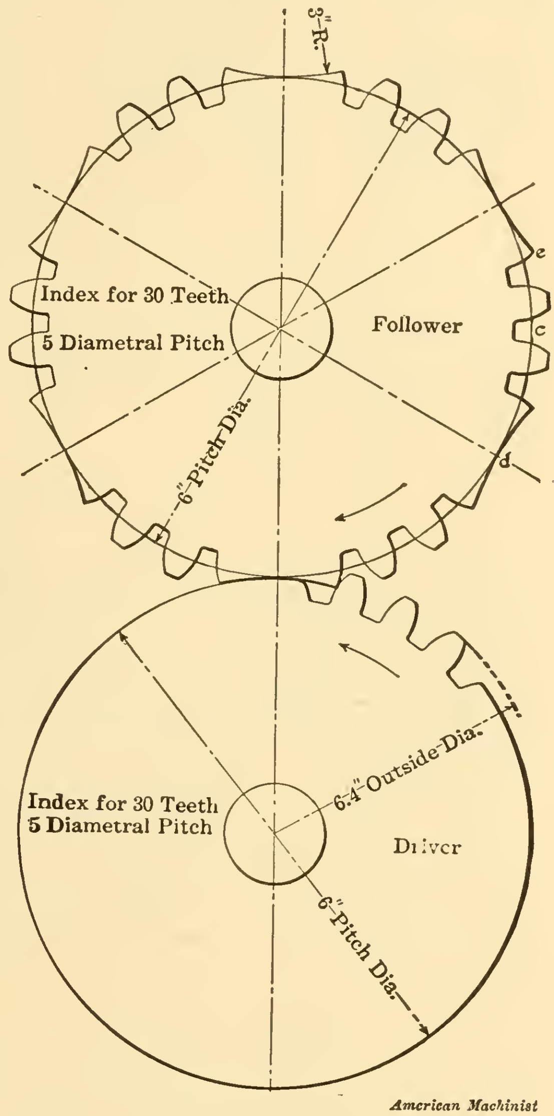


FIG. 194. INTERMITTENT GEARS WITH SIX STOPS.

point midway between two of the stops, and continue cutting toward one of the stops until the point of the stop touching the outside circumference of the blank is cut away, or, in other words, until there is no blank space on the gear between the last space cut and the point of the stop. The same number of teeth are then cut in the opposite direction until the same

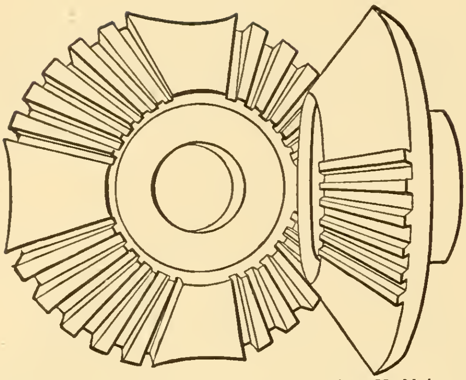


FIG. 195. INTERMITTENT BEVEL GEARS.

condition is met. If the stops are evenly spaced the cutting of the remaining teeth is a simple matter. If the stops are not evenly spaced the first tooth for each group must be located between each stop. The same number of teeth are then cut in the driver as there are spaces in the follower for each group and the remainder of the blank milled to the pitch line. For a pair of gears such as shown in Figs. 193 and 194, the cutting of the teeth by this process will be a simple matter.

The cutting of internal intermittent gears is a counterpart of the above. Bevel gears, while being more difficult to cut, are governed by the same rules (see Fig. 195).

MODIFICATIONS OF THE GENEVA STOP

The accompanying engravings illustrate three highly ingenious and extremely interesting modifications of the device used in watches to prevent overwinding which have been applied by Mr. Hugo Bilgram to various automatic machines constructed at his works. The constructions have a family resemblance in principle, though they are entirely unlike from a structural standpoint.

Three main features characterize the constructions: first, the intermittent motion of the Geneva stop; second, the entire absence of shock at engagement or disengagement; and third, the positive character of the movement—the parts being locked in position both when they are in motion and when they are idle.

Fig. 196 represents the smallest departure from the watch mechanism. The interrupted disk *a* is the driver and revolves continually. The driven piece is seen at *b*, and the requirements are that the driven piece shall remain at rest during three-fourths of a revolution of the driver, and shall then make one-quarter of a turn during the remaining quarter turn of the driver. The driver may revolve in either direction, but supposing it to turn in the direction of the arrow, a roller *c* attached to the driver is about to enter one of four radial slots in the face of the driven piece. During the succeeding quarter turn of the driver the parts will move together, the motion of the follower ceasing when groove *d* has reached the position occupied in the figure by groove *e*, and this movement of the follower will obviously occupy ninety degrees of

angle. It will be seen that the parts are so laid out that roller *c* enters and leaves the grooves tangentially, insuring absence of shock at both the commencement and the conclusion of the engagement.

At *f g h i* on the follower is a series of rollers raised above the faces surrounding the grooves, and the circular part of the driving disk carries a circular groove *j k l* at such a radial distance as to engage these rollers in succession

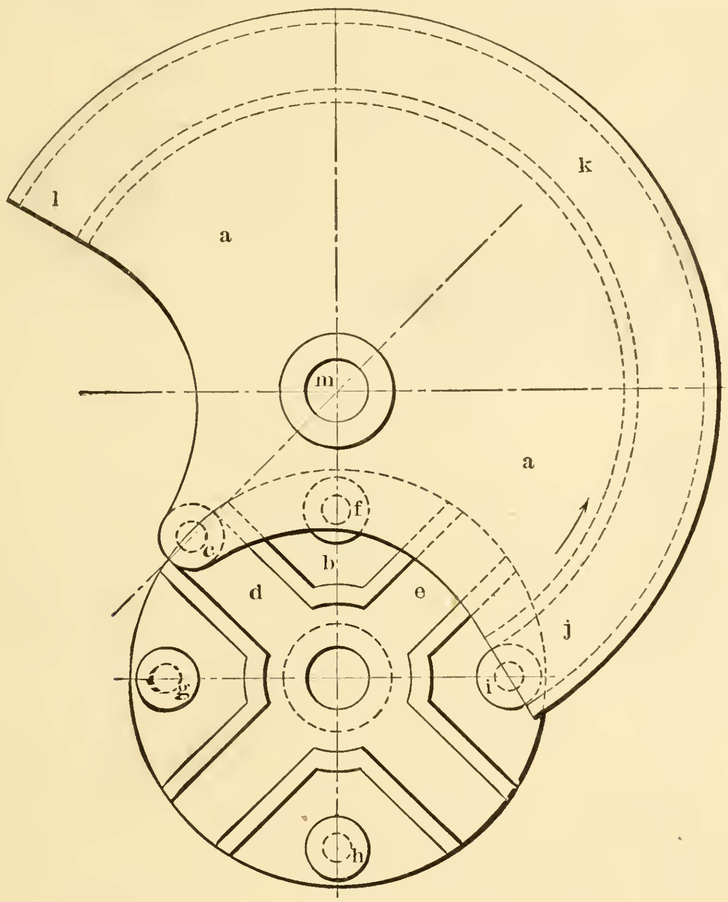


FIG. 196. GENEVA STOP—MODIFICATION NO. 1.

during the idle period of the follower and hence lock it in position. It is obvious that in the direction of motion supposed, this circular groove is just leaving roller *i*, and so disengaging it preparatory to movement by roller *c*. On the completion of the follower's movement, roller *f* will occupy the position of roller *g* in the illustration, while the end *l* of groove *j k l* will have turned to a position ready to embrace it and so lock the follower in position. With motion in the opposite direction, groove *j k l* in the position shown would be in the act of engaging roller *i* on the completion of the movement. Rollers *g* and *i* being at the same distance from the center *m* of the driver, both are

engaged by the groove during a revolution, the locking taking place with one and the unlocking with the other, both rollers being in the groove during most of the time.

A modification of this gear, which it is unnecessary to show, has five grooves in the driven wheel with corresponding modifications in the character of the movement.

In the second construction the motion of the follower is intermittent like the last, but with different relations between the idle and acting periods. The driver runs continuously, the relationship being:

During 5-6 turn of the driver the follower is at rest.

During 1 1-6 turn of the driver the follower makes one complete turn.

In other words the follower makes one turn to every two turns of the driver, but this revolution of the follower occupies little more than a turn of the driver.

Pinned to the face of the driven gear is the plate *a*, Fig. 197, the arm *b* of which is fitted to embrace a hub *f* on the driver shaft, whereby, until released, the follower is locked in its idle position. Revolving with this hub is an arm *c* carrying a roller *d*, which is fitted to engage the slot *g*. As it does so, the notch *e* in hub *f* comes opposite finger *h*, thus disengaging the locking mechanism. Roller *d* enters tangentially without shock and accelerates the motion of the follower until the roller reaches the line of centers, when the gears engage and the motion goes on. The completion of the revolution of the driver finds the roller *d* again on the line of centers, but engaging slot *i*, and the continuance of the motion brings the parts again to the positions shown. It should be noted that in this mechanism not only is the starting and stopping of the driver without shock, but at the instant the engagement of the gear teeth the roller *d* has brought the velocity of the follower up to that due to the gears, so that the transition of the motion from the pin to the gears and back again from the gears to the pin is also without shock.

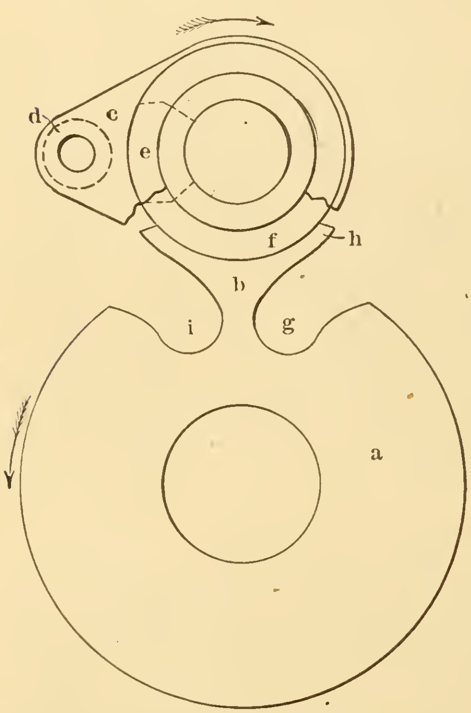


FIG. 197. GENEVA STOP—MODIFICATION NO. 2.

The most elaborate of these mechanisms is that shown in Figs. 198, 199,

and 200. In this the driver—turning about a—is required to turn through about 73 per cent. of a revolution, while the follower stands still, the follower then making a complete revolution during the remaining 27 per cent. of the revolution of the driver. Figs. 198, 199, and 200 are side views intended to show the action in a succession of positions.

The driver is an interrupted disk b c d, Fig. 198, having cam-shaped edges e f at the mouth of the notch. Slightly in the rear of this disk is a

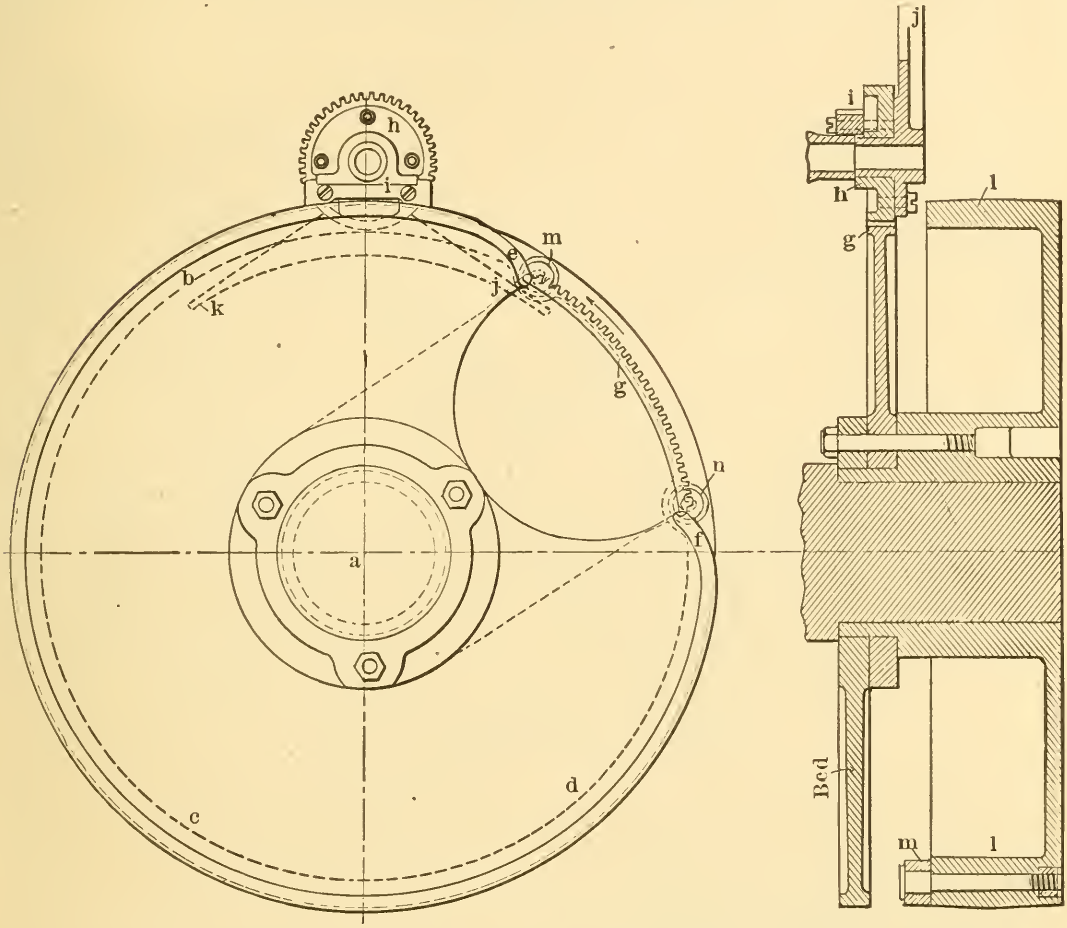


FIG. 198. GENEVA STOP—MODIFICATION NO. 3.

toothed sector g. The incomplete driven gear h meshes with g during the acting periods, and the purpose of the remainder of the mechanism is to start b in motion and throw the teeth in mesh as well as to lock the follower in position during the period that it stands still. Fig. 198 shows the parts in position at the beginning of the movement of the follower, which is still locked in the position which it occupies during its idle period. A bar i on the front face of gear h rides on, and up to this point has been locked in the idle position

by the disk *b c d*. Finger *j*—one of a pair *j k*—is attached to the rear of gear *h*, where it may turn freely between the sector *g* and the driving pulley *l*. This pulley *l* carries two rollers *m n* arranged to engage the fingers *j k* respectively. In the position of Fig. 198 the driving disk, moving in the direction of the arrow, has brought roller *m* into position, where it is about to engage finger *j*, the direction of the acting side of *j* being tangential to the motion of *m*, so that the movement begins without shock. To permit *j*, *b*, and *i* to turn, the edge *e* of the disk is dressed off, but to such a degree

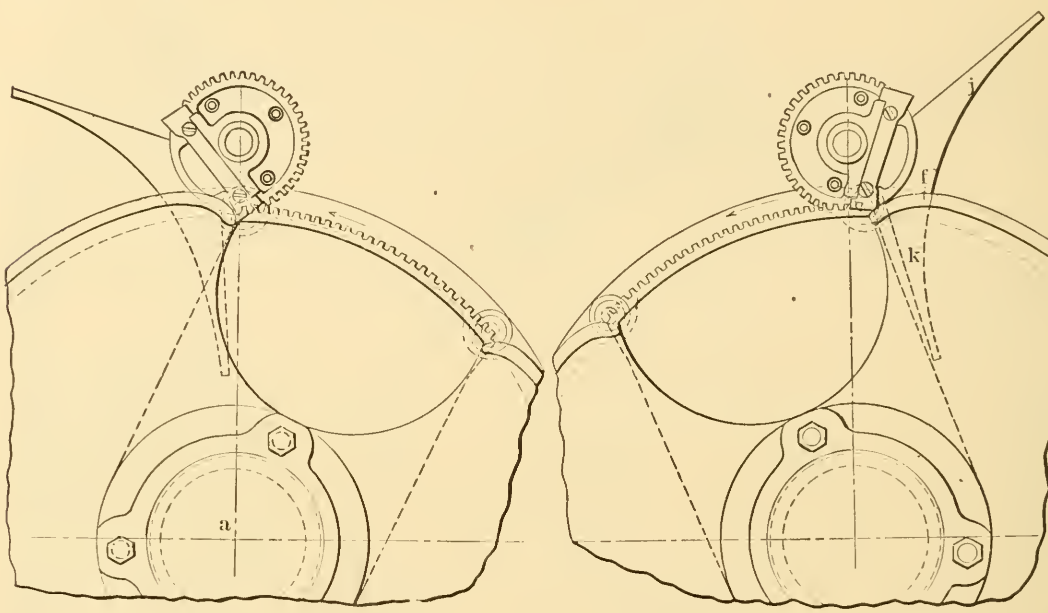


FIG. 199. FIG. 200.

GENEVA STOP—MODIFICATION NO. 3.

that contact is maintained between the right-hand end of *i* and the edge of the disk as the follower turns, so that the motion is positive without slack. As the movement progresses the speed on the follower increases until the position of Fig. 199 is reached, when—the roller being on the pitch circle of sector *g*—the speed of the follower is the same as that due to the gears and the teeth drop into mesh without shock. From this on the gears drive, and the arms *j k* turn completely over, the position when the gears go out of mesh being shown in Fig. 200. From this on the action is the reverse of that shown by Figs. 198 and 200, the driving piece being now the cam-shaped edge *f* of the disk, the finger *k* preserving the positiveness of the motion and preventing the driven pieces overrunning by momentum as they are brought to rest, the final stopping being accomplished as the roller slips out of action in a tangential direction, and again without shock. From the position of Fig. 200 to

that of Fig. 198 the bar *i* simply rides on the edge of the driver disk and the follower remains at rest.

"It should be remembered that these mechanisms are not models designed to embody a pretty movement invented beforehand, but they are parts of machines, some of which are made in considerable numbers, and have been devised, as occasion arose, to accomplish certain required results. As such, they represent the art of invention carried to a high degree of perfection."

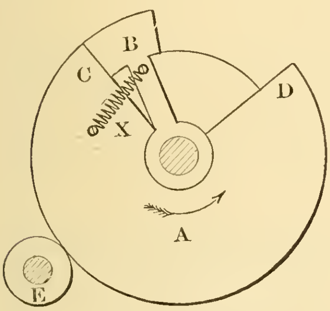


FIG. 201.

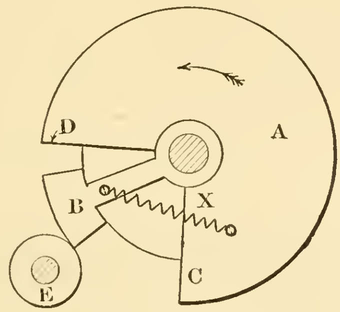


FIG. 202.

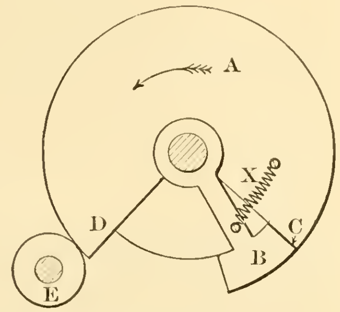


FIG. 203.

AN INTERMITTENT SPUR GEAR.

The line cuts (Figs. 201, 202, and 203) show a pair of intermittent spur gears in three positions. The peculiarity about this gear is that although a dwell occurs, the teeth of the gear and pinion are in mesh at all times.

The mutilated gear *A* is the driver and is secured to its shaft. A portion of its rim—dependent upon the length of dwell—is cut away. A segment *B* is mounted on the same shaft. This segment is free to swing in the cut-away portion of *A*, and is held in place against the side of *A* by a collar on the shaft. The teeth of *B* match with the teeth of *A* in both of its extreme positions. *B* is held against the face *C* by a spring *X*. This spring is elastic enough to allow *B* to move as far as *D*.

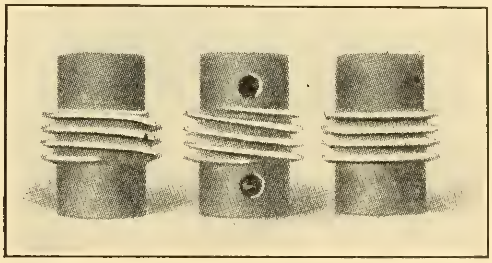


FIG. 204. AN INTERMITTENT WORM.

As the gear *A* moves in the direction of the arrow, it turns the pinion *E*. When *B* engages with *E*—the resistance of *E* being greater than the resistance of the spring *X*—the segment *B* remains stationary, while *A* moves till *D* comes in contact with *B*. During the time that *B* is at rest the pinion *E* is of course also at rest. As soon as *D* comes in contact with *B*, both *B* and *E* begin again to move. When *B* reaches a position where its teeth are no longer in engagement with the teeth of *E*, the spring *X* returns it to the face

C. These gears were used as a feed gear for paper, the paper being cut during the dwell.

The half-tone, Fig. 204, shows three views of an intermittent worm used in a looping machine. The pitch is 1-6 inch. The dwell is two-thirds of a turn, and the advance the remaining third. It was cut on an ordinary 16-inch lathe, using a mutilated change-gear.

AN INTERESTING PAIR OF SPIRAL INTERMITTENT GEARS

Figs. 205 and 206 show a pair of intermittent gears having the peculiar characteristics that, if the large gear be rotated continuously in one direction, the pinion will rotate alternately three-quarters of a turn in one direction and one-quarter of a turn in the opposite direction, with a rest or dwell between each

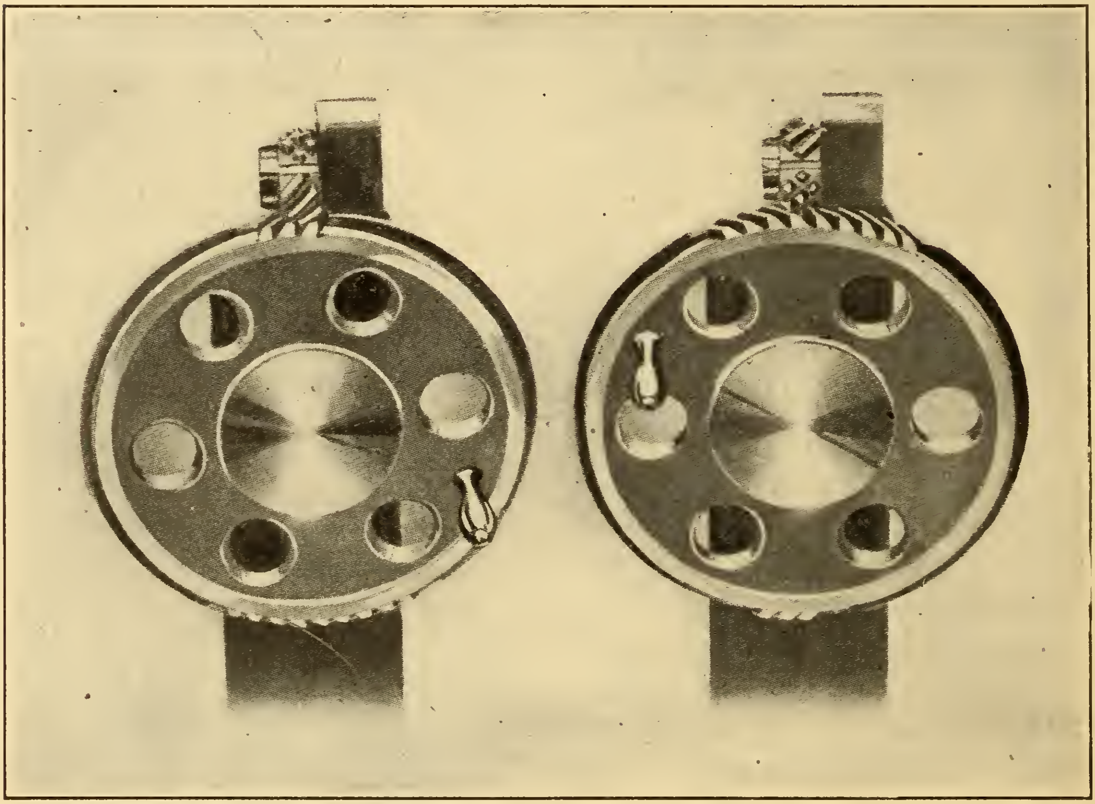


FIG. 205. THE RIGHT-HAND TEETH IN MESH WITH THE "PEG" SEGMENT.

FIG. 206. THE LEFT-HAND TEETH IN MESH.

movement. Similar gears have been made as part of a certain machine, the nature of which we are not at liberty to mention. The angle of spiral of both gears is 45 degrees, and under ordinary conditions either gear could be the driver. The large gear, however, has around portions of its periphery

two tongues—which are practically a continuation of certain of the teeth. These tongues fit into grooves in the pinion, and during their passage through the grooves lock the pinion at rest. Owing to this feature, the large gear must in this case be the driver.

The pinion has twelve teeth, divided into four groups of three teeth each, with one of the before-mentioned slots between each group. Two opposite groups of teeth are cut left hand, the two alternate groups are cut both left and right, leaving the teeth like a series of pegs.

Imagine the handle at a position opposite the pinion, then the left-hand teeth in the large gear will be at the left. There are nine teeth cut in this segment which, when the handle is turned—in the direction of the hands of a clock—engage first with the three teeth of a double-cut group on the pinion, then with the three left-hand full teeth of the next group, then with the three teeth of the other double-cut group. The pinion has then turned three-quarters of a revolution. The tongue then engages with the slot in the pinion and the rotation of the pinion is arrested. The large gear turns until the three right-hand teeth on its periphery come into mesh with the double-cut group first referred to, and reverse the direction of rotation of the pinion for a space of three teeth, or one-quarter turn, when the tongue again locks the pinion and the handle reaches the starting position. The large gear has thus made one turn and the pinion has advanced through three groups of three teeth each, equal to three-quarters of a turn, and has reversed through one group of three teeth, or one-quarter turn. Thus the total advance is but two groups of three teeth, or one-half turn, and to bring the gear and pinion into the same relative position as they were at the start the wheel must make another complete turn.

The blank for the large gear was turned to the extreme diameter across the top of tongue, mounted in the milling machine, and the left-hand teeth, which extend clear across the face, were gashed slightly below the level of the top of the tongue. The blank was then indexed halfway around from the central left-hand space, the table swung for right hand, and a deep right-hand gash made for locating. Then the blank was indexed halfway around from the central right-hand space, the table swung back for left hand, and the left-hand teeth section milled down to the proper gear diameter. Then the full-length left-hand teeth were cut, and also a gash made outside of the end teeth which join the tongue so as to get the curve of the tooth, but not going far enough to touch the tongue. The blank was then indexed back to the right-hand locating space; the table swung, blank milled down to the proper gear diameter, and the right-hand teeth were cut the same as the left-hand. The table was then set square, and an end mill was put into the

spindle, and the stock milled away—leaving the tongue. It will be noticed that it was not necessary to remount the blank at all. The outsides of the teeth joining the tongue were chipped out as smooth and true to curve as possible.

An important feature was to get the angle of the end of the tongue just right, and not file it too far back, which would have allowed the large gear to carry the small one somewhat beyond the point where the tongue should enter the slot, thus causing the end of the tongue to strike the side of the small gear and arrest the movement or perhaps cause breakage. The cutting

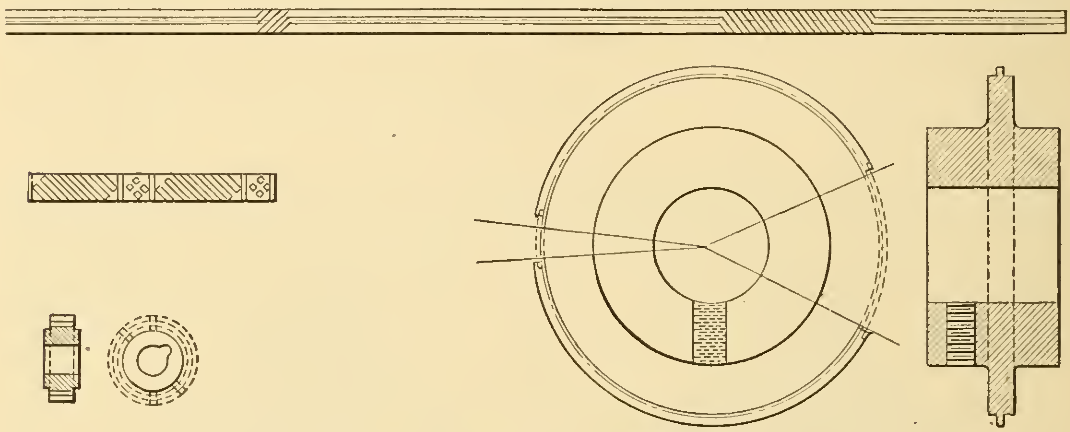


FIG. 207. LAYOUT OF A PAIR OF INTERMITTENT SPIRAL GEARS.

of the small gear was simply a matter of setting the cutter directly over the center of the blank and swinging the table for both hands; the slots were cut at the last machine operation, and then the little sharp projections which were left at the ends of the crosscut sections and near the slots were chipped off, as they were surplus and would interfere with the movement of the gears.

Fig. 207 shows layout of a set of the same style of gearing as Figs. 205 and 206, but of a different ratio.

The gears were cut by the Boston Gear Works, and were prepared preliminary to cutting into more expensive blanks. The work was accomplished by the usual methods, and the finished gearing accomplished the desired results and is now in successful use.

SECTION XI

Elliptical Gears

Elliptic gears are in general use on shapers, planers, slotters and similar machine tools to transmit a quick-return motion to the ram. The cost of production is more than for gears having circular pitch lines, but they are undoubtedly the cheapest quick-return motion among known mechanical movements. The method to be outlined is not new, but is as accurate as any and the cost of the tools is not high. This method is in general use in many shops to the exclusion of other methods not considered as good.

In order more clearly to describe the process it will be well to take an actual case and carry it through from start to finish. Assume that a pair of gears is ordered to transmit a 3 to 1 quick-return motion and the centers are 8 inches apart. About 3 diametral pitch is specified.

METHOD OF LAYING OUT*

Fig. 208 represents diagrammatically the method of laying out the gears. Lines $A\ A'$ and $B\ B'$ are first drawn perpendicular to each other. The major axis of the pitch line of the gear is the same as the given center distance, 8 inches, and is laid off as shown. The focus points $X\ X'$ are drawn in so that $A\ X$ is in the same ratio to $A'\ X$ as the given quick-return ratio. The points B and B' are located by setting the dividers at one-half the major axis $A\ A'$ and cutting the minor axis with arcs having centers at X and X'. Arcs having radii equal to $O\ B'$ and $O\ A'$ are drawn covering one quadrant, as shown. This quadrant is divided into six equal divisions, as shown by radii from the center O. These radii are marked a, a^1, a^2, etc. Next intersecting lines are drawn from the intersection points of a, a^1, a^2, etc., with the circles, as shown,

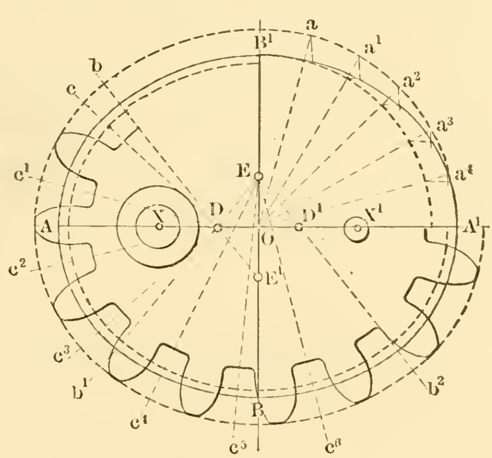


FIG. 208. LAYOUT FOR AN ELLIPTIC GEAR.

* W. E. Thompson.

and the intersection points of these perpendiculars are points on a perfect ellipse.

"With a center on A A^1 find an arc that will very nearly cut the points from a^4, a^3, a^2, and A^1. Also find a center on B B^1 about which an arc can be drawn cutting points B^1, a, a^1, and a^2. These centers are used when cutting the teeth and should be laid out as accurately as possible. After finding two centers and measuring the respective distances from O, the other two centers

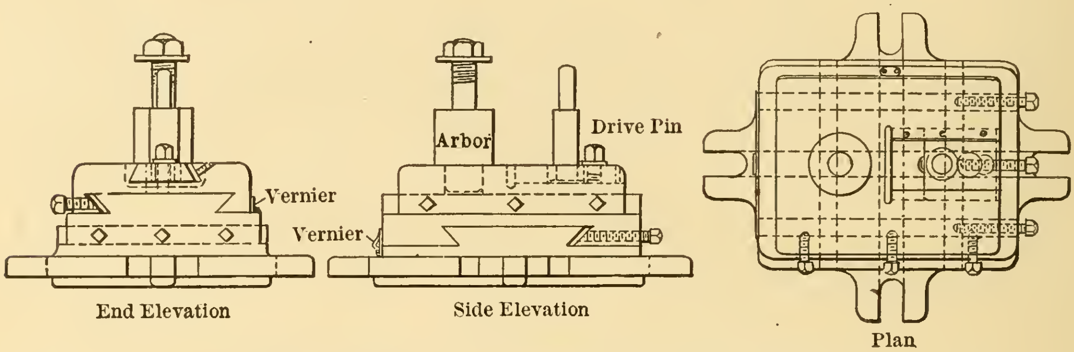


FIG. 209. FIXTURE FOR CUTTING ELLIPTICAL GEARS.

may be put in and an ellipse drawn, as shown. This line is then stepped off with dividers set at the corrected tooth thickness for the desired pitch and the number of divisions noted.

"It is preferable to have an odd number of teeth for convenience in cutting, so if the pitch line does not divide into an even number of divisions, half of which is an odd number, when the dividers are set to the chordal-tooth thickness of the desired pitch, the pitch line may be reduced or enlarged, as is found necessary. When the pitch is given, as it was in this case, the pitch line may be divided and the divider division measured. The corresponding pitch is used in selecting a cutter. In this case the line was divided into 30 divisions which corresponded very close to 2¾ pitch, so these cutters were used.

"Radial lines common to two centers of the elliptic arcs are drawn, as shown at b, b^1, b^2, and in the other quadrant not drawn. These lines are the dividing points between two different arcs and are drawn before laying out or cutting the teeth. Radial lines from the four centers are drawn, through the centers of the space divisions, as shown at c, c^1, c^2, etc. These lines are for the purpose of starting the first cut and checking the rest to prevent large errors.

"The base circles of the two different curves are drawn, as shown, and a few teeth laid out by an accurate odontograph on each curve, as shown, or cutters may be selected by measuring the pitch radii D A and E B and figuring the proper number of teeth to cut for. Cutters are selected by one or the other of

these methods and the gear, previously having the shaft and driving pin holes X and X^1 bored and the addendum outline roughed, is ready for cutting.

METHOD OF CUTTING

"The fixture shown at Figs. 209 to 211 is bolted onto a circular milling attachment. The fixture consists of a base plate carrying two slides moving at right angles to each other. These slides are equipped with verniers registering zero when the center of the arbor is in line with the center of rotation of the milling attachment. The drive pin is riveted into a separate slide that is adjustable and is in line with the arbor and line of motion of the large slide. This fixture is placed in position on the machine and the rough-gear blank clamped in place.

The point D^1, Fig. 208, is then set over the center of rotation of the attachment by means of the verniers and the center of the cutter brought into the line A A^1.

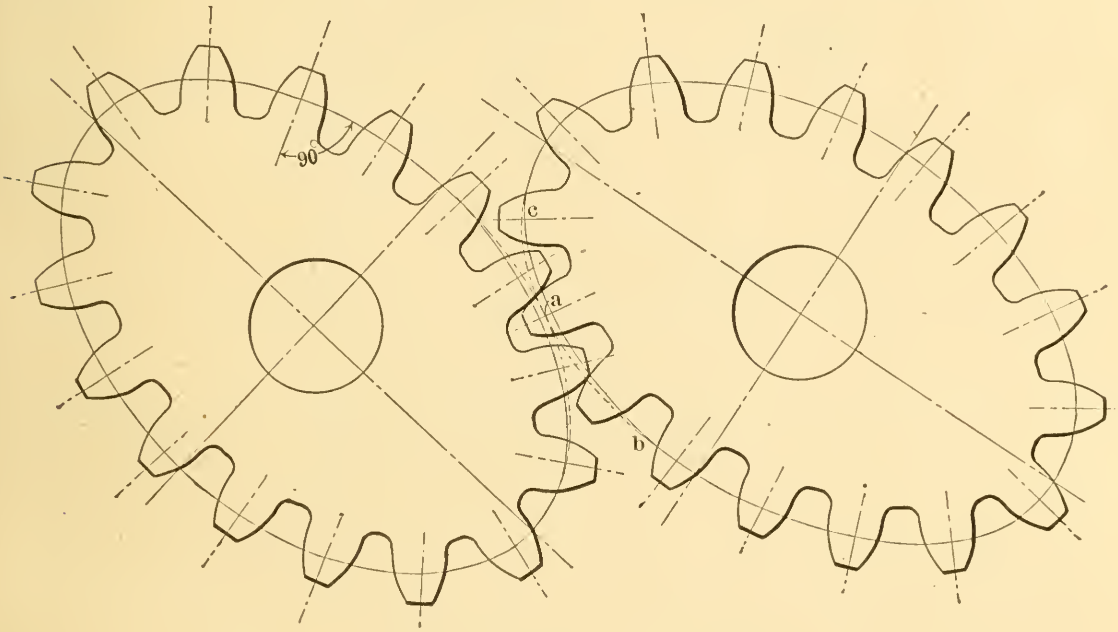


FIG. 210. ELLIPTICAL GEARS WITH BORE IN CENTER, SHOWING SEPARATION OF TRUE ELLIPTICAL PITCH LINES.

In this case the cutter is an end mill used to finish the outside of the gear. By moving around each quadrant setting the corresponding centers over the center of rotation for each quadrant the outside is milled true with the pitch line. A cutter for the ends is then put in the machine and in line with A A^1. The first space may then be cut to depth with two or more cuts, depending on the tests for alignment. A piece of glass having a center line and a number of short lines scratched equal distances apart and from the center line is used

for testing the cuts. The center line is placed over the line A A^1 on the first cut, and the short lines brought intersecting the pitch line. By using a glass a variation of 0.001 inch may be readily seen and remedied. After the first

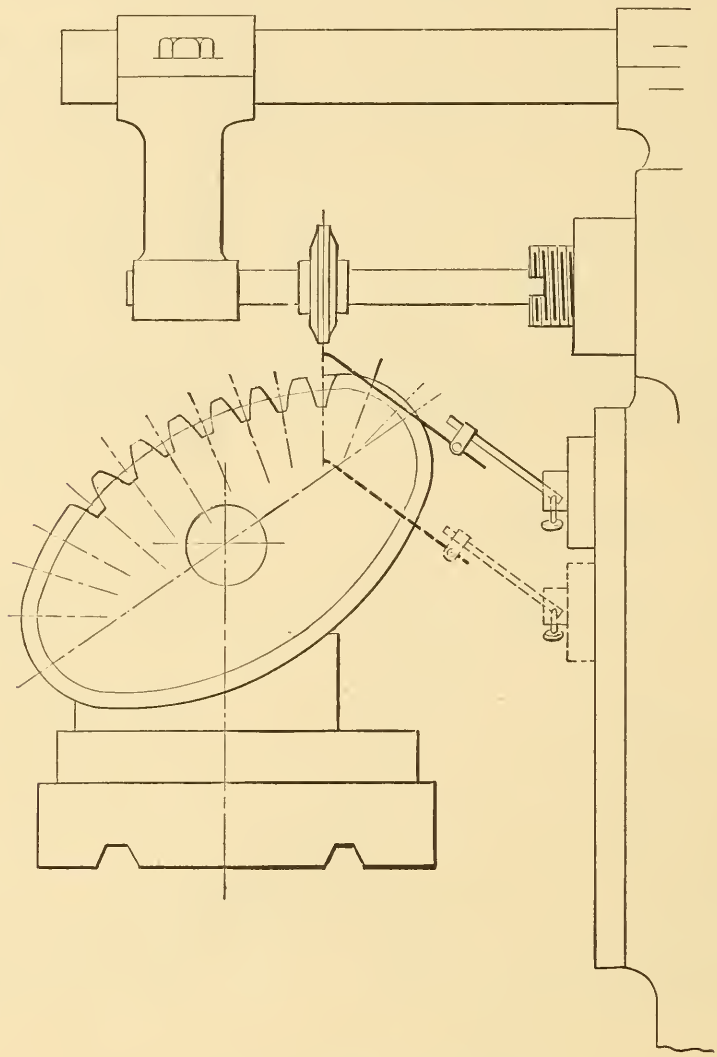


FIG. 211. CUTTING THE TEETH.

space is cut the gear-tooth calipers may be set and used to check the cutting. By checking with both the radial center lines and the tooth calipers errors are reduced to a minimum. After the first gear is cut it may be used as a templet for all others, or the indexing may be noted and repeated."

Several years ago the author had an experience cutting elliptical gear that will be of interest. The driven gear was required to have two variations of speed per revolution; therefore the bore was put in the center of the gears instead of at the foci, which is usual. The gears cut are shown in Fig. 214. When

the ellipse is very flat the four-arc method, described above, cannot be employed, as the pitch line cannot be described even approximately correct by four arcs. Therefore the gears were cut as illustrated by Fig. 211, the blank being set for each tooth cut. The teeth were first located on a templet which was secured to the face of one of the gears; they were both cut at one time. The blank was manipulated until the center line of each tooth space was brought in line with the center line of the cutter.

This is accomplished with a surface gauge placed against the front ways of the milling machine as illustrated in Fig. 211. The depth of the teeth was obtained by first bringing the cutter to the outside of the templet and raising

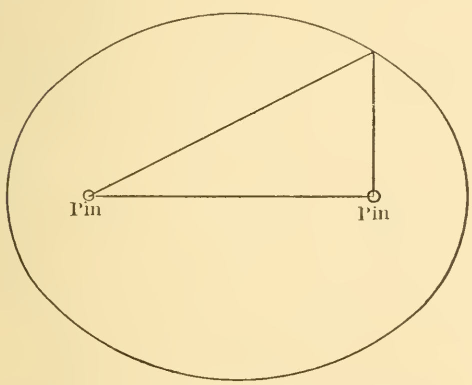


FIG. 212. THE GARDENERS' ELLIPSE.

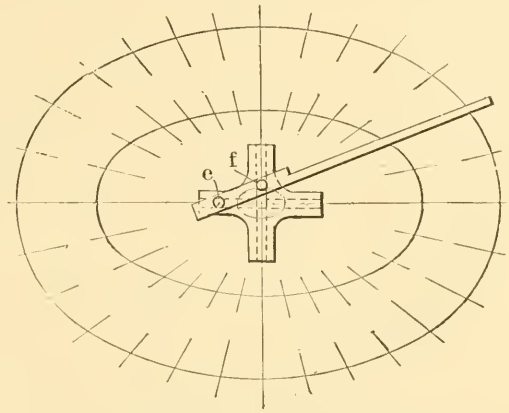


FIG. 213. THE METHOD EMPLOYED IN LAYING OUT GEARS SHOWN IN FIGS. 214 AND 215.

the table a distance equaling the depth of the tooth, or by locating the pitch line of the templet with another surface gauge from the cutter spindle.

A cutter made to finish the outside diameter as the teeth are cut is a decided improvement over first milling or slotting the outer surface, although milling off the points of the teeth after they are cut is the next best plan. When the gears were mounted as shown in Fig. 210, it was found that the pitch lines separated at four points of the ellipse, as between *b* and *c*, therefore there was excessive backlash between the teeth at these points.

The gears had been laid out with a gardeners' ellipse, using a piece of silk thread looped around pins set in the foci; the loop being adjusted until the describing point passed through the intersections of both the major and minor axes as shown in Fig. 212. Of course, we all thought the gardeners' ellipse was at fault, and it was decided to employ this process only on "tulip patches" in the future.

A proper instrument was secured for the next attempt and everything done according to Hoyle. This time, instead of first cutting the gears and invest-

igating afterwards, two templets were laid out to represent the pitch lines of the gears. These templets developed the same error found in the first gears, so the pitch lines were corrected as per dotted lines in Fig. 210 and the gears were satisfactory.

Later on several gears of the same size were required and a search was made for a simpler method of laying them out. Noticing the attachment described by Mr. George B. Grant in his "Treatise on Gears," section 150, an attempt

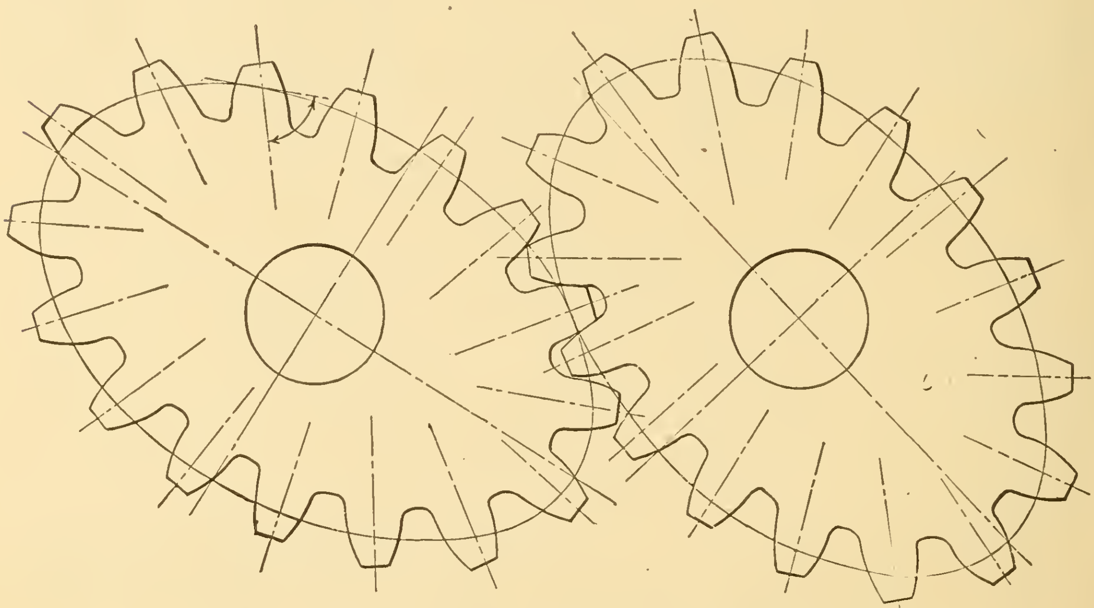


FIG. 214. ELLIPTICAL GEARS LAID OUT AS SHOWN IN FIG. 213.

was made to apply this principle. A circle whose radius equalled the radii of both the major and minor axes was first drawn and divided into the number of teeth to be cut in the gear. The ellipse was then described from the same center. The center lines of the teeth were then projected from the points located on the outer circle to the ellipse, the edge of the blade being on a line with the points *e* and *f* as shown in Fig. 213. The center line of the tooth thus projected did not always cross the pitch line at right angles, except at the major and minor axes, and doubts were expressed as to the success of gears cut on these lines, but it was thought worth a trial at least. The pitch lines were corrected in the same manner as before and the gears were cut.

They not only ran better than the first gears, but it was found necessary to remove part of the correction between the points *b* and *c*. This is accounted for by the fact that the circular thickness of the teeth increased between these points owing to the obliquity of the teeth; the thickness of the tooth being measured on the normal section (see Fig. 214).

Gears with the bore located at the foci will not operate when cut in this manner; the center lines of the teeth must be at right angles to the pitch line at all points. The interference of the teeth when thus mounted is shown in Fig. 215.

When the bore is in the center of an elliptical gear it is better to have an

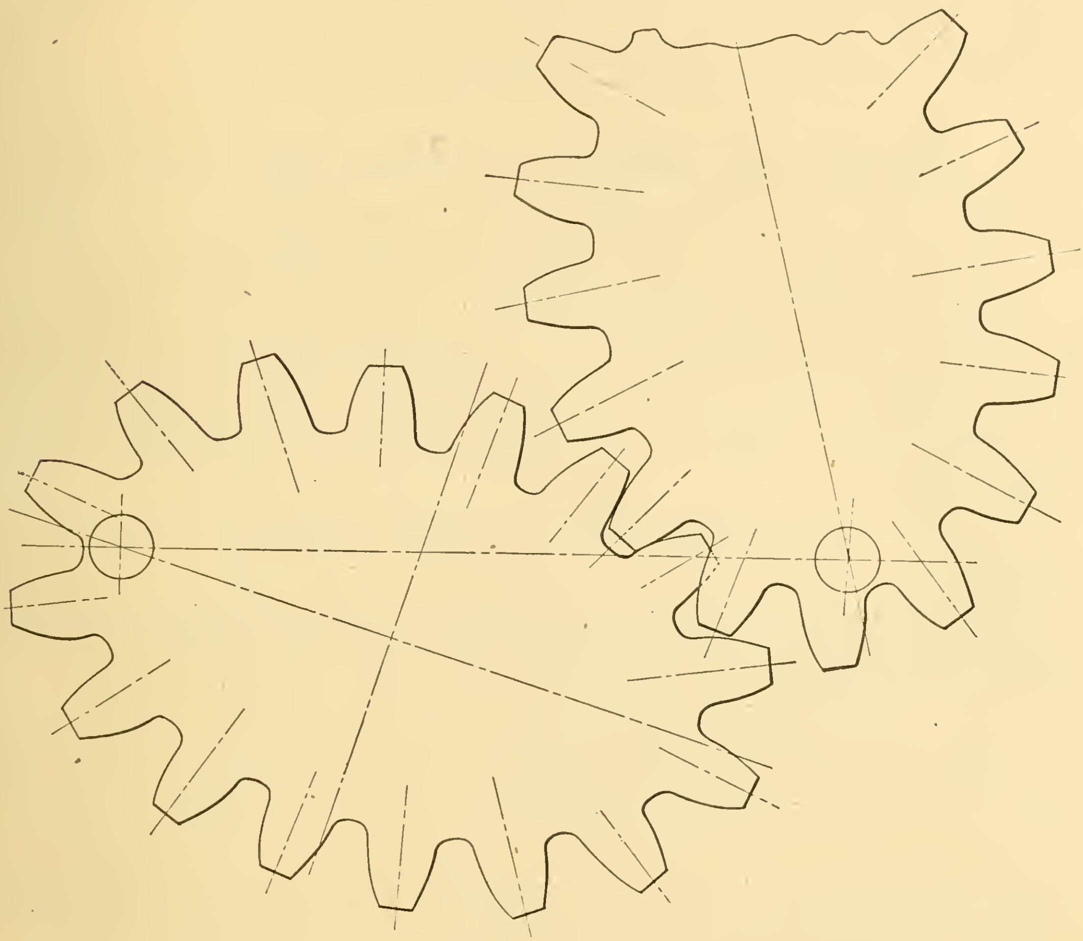


FIG. 215. INTERFERENCE OF GEARS SHOWN IN FIG. 214, WHEN BORE IS AT FOCI.

even number of teeth, otherwise the gears must be cut separately. For an odd number of teeth the tooth centered on the major axis must engage a space located at the minor axis of the engaging gear. It is better to use an even number and place the edge of the teeth on the major and minor axes, for gears of this type.

For a complete treatment of the subject of elliptical gears Grant's "Treatise on Gear Wheels" is to be recommended.

SECTION XII

Epicyclic Gear Trains

CALCULATIONS RESPECTING EPICYCLIC GEAR TRAINS

DERIVATION OF FORMULAS FOR SEVERAL USUAL TYPES, AND EXTENSION OF THE METHOD OF ANALYSIS TO A SOMEWHAT COMPLEX EPICYCLIC TRAIN *

This form of gearing, which is really that in which one gear revolves around the center of the one with which it is in contact, has received considerable attention, and one notices its use in several directions. We will therefore look into some of the calculations respecting it, leading from the simpler to the more complex.

SIMPLE PAIR OF GEARS IN FIXED BEARINGS

Example I.

If in Fig. 216 R and N are two gears in mesh, r and n being their respective numbers of teeth, their bearings being fixed, then:

$$\frac{\textit{Velocity of drive N gear N}}{\textit{Velocity of drive R gear R}} = \frac{r}{n};$$

$$\text{or, } \textit{N's velocity} = \textit{R's velocity} \times \frac{r}{n}.$$

If, however, R revolves in a positive direction, n must revolve in the opposite, that is, in a negative direction.

$$\therefore \textit{N's velocity} = \textit{R's velocity} \times \frac{r}{n}. \qquad (1)$$

In all these calculations it is essential that great care be taken in order to obtain the correct sign of the resulting velocity.

GEARS IN FIXED BEARINGS, WITH AN IDLER

Example II.

An intermediate gear I is placed in contact with both N and R, Fig. 217. The effect will be that of giving N motion in the same direction as R.

$$\therefore \textit{N's velocity} = \textit{R's velocity} \times \frac{r}{n}. \qquad (2)$$

* Francis J. Bostock.

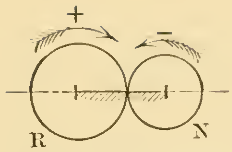


Fig. 216 Simple Pair of Gears in Fixed Bearings.
Eq. 1. N's V. = R's V. x $\frac{r}{n}$

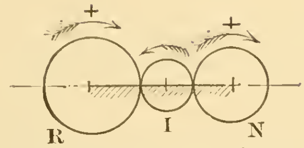


Fig. 217 Gears in Fixed Bearings. with an Idler
Eq. 2. N's V. = R's V. x $\frac{r}{n}$

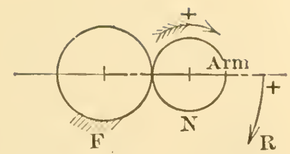


Fig. 218 Simple Epicyclic Train
Eq. 3. N's V. = R's V. x $(1+\frac{f}{n})$

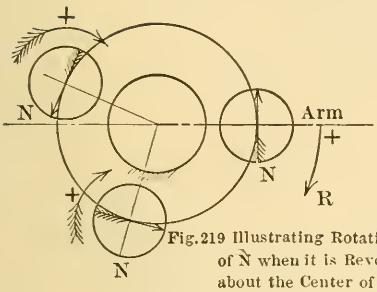


Fig. 219 Illustrating Rotation of N when it is Revolved about the Center of F.

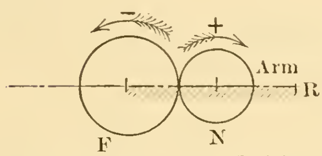


Fig. 220 Second Stage in Deriving Equation 3: Arm assumed to be fixed, F turned backward.

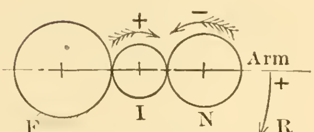


Fig. 221 Epicyclic Train with an Idler
Eq. 4. N's V. = R's V. x $(1-\frac{f}{n})$

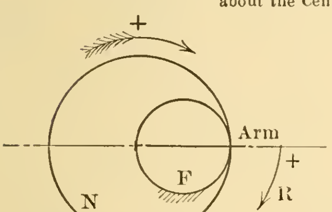


Fig. 222 Simple Epicyclic Train with Internal Gear
Eq. 5. N's V. = R's V. x $(1-\frac{f}{n})$

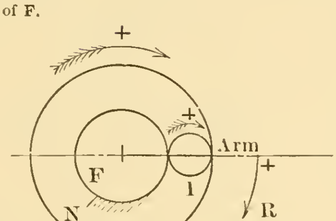


Fig. 223 Internal Gear Train with intermediate Gear: the Arm Driving
Eq. 6. N's V. = R's V. x $(1+\frac{f}{n})$

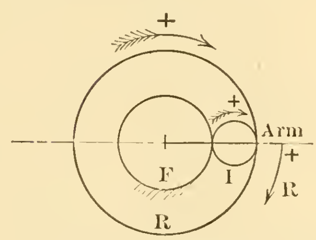


Fig. 224 Same Train as Fig. 223 but with the Internal Gear Driving
Eq. 7. N's V. = R's V. x $(\frac{r}{r+f})$

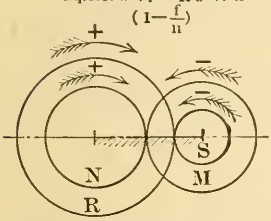


Fig. 225 Compounded Gears in Fixed Bearings
Eq. 8. N's V. = R's V. x— $\frac{rm}{sn}$

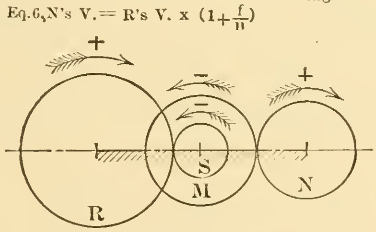


Fig. 226 Compounded Gears in Fixed Bearings
See Equation 8, Fig. 225

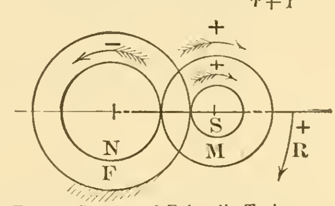


Fig. 227 Compound Epicyclic Train
Eq. 9. N's V. = R's V. x $(1-\frac{fm}{sn})$

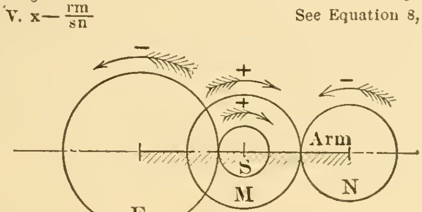


Fig. 228 Second Stage in Deriving Equation 9 Arm assumed to be fixed, F turned backward

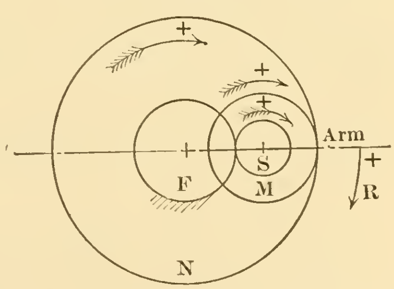


Fig. 229 Compound Epicyclic Train with One Internal Gear
Eq. 10. N's V. = R's V. x $(1+\frac{fm}{sn})$

FIGS. 216 TO 229.

EPICYCLIC GEAR TRAINS WITH CORRESPONDING VELOCITY RATIO FORMULAS.

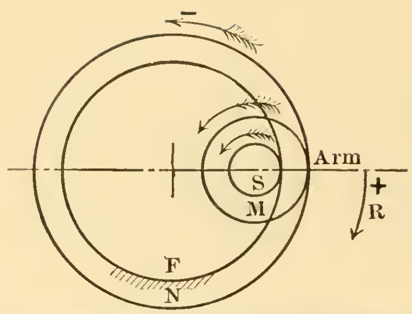


FIG. 230. Compound Epicyclic Train with Two Internal Gears See Eq. 9, same as Fig. 227

NOTATION

R = Denotes Driving Gear, or, in some cases, Arm.
r = Number of Teeth in Driving Gear.
N = Denotes Driven Gear or Arm.
n = Number of Teeth in Driven Gear.
I, S and M denote Intermediate Gears.
F = Denotes Fixed Gear.
f = Number of teeth in it.
V = Angular Velocity.
//////// Denotes part which is fixed.

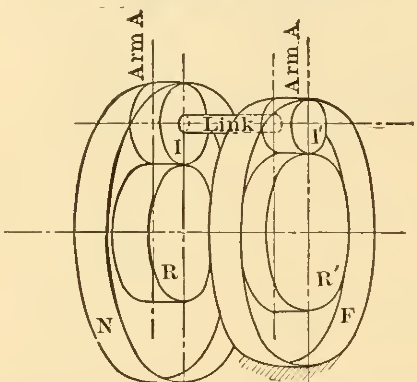


FIG. 231 An Epicylic Train Consisting of Two Central Gears, One Arm carrying Two Planetary Gears, and Two Internal Gears, One of which is Fixed.

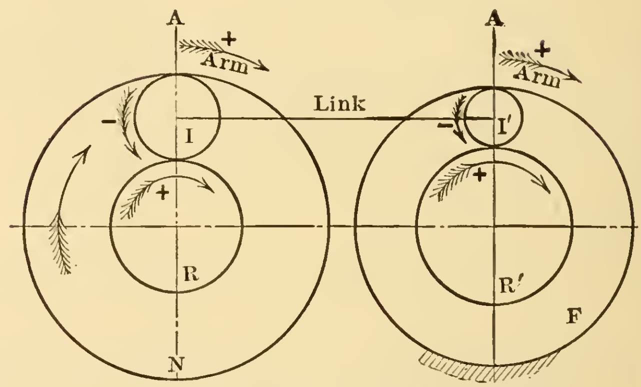


FIG. 232 Diagram of the Train of Fig 231
Eq. 11. N's V. = R's V. x $\left(\frac{r'n+rf}{n(r+f)}\right)$

FIGS. 230 TO 232.

EPICYCLIC GEAR TRAINS WITH CORRESPONDING VELOCITY RATIO FORMULAS.

SIMPLE EPICYCLIC TRAIN

Example III.

Two gears, *F* and *N*, are in mesh, the centers of which are on the arm *R*, which is capable of revolving around the center of *F*. It is required to find the velocity ratio between *R* and *N* when *R* revolves around the fixed gear *F*; Fig. 218 shows the arrangement. The gear *N* is subject to two motions due to the following two conditions:

a. The fact of its being fixed to the arm *R*.
b. The fact that it is in contact with the gear *F*.

We will therefore in the first place suppose that they are not in gear, and that *N* cannot rotate on the arm *R*. Then if *R* makes one revolution around *F* it is obvious that *N* must also make one revolution around *F*, as in Fig. 219.

: N's velocity, due to condition a, = R's velocity,

the direction being the same as *R's*.

Secondly, if instead of R making one revolution around F in a + direction, we cause F to make one in the opposite, that is, negative direction, we shall have exactly the same effect. Therefore place F and N in mesh, and fix the arm R, as in Fig. 220.

Then if F makes -1 revolution, N will make $+\frac{f}{n}$ revolutions. (According to Equation 1.)

But -1 of $F = +1$ of R.

$$\therefore 1 \text{ revolution of } R = \frac{f}{n} \text{ revolutions of } N,$$

$$\text{or, } N\text{'s velocity due to conditions } b = \frac{f}{n} R\text{'s velocity}.$$

By addition we obtain the total impulses given in N, that is:

$$N\text{'s velocity} = R\text{'s velocity} + \frac{f}{n} R\text{'s velocity}$$

$$= R\text{'s velocity}\left(1 + \frac{f}{n}\right). \tag{3}$$

EPICYCLIC TRAIN WITH AN IDLER

Example IV.

If an intermediate gear I be inserted between F and N, as in Fig. 221, we have a similar case to the above; but the intermediate gear has the effect of changing the direction of revolution of N (Equation 2), due to its contact with F through I.

$$\therefore N\text{'s velocity} = R\text{'s velocity} \times \left(1 - \frac{f}{n}\right) \tag{4}$$

It will be seen that if $f=n$, N will not have any motion of rotation at all; and it will have a positive one if $f < n$ and negative if $f > n$. Thus by the adjustment of f and n one can obtain great reduction in speed by means of few moving parts.

SIMPLE EPICYCLIC TRAIN WITH INTERNAL GEARS

Example V.

Instead of the driven gear N being external, it might have been internal, as shown in Fig. 222. The effect will be the same as inserting an intermediate gear in Example III, giving the same result as Case IV, namely:

$$N\text{'s velocity} = R\text{'s velocity} \times \left(1 - \frac{f}{n}\right). \tag{5}$$

In this case $n > f$.

$\therefore$ The final direction is always +.

INTERNAL GEAR EPICYCLIC WITH INTERMEDIATE GEAR

Example VI.

Fig. 223 shows a still further modification of this condition, I being an intermediate gear. The result is:

$$N\text{'s velocity} = R\text{'s velocity} \times \left(1 + \frac{f}{n}\right). \qquad (6)$$

THE SAME TRAIN WITH THE INTERNAL GEAR DRIVING

Example VII.

With the above type, one often arranges the outer internal gear to be the driver, imparting motion to the arm carrying the intermediate gear (see Fig. 224).

We have seen by equation 6 that:

$$\frac{N\text{'s velocity (driven)}}{R\text{'s velocity (driver)}} = \frac{1}{1 + \frac{f}{n}}$$

$$\therefore N\text{'s velocity} = R\text{'s velocity} \div \left(1 - \frac{f}{r}\right)$$

$$= R\text{'s velocity} \times \left(\frac{r}{r + f}\right). \qquad (7)$$

The last two examples constitute what is known as the "Sun and Planet" gear, which is largely used in many mechanisms. All the above examples show "simple" gearing, but they can be compounded with great advantage.

COMPOUNDED GEARS IN FIXED BEARINGS

Example VIII.

Gears compounded together are shown in Figs. 225 and 226, 226 being a diagram of 225. One repeats the well-known rule that:

$$\frac{\text{Velocity of driven gear}}{\text{Velocity of driver gear}} = \frac{\text{Product of number of teeth of driver gears}}{\text{Product of number of teeth of driven gears}}$$

$$\text{or, } N\text{'s velocity} = R\text{'s velocity} \times \frac{r \times m}{s \times n}. \qquad (8)$$

The direction is the same as N's namely, $+$.

COMPOUND EPICYCLIC TRAIN, WITHOUT INTERNAL GEAR

Example IX.

We will now arrange to fix one of the gears F, and by means of the arm R revolve the others around it, thereby causing N to revolve as shown in Figs. 227 and 228. As before, we will assume the gears M and S to be out of mesh, so that when the arm R, carrying with it the gear N, makes one revolution

around F, N must also make one revolution relatively to F. Also when they are in mesh, the arm R being fixed and F makes one revolution in a negative direction (see Fig. 228), N will make $-\frac{f\,m}{s\,n}$ revolutions. (Equation 8.)

Now the total motion imparted to N must be the sum of these two, namely:

$$1 \textit{ revolution of } R = 1 - \frac{f\,m}{s\,n} \textit{ revolutions of } N,$$

or,

$$N\text{'s velocity} = R\text{'s velocity} \times \left(1 - \frac{f \times m}{s \times n}\right). \qquad (9)$$

COMPOUND EPICYCLIC TRAIN WITH ONE INTERNAL GEAR

Example X.

Fig. 229 shows a slight modification of the last case, N being an internal instead of an external gear. Obviously the only difference will be in the direction of N's motion, that is:

$$N\text{'s velocity} = R\text{'s velocity} \times \left(1 + \frac{f\,m}{s\,n}\right). \qquad (10)$$

COMPOUND EPICYCLIC TRAIN WITH TWO INTERNAL GEARS

Example XI.

A further modification, however, is one in which both F and N are internal gears (Fig. 230), the effect of such being a change of sign in the equation.

$$\therefore N\text{'s velocity} = R\text{'s velocity} \times \left(1 - \frac{f\,m}{s\,n}\right). \qquad (9)$$

The type shown in Figs. 227 and 230 is, perhaps, one of the best methods of obtaining a good reduction of speed in an easy and cheap manner.

There are several combinations of the examples shown, but as they are all somewhat similar we will take another typical case as a guide for future calculations.

AN EPICYCLIC TRAIN CONSISTING OF TWO CENTRAL GEARS, ONE ARM CARRYING TWO PLANETARY GEARS, AND TWO INTERNAL GEARS, ONE OF WHICH IS FIXED

Example XII.

The writer has successfully used the arrangement shown in Figs. 231 and 232, in which R and R' are two spur gears mounted on one shaft; I and I' are two "planet" pinions, while F and N are two internal gears, the former being fixed. R and R' are made to revolve, which has the effect of giving N a very slow speed.

A SCHEME FOR FINDING THE VELOCITY RATIO

As this is somewhat complicated, we will work it out in stages:

1. Obtain the revolutions of the arm A when R' makes one revolution, F, of course, being fixed.

2. Obtain N's revolutions when the arm A is fixed and R makes one revolution.

3. Assume R fixed, and that the arm makes one revolution; obtain, then, N's revolutions.

4. Then if N makes so many revolutions to one of the arm, as given by stage 3, we can by proportion obtain how many will be caused by the amount given by Stage 1.

5. Add the results of 2 and 4 together, and obtain the motion given to N by one revolution of R, which is the desired result.

THE SCHEME WORKED OUT

Working the above out we obtain:

1. When F is fixed and R' makes one revolution, the arm A must make $+\frac{r'}{r'+f}$. (According to Equation 7.)

2. R makes one revolution, arm A being fixed; then N must make $-\frac{r}{n}$ revolutions. (According to Equation 2. Negative sign used because of the internal gear.)

3. When R is fixed and arm A makes one revolution, N will make $+\left(1+\frac{r}{n}\right)$ revolutions. (According to Equation 6.)

4. With one revolution of arm, N makes $1+\frac{r}{n}$ revolutions, from Stage 3; $\therefore$ *with* $\frac{r'}{r'+f}$ revolutions of the arm, as derived in Stage 1, N will make

$$\left(1+\frac{r}{n}\right)\times\left(\frac{r'}{r'+f}\right).$$

5. The aggregate is the sum of the effects derived in Stages 4 and 2, namely, to one of R, N makes:

$$\left(1+\frac{r}{n}\right)\times\left(\frac{r'}{r'+f}\right)+\left(-\frac{r}{n}\right)=\frac{(n+r)\,r'}{n\,(r'+f)}-\frac{r}{n}$$

$$=\frac{rr'+r'n-rr'-rf}{n\,(r'+f)}=\frac{r'n-rf}{n\,(r'+f)}.$$

The final direction of revolution of N will depend upon the relation which

$r'\,n$ bears to $r\,f$; if the former be greater, then the direction will be positive (+), and *vice versa*. The formula for this combination is then:

$$N\text{'s velocity} = R\text{'s velocity} \times \left(\frac{r'\,n - r\,f}{n\,(r' + f)}\right). \qquad (11)$$

SOME NUMERICAL EXAMPLES IN EPICYCLIC GEARING

In order to illustrate the above examples we will take one or two cases.

If in Example and Fig. 218, $f = 30$, $n = 25$, then to one revolution of R, N will make $\left(1 + \frac{f}{n}\right) = 1 + \frac{30}{25} = 2\frac{1}{5}$ revolutions.

It will be obvious that with $f = n$, N would revolve at twice the speed of R.

In the type shown in Fig. 7, $f = 60$, $n = 65$;

then

$$\frac{\text{Velocity of } N}{\text{Velocity of } R} = \frac{1 - \frac{f}{n}}{1} = \frac{1 - \frac{60}{65}}{1} = \frac{5}{65} = \frac{1}{13}.$$

The arrangement of Fig. 227 is much used. Let $n = 60$, $f = 61$, $s = 40$, $m = 41$.

Then the velocity ratio between N and R is $1 - \frac{f\,m}{s\,n} : 1$

$$= 1 - \frac{61 \times 41}{40 \times 60} = 1 - \frac{2501}{2400} = \text{say } 1:24, \text{ in a minus direction.}$$

Illustrating Example XII, Fig. 231, let $r = 90$, $r' = 91$, $f = 120$, $n = 121$.

$$\frac{\text{Velocity of } N}{\text{Velocity of } R} = \frac{r'\,n - r\,f}{n\,(r' + f)} = \frac{91 \times 121 - 90 \times 120}{121\,(91 + 120)}$$

$$\frac{11{,}011 - 10{,}800}{121 \times 211} = \frac{211}{121 \times 211} = \frac{1}{121}.$$

DIRECTION OF ROTATION OF GEARS

The following, in reference to epicyclic gear calculations, is by Oscar J. Beale, American Machinist, July 9, 1908:

"A very valuable article relative to epicyclic gears is by Prof. A. T. Woods in the American Machinist for February 14, 1889. This article is well-nigh perfect. It is so clear and comprehensive that it was of great help to me. I have read a number of later articles, and I have always gone back to this in order to clear up the subject. I think that many of your readers would like to see it reprinted.

"About the best way to determine the direction of rotation of epicyclic gears is by careful inspection of the position of the members; then, if you make a

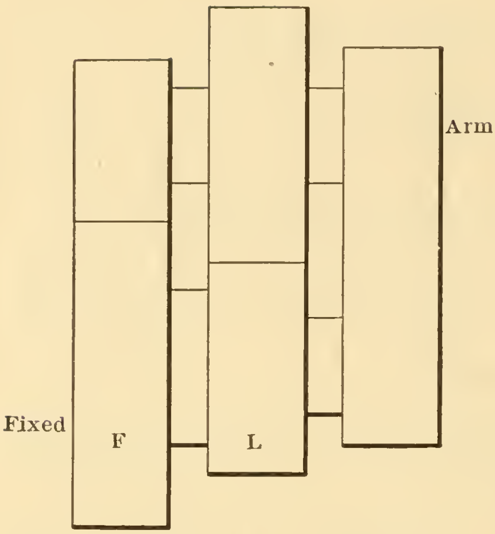


FIG. 233. DIAGRAM OF AN EPICYCLIC TRAIN

correct statement of the effect of each member, Professor Woods' methods will bring the answer right. One must be sure to give the result the proper sign; that is, one must be able to add and to subtract algebraically.

"I have sometimes used a sort of mental key in cases like this sketch, Fig. 233. If the pitch circle of *L* is smaller than the pitch circle of *F*, the rotation of *L* will be opposite that of the arm. If *L* is greater than *F*, the rotation of *L* will be the same as that of the arm.

"This 'mental key' may help some; but after all, it is usually better to reason mathematically as in Professor Woods' article."

We reprint herewith Professor Woods' article to which Mr. Beale refers because of its value in determining the direction of rotation of epicyclic trains.

EPICYCLIC TRAINS *

" An epicyclic train consists of a number of gear wheels, or pulleys, and belts, some of which are carried upon a revolving arm. For example, in Fig. 236 the wheel F is fastened to the shaft *B*, about which arm *A* turns. This arm carries the axes of *C* and *L*, *C* being an idle wheel gearing with *F* and *L*. The motion of the wheel *L* is thus composed of three motions: (1) that which it has by reason of its revolution about *B* as a center, (2) that due to the revolution of the arm *A* about *F*, and (3) that due to its connection with *F* by means of the wheel *C*. We will consider the effect of these motions separately, and will begin with the simplest possible arrangement. In Fig. 234 let *A* be an arm which revolves about a center *B*, and carries a wheel *L*, which we will suppose to be fastened to it. If the arm be turned through one revolution, the wheel *L* will in effect revolve once about its own center. This will be clear by an examination of the successive positions shown in dotted lines, the revolution of the arm being in the direction of the arrow. For example, follow the motion of a point such as *P*; at 1 it is to the right of the center, at 2 below it, at 3 to the left, and at 4 above the center, finally returning to its first position on the right. We thus see that *L* has practically made one revolution about its own center, just as it would have if it had been fixed at *B* concentric

* Prof. A. T. Woods.

with the arm. If L has not revolved by reason of the revolution of A, the point P would have remained horizontally to the right of the center during the revolution of A. This is, of course, the same motion as that of a crank-pin and crank, and will be still more clear if we remember that, if the pin did not in effect revolve about its own center, it would not turn in the brasses, and they could be dispensed with.

Now, considering the second motion of L, that due to the revolution of the arm about F, refer to Fig. 235, and let the wheel F be fixed, or a dead wheel, con-

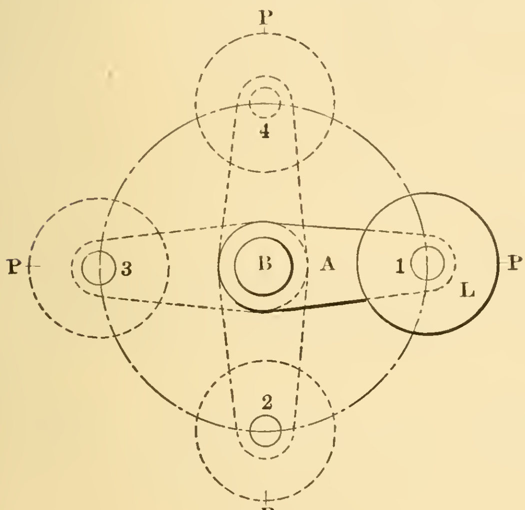


Fig. 234

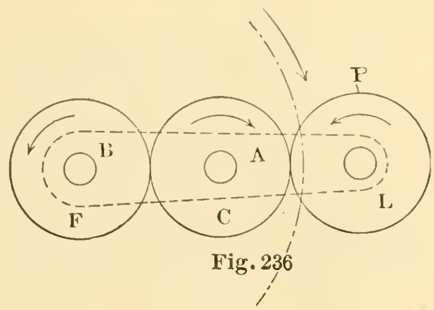


Fig. 236

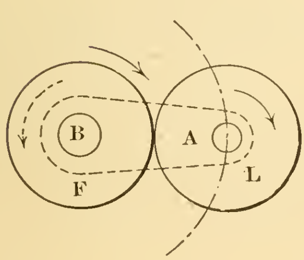


Fig. 235

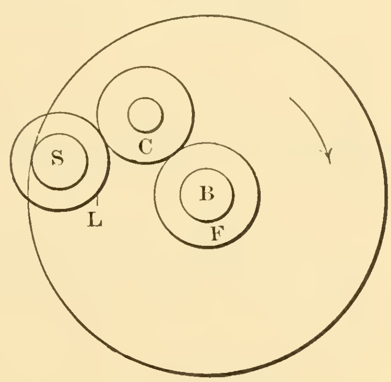


Fig. 237

DIAGRAMS OF EPICYCLIC GEARS

centric with the arm A. Let F and L have the same number of teeth. Then while the arm revolves once in the direction indicated, L will revolve in the same direction, the result being the same as if the arm had remained fixed, and F had been revolved once in the opposite direction. The final motion of L, while the arm revolves once, is therefore one revolution, as in Fig. 234, supposing F to be removed, and one revolution, as in Fig. 235, supposing F to be in place and fixed; or, while the arm revolves once, L revolves twice in the same direction.

Now, instead of F being fixed, let it revolve once in the direction of the dot-

ted arrow. The effect of this will be to give one additional revolution to L, resulting in three revolutions of L to one of the arm. In order, then, to get at the resultant motion of the last wheel in epicyclic trains, we must consider the three independent motions separately: *first*, suppose the first wheel F, which is concentric with the arm, to be removed; *second*, suppose the first wheel to be in place and fixed; and *third*, suppose the arm to be fixed and the first wheel to revolve as intended. The final motion of L is the sum of these three motions.

For the sake of brevity we will designate revolution in the direction of the hands of a watch ahead, or $+$; and that in the opposite direction backward, or $-$. Thus expressed, the revolutions of L in Fig. 235, as we have just discussed it, will be:

(a) $+ 1$ due to the revolution of the arm,
$+ 1$ due to the revolution about F'',
$+ 1$ due to the revolution of F.
$+ 3$ revolution of L to one of the arm.

As a further illustration assume that in Fig. 235, F has 40 teeth and L 30, then if F is a fixed wheel, L will revolve:

(b) $+ 1$ due to the revolution of the arm,
$+ \frac{40}{30}$ due to the revolution about F,
0 due to the revolution of F,

$+ \frac{7}{3}$ revolutions of L to one of the arm.

If F makes one revolution backward while the arm makes one ahead, we will have for L:

(c) $+ 1$ due to the revolution of the arm,
$+ \frac{40}{30}$ due to the revolution about F,
$+ \frac{40}{30}$ due to the revolution of F,

$+ \frac{11}{3}$ revolutions of L to one of the arm.

If F makes one revolution ahead, or in the same direction as the arm, the result is to balance the effect of the revolution about F, and we have for L:

(d) $+ 1$ due to the revolution of the arm,
$+ \frac{40}{30}$ due to the revolution about F,
$- \frac{40}{30}$ due to the revolution of F,

$+ 1$ revolution of L to one of the arm, or the same as in Fig. 234. Similarly, if we take the conditions the same as (c) and let F have 30 teeth and L 40, we will have $+ 2\frac{1}{2}$ revolutions of L to one of the arm.

We will now consider the effect of introducing an idle wheel, as shown in Fig. 236. In the first place, let L equal F, and let F be a fixed wheel. The revolutions of L will be:

(e) $+$ 1 due to the revolution of the arm,
$-$ 1 due to the revolution about F,
0 due to the revolution of F,

---

0 revolution of L, or, in other words: a point P, which is, say, vertically over the center of L, will remain so throughout the revolution of the arm. If we assume the same condition as (a), the resulting revolution of L will be:

(f) $+$ 1 due to the revolution of the arm,
$-$ 1 due to the revolution about F,
$-$ 1 due to the revolution of F,

$-$ 1 revolution of L to one of the arm.

If we let F have 40 teeth and L 30, and let F be a dead wheel, we will have for L:

(g) $+$ 1 due to the revolution of the arm,
$-\frac{40}{30}$ due to the revolution about F,
0 due to the revolution of F,

$-\frac{1}{3}$ revolution of L to one of the arm.

Or, reversing the position of the wheels, making $F = 30$ and $L = 40$, the revolutions of L will be:

(h) $+$ 1 due to the revolution of the arm,
$-\frac{40}{30}$ due to the revolution about F,
0 due to the revolution of F,

$+\frac{1}{4}$ revolution of L to one of the arm.

The arrangement shown at (e) is used in one form of rope-making machinery. The "arm" A, Fig. 236, is then the revolving frame which carries the bobbins on which the strands or wire have been wound. B is the center of this frame, and on it the wheel F is fixed. A small yoke or frame, which carries a bobbin, is fixed on the axis of L, there being as many of these wheels and bobbins as there are to be strands in the rope. Then if F and L have the same number of teeth as at (e), the axes of the bobbins always point in one direction, and the rope is laid up without twisting the separate strands. If L has a few less teeth than F, the strands will be given a slight twist, making the rope harder.

Arrangements such as Figs. 235 and 236 are applicable to boring bars having sliding head. In such cases *B* would be the dead center on which the bar turns and on which the wheel *F* is fastened, being, therefore, a dead wheel. The wheel *L* is fastened to the end of the feed-screw in the bar, as shown at *S*, in Fig. 237, which represents the end view of the bar. While the arrangement is an epicyclic train, such as we have discussed, the explanation of it is extremely simple, because the motion to be determined is that of the screw *S with regard to the bar*, not with regard to the lathe, or any stationary object.

As *F* is fixed, the effect on *L* of one revolution of the bar is the same as if the bar remained stationary and *F* revolved once. Thus, if *F* has 20 teeth and *L* 40, the screw *S* will make, in the bar, $\frac{20}{40} = \frac{1}{2}$ of a revolution, while the bar revolves once. And if the screw has four threads to the inch, the feed will be $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ inch. The effect of the idle wheel (shown in Fig. 237) is simply to change the direction of the feed.

Another form of epicyclic train is that shown in Fig. 238 in which the last wheel is concentric with the arm and first wheel. This does not change the resultant motion of *L* in any way, but only makes a more convenient form for transmitting motion from *L* to other parts of the machine. If *F* and *L* have 40 and 30 teeth, while the wheels *C* and *D* are equal, we will have the same motion as at (g) and (h), supposing *F* to be a dead wheel. A recent and novel application of this form of train is to be found in the Waterbury watch, the principle of which is shown in Fig. 239. In this figure *a b* is the face of the watch and *c d* the frame which carries the principal train of wheels, that from *C* to the balance wheel *g*. This frame turns about the center shown below it and the bearing in the face. It is driven by the spring *e* and carries the minute hand *M*, and hence revolves once each hour. Between the frame and the face is a pinion *C*, having 8 teeth, which is connected to the balance wheel *g* by the train of wheels shown, and itself gears with two wheels *F* and *L*, having, respectively, 44 and 48 teeth. *F* is fastened to the face, and so is a dead wheel, and *L* is fastened to the hour hand *H* by the tube, as shown. It remains to show that the hour hand will revolve once in 12 hours, as required by means of this connection. We have here an epicyclic train *F C L* in which the first wheel is fixed. The revolu-

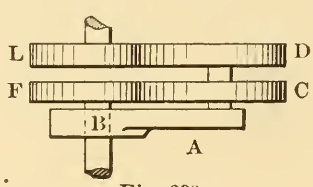


Fig. 238

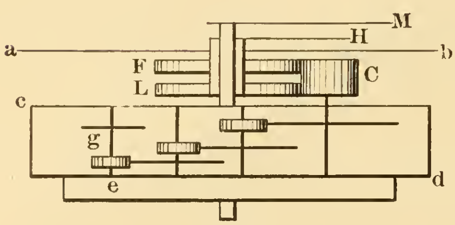


Fig. 239

tions of L during one revolution of the arm, as we have called it, or the frame $c\ d$ are, therefore:

$+ 1$ due to the revolution of the arm,
$- \frac{44}{48}$ due to the revolution about F,
0 due to the revolution of F

$+ \frac{4}{48} = + \frac{1}{12}$ revolution during one revolution of the arm or minute hand, which is, of course, as it should be. The remainder of the train of wheels in this watch do not differ in principle from that ordinarily employed, a peculiarity being, however, that the entire "works," held in the frame $c\ d$, revolve within the case every hour.

Another peculiar adaptation of epicyclic trains is for the production of very slow velocities, using a small number of wheels. For example, in Fig 238, let the numbers of teeth on the several wheels be $F = 19$, $C = 20$, $D = 21$, and $L = 20$, and let F be a dead wheel. Working out this train as we have the others, it will be found that, while the arm A makes one revolution, the last wheel l will make but $\frac{1}{400}$ of a revolution. In the same way, if we take the number of teeth in order as above, as 27, 40, 37 and 25, the last wheel will make but one revolution, while the arms makes 1000.

We have chosen examples in which the first wheel is the dead wheel, as these are the simplest and most common. By adjusting the speed of the first wheel, however, it becomes possible to transmit velocities by means of epicyclic trains, which would be practically impossible by ordinary means. As an illustration, suppose it is required to have one shaft make 641 revolutions to one of another. As 641 is a prime number, this ratio could not be transmitted exactly by ordinary gearing on account of the large number of teeth required for a single wheel; but by means of an epicyclic train it can be readily accomplished. Of course, the necessity for such ratios as this rarely occurs in machinery.

A method of solving problems involving epicyclic trains, which will be more convenient for many than that which we have followed, is by means of a general formula. Let v = the value of the train of gears, or the product of the number of teeth on the drivers divided by the products of the numbers of teeth on the followers, which would be, in Fig. 238,

$$\frac{F \times D}{C \times L}.$$

In case of pulleys, v = the product of the diameters of the drivers divided by the product of the diameters of the followers. Let f, l and a represent the number of revolutions of the first wheel, last wheel, and arm, respectively, in the same time.

Then,

$$v = \frac{l - a}{f - a}$$

If one direction is represented as +, the other will be represented as —. If the last wheel is to revolve in the same direction as the first, supposing the arm to be fixed, v is +, and if the opposite direction, it is −. For example, take the data as at (b); then,

$$v = \tfrac{40}{30} = \frac{l - a}{0 - a}, \text{ whence } l = \tfrac{7}{3}\, a.$$

Again, let it be required that L shall make one revolution to 1000 of A (Fig. 238),

$$v = \frac{1 - 1000}{0 - 1000} = + \frac{999}{1000} = \frac{27 \times 37}{40 \times 25} = \frac{F \times D}{C \times L}.$$

TABLE OF PROPORTIONS OF DIFFERENTIAL BACK-GEARS

The following table, originally published in AMERICAN MACHINIST by Ernest J. Lees, gives data for ready reference. For a back-gear on drill presses and

SIZE NO.	1	2	3	4	5	6	7	8	9	10	11	12
Diameter of Pulley	8	8	10	10	12	12	15	15	15	15	18	18
Face of Pulley	3	3	$3\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$
Width of Belt	$2\frac{1}{2}$	$2\frac{1}{2}$	3	3	4	4	5	5	6	6	7	7
Approximate H. P. at 300 r. p. m.	$2\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{4}$	$4\frac{1}{4}$	$7\frac{1}{4}$	$7\frac{1}{4}$	11	11	13	13	18	18
Shaft Diameter D	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{3}{4}$	2	2	3	3
Pitch Diameter pinion A	$2\frac{1}{4}$	$4\frac{1}{2}$	$2\frac{4}{7}$	$5\frac{1}{7}$	$2\frac{4}{7}$	$5\frac{1}{7}$	$2\frac{4}{7}$	6	3	9	5	13
Number of teeth in A	18	36	18	36	18	36	18	42	18	54	15	39
Pitch Diameter Internal Gear B	9	9	$10\frac{2}{7}$	$10\frac{2}{7}$	$10\frac{2}{7}$	$10\frac{2}{7}$	$12\frac{6}{7}$	$12\frac{6}{7}$	18	18	25	25
Number of teeth in B	72	72	72	72	72	72	90	90	108	108	75	75
Pitch Diameter Idler C	$3\frac{3}{8}$	$2\frac{1}{4}$	$3\frac{6}{7}$	$2\frac{4}{7}$	$3\frac{6}{7}$	$2\frac{4}{7}$	$5\frac{1}{7}$	$3\frac{3}{7}$	$7\frac{1}{2}$	$4\frac{1}{2}$	10	6
Number of teeth in C	27	18	27	18	27	18	36	24	45	27	30	18
Number of Idlers	3	3	3	3	3	3	3	3	3	3	3	3
Diameter Pitch of Gears	8	8	7	7	7	7	7	7	6	6	3	3
Face of Gears	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{3}{4}$	2	2	3	3
Ratio	5 to 1	3 to 1	5 to 1	3 to 1	5 to 1	3 to 1	6 to 1	3.142 to 1	7 to 1	3 to 1	6 to 1	2.923 to 1

TABLE 29. PROPORTIONS OF DIFFERENTIAL BACK-GEARS.

other light machinery, the differential back-gear as originally designed. For a heavy drive and continuous service there is a better method of arrangement. This consists of using three idlers in place of one, these being equally spaced in order to retain the balance of the whole when locked up and driving direct. It will be readily seen that this arrangement calls for the following conditions

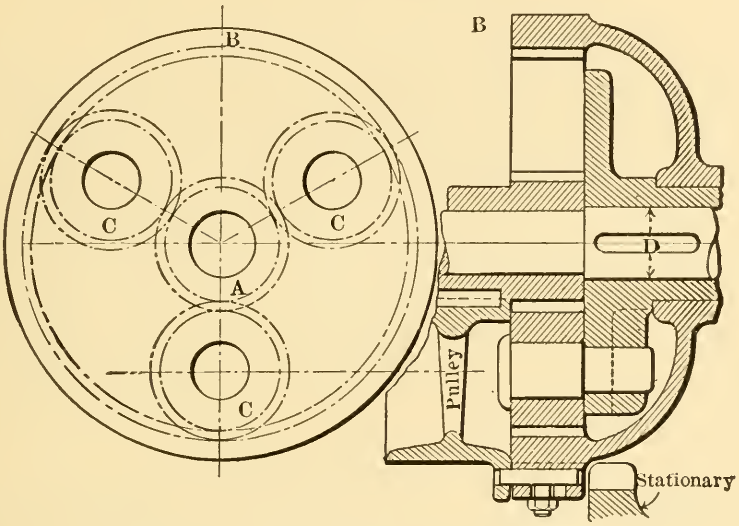


FIG. 240. DIFFERENTIAL BACK-GEARING.

in the gearing: That the number of teeth in pinion, idler, and internal gear must each be divisible by three, at the same time having correct diameters and pitch.

A table is given herewith showing drives from $2\frac{1}{2}$ to 18 horse-power in 12 sizes. This is arranged so that there are only 6 different diameters of internal gears, and by using different pinion and idlers one size can be used for two different ratios.

Fig. 240 shows the principle on which these back-gears are operated. The locking device should not be copied, however, as it would be rather inconvenient to use, and as there are various methods of operating clutches the writer has not gone into detail on this point, but gives the general dimensions, leaving the rest to be worked out by the designer to suit the requirements and conditions of the machine on which it is to be used.

SECTION XIII

Friction Gears *

A friction drive, as the term is here employed, consists of a fibrous or somewhat yielding driving wheel working in rolling contact with a metallic driven wheel. Such a drive may consist of a pair of plain cylindered wheels mounted upon parallel shafts, or a pair of beveled wheels, or of any other arrangement which will serve in the transmission of motion by rolling contact. The use of such drives has steadily increased in recent years, with the result that the so-called paper wheels have been improved in quality, and a considerable number of new materials have been proposed for use in the construction of fibrous wheels.

THE WHEELS TESTED

Choosing materials which have been used for such purposes, driving wheels of each of the following materials have been tested: straw fiber, straw fiber with belt dressing, leather fiber, leather, leather faced iron, sulphite fiber, tarred fiber.

The straw fiber wheels are worked out of the blocks which are built up usually of square sheets of straw board laid one upon another with a suitable cementing material between them and compacted under heavy hydraulic pressure. In the finished wheel the sheets appear as disks, the edges of which form the face of the wheel. The material works well under a tool, but it is harder and heavier than most woods and takes a good superficial polish. The wheel tested was taken from stock.

The wheel of straw fiber with belt dressing was similar to that of straw fiber, except that the individual sheets of straw board from which it was made had been treated, prior to their being converted into a block, with a "belt dressing" the composition of which is unknown to the writer.

The leather fiber wheel was made up of cemented layers of board, as were those already described; but in this case the board, instead of being of straw fiber, was composed of ground sole leather cuttings, imported flax and a small percentage of wood pulp. The material is very dense and heavy.

The leather wheel was composed of layers or disks of sole leather.

* Abstract of paper presented to the American Society of Mechanical Engineers, December, 1907, by W. M. Goss, Professor, University of Illinois.

The leather faced iron wheel consisted of an iron wheel having a leather strip cemented to its face. After less than 300 revolutions the bond holding the leather face failed and the leather separated itself from the metal of the wheel. This wheel proved entirely incapable of transmitting power and no tests of it are recorded.

The wheel of sulphite fiber was made up of sheets of board composed of wood pulp. The sulphite board is said to have been made on a steam-drying continuous process machine in the same way as is the straw board.

The tarred fiber wheel was made up of board composed principally of tarred rope stock, imported French flax, and a small percentage of ground sole leather cuttings.

Each of the fibrous driving wheels was tested in combination with driven wheels of the following materials: iron, aluminum, type metal. All wheels tested, both driving and driven, were 16 inches in diameter. The face of all driving wheels was $1\frac{3}{4}$ inches while that of all driven wheels was $\frac{1}{2}$ inch.

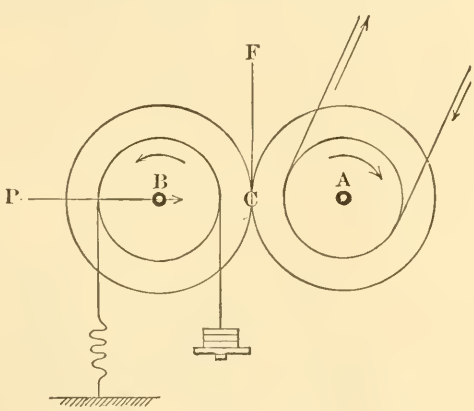


FIG. 241. DIAGRAM OF TESTING MACHINE FOR FRICTION WHEELS.

The purpose of the experiments was to secure information which would permit rules to be formulated defining the power which may be transmitted by the various combinations of fibrous and metallic wheels already described. To accomplish this it was necessary to determine for each combination of driving and driven wheel the coefficient of friction under various conditions of operation; also the maximum pressures of contact which can be withstood by each of the fibrous wheels.

The testing machine used is shown diagrammatically by Fig. 241. The principles involved will be made clear by assigning the functions of the actual machine to the several parts of this figure. The shaft A runs in fixed bearings and carries the fibrous friction wheel. This wheel is the driver. Its shaft A carries, besides the friction wheel, two belt pulleys, one on either side to which, from any convenient source of power, serve to give motion to the driver. The shaft B carries the driven wheel, which in every case was of metal. The bearings of this shaft are capable of receiving motion in a horizontal direction and by means of suitable mechanism connected therewith, the metal driven wheel may be made to press against the fibrous driver with any force desired. The pressure transmitted from B to A is hereinafter referred to as the "pressure of contact" and is frequently represented by the symbol P. The tangential

forces which are transmitted from the driver to the driven wheel are received, absorbed and measured by a friction brake upon the shaft *B*. In action, therefore, the driven wheel always works against a resistance, which resistance may be modified to any desired degree by varying the load upon the brake. The theory of the machine assumes that the energy absorbed by the brake equals that transmitted from the driver to the driven wheel at the contact point *C*.

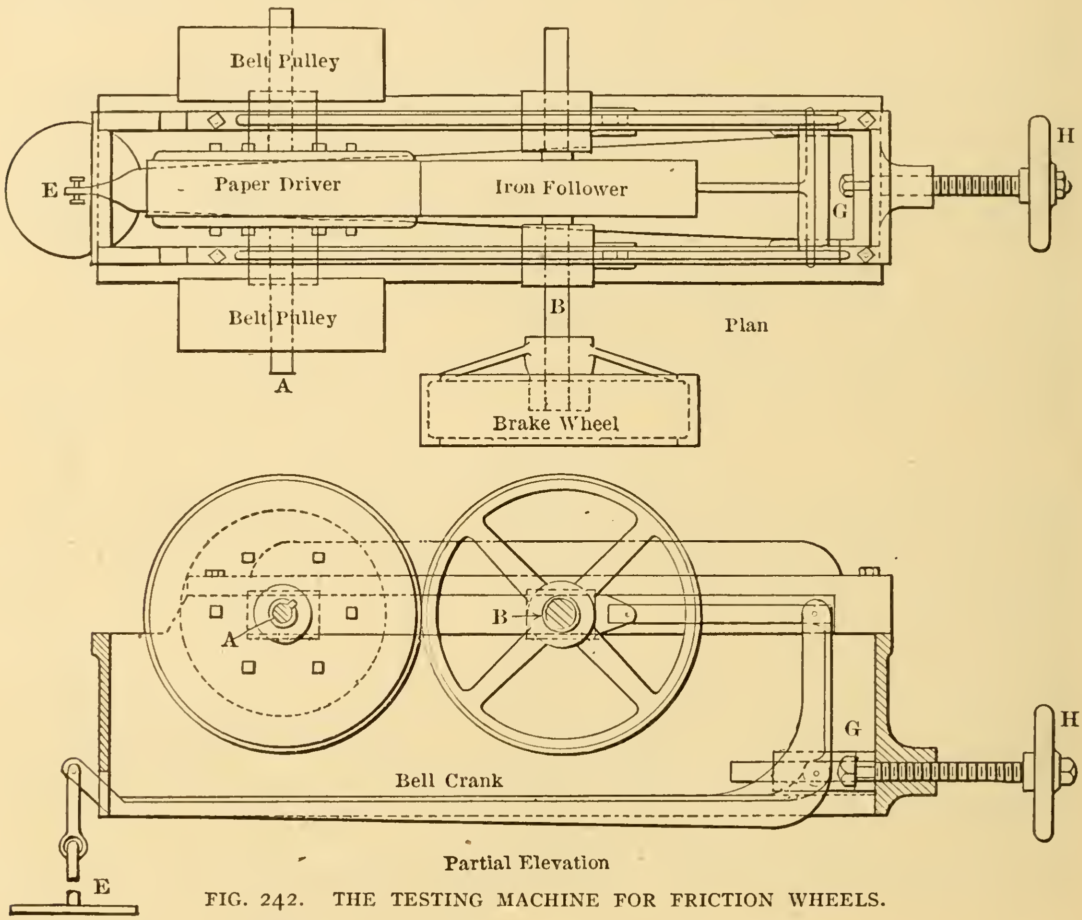


FIG. 242. THE TESTING MACHINE FOR FRICTION WHEELS.

Accepting this assumption, the forces developed at the periphery of the brake wheel may readily be reduced to equivalent forces acting at the circumference of the driven wheel. The force, which is directly transmitted from the driver to the driven wheel, is hereinafter designated by the symbol F. It will be apparent from this description that the functions of the apparatus employed are such as will permit a study of the relationship existing between the contact pressure P and the resulting transmitted force F, which relation is most conveniently expressed as the coefficient of friction. It is,

$$f = \frac{F}{P}.$$

It is obvious, in comparing the work of two friction wheels, that the one which develops the highest coefficient of friction, other things being equal, can be depended upon to transmit the greatest amount of power.

The actual machine as used in the experiments is shown by Fig. 242. Its construction satisfies all conditions which have been defined except that shaft B, Fig. 241, does not run in bearings which are absolutely frictionless, as is required by a rigid adherence to the theoretical analysis already given. These bearings, however, are of the "standard roller bearing" type and of ample size, and it is believed that the friction actually developed by them is so small compared with the energy transmitted between the wheels that it may be neglected.

The bearings of the fixed shaft A are secured to the frame of the machine. The bearings of the axle B are free to move horizontally in guides to which they are well fitted. Those bearings are connected by links to the short arm of a bell crank lever, the arm of which projects beyond the frame of the machine at the right-hand end and carries the scale pan and weights E. The effect of the weights is to bring the driven wheel in contact with the driver under a predetermined pressure, the proportions of the bell crank lever being such as to make this pressure in pounds equal,

$$P = 10\,W + 73,$$

where W is the weight on the scale pan E.

The fulcrum of the bell crank lever is supported by a block G which may be adjusted horizontally by the hand wheel H at the rear of the machine, so that whatever may be the diameter of the driven wheel, the long arm of the bell crank may be brought to a horizontal position. The constants employed in calculating the coefficient of friction from observed data are as follows:

Diameter of friction wheels (inches)	16
Effective diameter of brake (inches)	18.35
Ratio of diameter of friction wheel to that of brake wheel	1.145
Effective load on brake	F'
Coefficient of friction	$1.145\,\frac{F'}{P}$

The slippage between the friction wheels was determined from the readings taken from the counters connected to each one of the shafts.

THE TESTS

In proceeding with a test, load was applied to the scale pan E, Fig. 242, to give the desired pressure of contact, after which the hand wheel H at the back of the machine was employed to bring the bell crank to its normal position.

This accomplished, with the driving wheel in motion, the driven wheel would roll with it under the desired pressure of contact. A light load was next placed upon the brake to introduce some resistance to the motion of the driven shaft, and conditions thus obtained were continued constant for a considerable period. Readings were taken simultaneously from the counters and time noted. After a considerable interval the counters were again read, time again noted, and the test assumed to have ended. From the readings of the counters and from the known diameters of the wheels in contact, the percentage of slip attending the action of the friction wheels was calculated. Three facts were thus made of

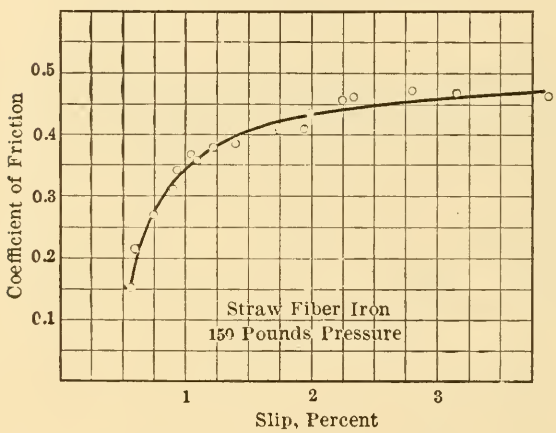


FIG. 243

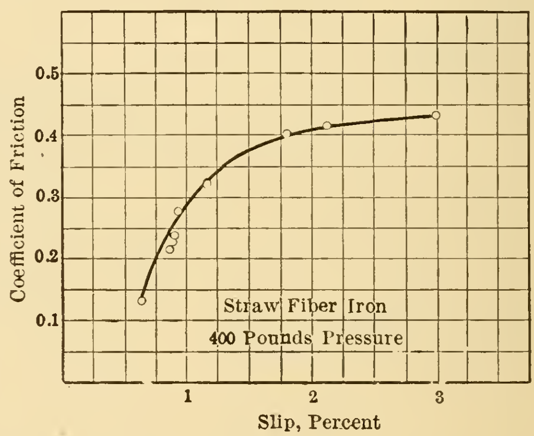


FIG. 244

CURVES FOR STRAW FIBER AND IRON, TYPICAL FOR ALL CURVES PLOTTED FROM THE FRICTION TESTS

record, namely: (*a*) The pressure of contact, (*b*) the coefficient of friction developed, and (*c*) the percentage of slip resulting from the development of said coefficient of friction.

This record having been completed, the load upon the brake was increased and observations repeated, giving for the same pressure of contact a new coefficient of friction and a higher percentage of slip. This process was continued until the slippage became excessive and in consequence thereof the rotation of the driver ceased. By this process a series of tests was developed disclosing the relation between slip and coefficient of friction for the pressure in question. Such a series having been completed, the load upon the weight holder *E* was changed, giving a new pressure of contact, and the whole process repeated. As the work proceeded, curves showing the relation of coefficient of friction and slip for pressures per inch width of face in contact of 150 pounds and 400 pounds, respectively, were secured. The curves shown by Figs. 243 and 244 for the straw fiber driving wheel in contact with the iron driven wheel are typical in their general form of those obtained from all combinations of

wheels, but the curves of no two combinations were alike in their numerical values.

Having completed this series of tests at constant pressure, a series was next run for which the coefficient of slip was maintained constant at 2 per cent., and the pressure of contact varied from values which were low to those which are judged to be near the maximum for service conditions, with the result, which in all cases were similar in character with those given for the straw fiber and iron wheels, as set forth by Fig. 245. The numerical values of the points for other combinations were not the same as those shown by Fig. 245, but in the case of most of the combinations the coefficient of friction at constant slip gradually diminishes as the pressure of contact is increased.

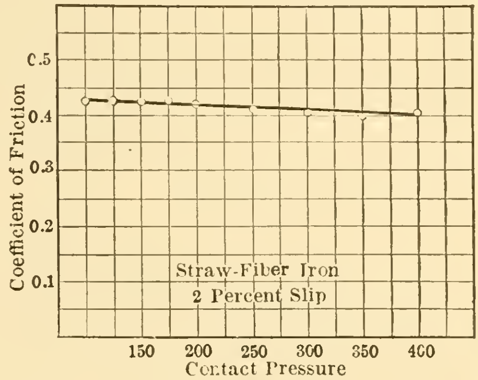


FIG. 245. CURVE FOR STRAW FIBER AND IRON WITH CONSTANT SLIP

As the series of tests involving each combination of wheels proceeded, the increase in pressure of contact was discontinued when the markings made upon the driving wheel by the metallic follower became so distinct as to suggest that a safe limit had been reached; but when all other data had been secured, tests were run for the purpose of determining the ultimate resistance of the fibrous wheel to crushing. The details of these will be described later.

COEFFICIENT OF FRICTION DEVELOPED BY THE SEVERAL COMBINATIONS OF WHEELS—STRAW FIBER AND IRON

The results of experiments involving a straw fiber driver and an iron driven wheel are shown graphically in Figs. 243, 244, and 245. Figs. 243 and 244 illustrate the relation between slip and coefficient of friction when the two wheels are working together under pressures per inch width of 150 and 400 pounds, respectively.

The figures show that although the values of the coefficient of friction are slightly lower than corresponding ones for 150 pounds pressure, the curves are sufficiently similar to establish the fact that the law governing change in coefficient friction with slip is independent of the pressure of contact. When the slippage is 2 per cent. the coefficient of friction is 0.425 for a contact pressure of 400 pounds. That the coefficients of friction for all pressures between the limits of 150 pounds and 400 pounds are practically constant is well shown by the diagram Fig. 245. The pressure of 400 pounds is the maximum at which tests

of this combination of wheels were run, though straw fiber was successfully worked up to a pressure of 750 pounds.

STRAW FIBER AND ALUMINUM

By curves plotted from values for a straw fiber driver and aluminum driven wheel, it can be shown that when the working pressure is 150 pounds per inch width and the slippage is 2 per cent. the coefficient of friction is 0.455; also, that for all pressures ranging from 100 to 400 pounds, the coefficient of friction is practically constant when the rate slip is constant. The maximum pressure at which tests involving this combination of wheels were run was 400 pounds per inch width.

STRAW FIBER AND TYPE METAL

By curves plotted from values for a straw fiber driver and a type metal driven wheel it can be shown that when the two wheels are operated under a pressure of contact of 150 pounds per inch width and when the slip is 2 per cent. the coefficient of friction is 0.310; also, that for all pressures of contact ranging from 100 to 400 pounds, the coefficient of friction is practically constant when the slip is constant.

STRAW FIBER WITH BELT DRESSING AND IRON

Curves plotted from values for a straw fiber driver treated with belt dressing, and an iron driven wheel show that when the two wheels are worked together under a pressure of 150 pounds per inch width and when the slip is 2 per cent. the coefficient of friction is 0.12; also, that for all pressures up to 400 pounds per inch width, the coefficient of friction remains constant. The greatest pressure at which tests of this combination of wheels were run was 500 pounds per inch width.

LEATHER FIBER AND IRON

Curves plotted from the results of tests involving a leather fiber driver and an iron driven wheel show that when the two wheels are worked together under pressure of 150 pounds per inch in width and when slip is 2 per cent. the coefficient of friction is 0.515. When the contact pressure is 300 pounds per inch width, the coefficient of friction is 0.510. The greatest pressure at which tests of this combination of wheels were run was 350 pounds per inch width, although leather fiber was successfully worked up to a pressure of 1200 pounds per inch width.

LEATHER FIBER AND ALUMINUM

Curves plotted from the results of experiments involving a leather fiber driver and an aluminum driven wheel show that under a contact pressure of 150 pounds per inch width and a slip of 2 per cent. the coefficient of friction is 0.495.

This value remains practically constant under all pressures. The maximum pressure used in tests of this combination of wheels was 400 pounds.

LEATHER FIBER AND TYPE METAL

Curves plotted from the results of experience involving a leather fiber driver and a type metal driven wheel show that when the wheels are operated under a contact pressure of 150 pounds per inch width and when the slip is 2 per cent. the coefficient of friction remains constant for all pressures up to 400 pounds per inch width.

TARRED FIBER AND IRON

Curves plotted from the results of the experiments involving a tarred fiber driver and an iron driven wheel show that the change in the value of the coefficient of friction with change of slip is practically independent of the pressure of contact. When the slip is 2 per cent., the coefficient of friction is 0.220 for a pressure of contact of 150 pounds and 0.250 for a pressure of contact of 400 pounds per inch width.

Tests of this combination were made also under different speeds when the wheels were working together under a pressure of contact of 250 pounds per inch width and when the slip was 2 per cent., with the result that the coefficient of friction was found to remain nearly constant for speeds of 450 and 3350 feet per minute, respectively. The greatest pressure at which tests of this combination of wheels were run was 400 pounds per inch width, although tarred fiber was successfully worked up to a pressure of 1200 pounds per inch width.

TARRED FIBER AND ALUMINUM

Curves plotted from the results of experiments involving a tarred fiber driver and an aluminum driven wheel show that when the slip is 2 per cent. and the pressure of contact 150 pounds per inch width, the coefficient of friction is 0.305; also, that for a pressure of 400 pounds per inch width, the coefficient of friction is 0.295. The greatest pressure at which tests of this combination were run was 400 pounds per inch width.

TARRED FIBER AND TYPE METAL

Curves plotted from the results of experiments involving a tarred fiber driver and a type metal driven wheel show that when the slip is 2 per cent. the coefficient of friction developed under 150 pounds pressure per inch width is 0.275; and under 400 pounds pressure per inch width, the coefficient of friction is 0.270. The maximum pressure at which tests of this combination of wheels were run was 400 pounds per inch width.

LEATHER AND IRON

Curves plotted from the results of experiments involving a leather driver and an iron driven wheel show that when the slip is 2 per cent. the coefficient of friction under a pressure of contact of 150 pounds per inch in width is 0.225 and under a pressure of 400 pounds, 0.215. The maximum pressure at which tests of this combination of wheels were run was 400 pounds per inch width, although the leather driver was successfully operated up to a pressure of 750 pounds per inch width.

LEATHER AND ALUMINUM

Curves plotted from the results of experiments involving a leather driver and an aluminum driven wheel show that when the pressure is 150 pounds per inch in width and the slip is 2 per cent. the coefficient of friction is 0.260; and when the pressure is 300 pounds per inch in width, the coefficient of friction is 0.295. The maximum pressure at which tests of this combination of wheels were made was 350 pounds per inch width.

LEATHER AND TYPE METAL

Curves plotted from the results of the experiments involving a leather driver and a type metal driven wheel show that when the slip is 2 per cent. and the contact pressure 150 pounds per inch width, the coefficient of friction developed is 0.410. The greatest pressure at which tests of this combination of wheels were run was 350 pounds per inch width.

SULPHITE FIBER AND IRON

Curves plotted from the results of the experiments involving a sulphite fiber driver and an iron driven wheel show that when the slip is 2 per cent. and the pressure 150 pounds per inch width, the coefficient of friction is 0.550. The maximum pressure at which tests of this combination of wheels were run was 350 pounds per inch width, although the sulphite fiber wheel was successfully operated up to a pressure of 700 pounds per inch width.

SULPHITE FIBER AND ALUMINUM

Curves plotted from the results of the experiments involving a sulphite fiber driver and an aluminum wheel show that when the slip is 2 per cent. and the pressure 150 pounds per inch width, the coefficient of friction developed is 0.410. The greatest pressure used in tests of this combination of wheels was 350 pounds per inch width.

SULPHITE FIBER AND TYPE METAL

Curves plotted from the results of the experiments involving a sulphite fiber driver and a type metal driven wheel show that when the slip is 2 per cent. and

the contact pressure 150 pounds per inch width, the coefficient of friction is 0.515. The maximum pressure used in tests of this combination of wheels was 350 pounds per inch width.

RESISTANCE TO CRUSHING

Upon the completion of tests designed to disclose the frictional qualities of the several combinations; each fibrous wheel was subjected to test for the purpose of determining the maximum pressure per inch width of the face which could be sustained by it. This was accomplished by placing the wheel to be tested in the machine under a pressure of contact of 200 pounds per inch width. The load on the brake was then adjusted to give a 2 per cent. slip, and this brake load was maintained without change throughout the remainder of the tests. Thus adjusted, the machine was operated until the driver had completed 15,000 revolutions. This accomplished, and for the purpose of determining the reduction, if any, in the diameter of the fibrous wheel, the brake load was removed and the operation of the machine continued without load for a period of 6000 revolutions, the readings of the counters being taken at the beginning and at the end of the period. Under conditions of no load, the actual slip was assumed to be zero and the apparent slip observed was used for determining the reduction in diameter of the fibrous wheel which had been brought about by the previous running under pressure. This accomplished, the pressure of contact was increased, usually by 100 pound increments, and the whole operation repeated. This process was continued until failure of the fibrous wheel resulted. It will be seen that the ultimate resistance to crushing, as found by the process described, is that pressure which could not be endured during 15,000 revolutions.

A summary of results is as follows:

A CONCLUSION AS TO METAL WHEELS

An examination of Table 30, which presents a comparison of values representing the coefficient of friction of the several combinations of wheels tested, reveals the fact that the relative value of the metal driven wheels is not the same when operated in combination with different fibrous driving wheels. It appears that those driving wheels which are the more dense work more efficiently with the iron follower than with either the aluminum or type metal followers; but in the case of the softer and less dense driving wheels, and especially in the case of those in which an oily substance is incorporated, driven wheels of aluminum and type metal are superior to those of iron. Finely powdered metal which is given off from the surface of the softer metal wheels seems to account for this effect, and the character of the driving wheels is perhaps the only factor neces-

sary to determine whether its presence will be beneficial or detrimental. Finally, with reference to the use of soft metal driven wheels, it should be noted that no combination of such wheels with a fibrous driver appears to have given high frictional results. Except when used under very light pressures, the wear of the type metal was too rapid to make a wheel of its material serviceable in practice.

CONCLUSIONS AS TO FIBROUS WHEELS

The relative value of the different fibrous wheels when employed as drivers in a friction drive may be judged by comparing their frictional qualities as set forth in Table 30 and their strength as set forth in Table 31. The results show at once that the addition of belt dressing to the composition of a straw fiber wheel is fatal to its frictional qualities. The highest frictional qualities are possessed by the sulphite fiber wheel, which, on the other hand, is the weakest of all wheels tested. The leather fiber and tarred fiber are exceptionally strong; and the former possesses frictional qualities of a superior order. The plain straw fiber, which in a commercial sense is the most available of all materials dealt with, when worked upon an iron follower possesses frictional qualities which are far superior to leather, and strength which is second only to the leather fiber and the tarred fiber.

THE POWER CAPACITY OF FRICTION GEARS

A review of the data discloses the fact that several of the friction wheels tested developed a coefficient of friction which in some cases exceeded 0.5. That is, such wheels rolling in contact have transmitted from driver to driven wheels a tangential force equal to 50 per cent. of the force maintaining their contact. These wheels also were successfully worked under pressures of contact approaching 500 pounds per inch in width. Employing these facts as a basis from which to calculate power, it can readily be shown that a friction wheel a foot in diameter, if run at 1000 revolutions per minute, can be made to deliver in excess of 25 horse-power for each inch in width. It is certainly true that any of the wheels tested may be employed to transmit for a limited time an amount of power which, when gauged by ordinary measures, seems to be enormously high; but obviously, performance under limiting conditions should not be made the basis from which to determine the commercial capacity of such devices. In view of this fact, it is important that there be drawn from the data such general conclusions with reference to pressures of contact and frictional qualities as will constitute a safe guide to practice.

	Coefficient of friction when contact pressure is 150 pounds per inch		
	Iron	Aluminum	Type Metal
Sulphite Fiber	0.550	0.530	0.515
Leather Fiber	0.515	0.495	0.350
Straw Fiber	0.425	0.455	0.310
Tarred Fiber	0.250	0.305	0.275
Leather	0.225	0.360	0.410
Straw Fiber with belt dressing	0.120		

TABLE 30—COEFFICIENT OF FRICTION.

	Load in pounds	Decrease in diameter	
Straw Fiber	200	0.000	Wheel failed before running 15,000 revolutions under 750 pounds pressure.
	650	0.053	
	750	0.125	
Leather Fiber	200	0.000	Wheel failed before running 15,000 revolutions under 1200 pounds pressure.
	300	0.005	
	400	0.013	
	500	0.021	
	600	0.027	
	700	0.040	
	800	0.051	
	900	0.068	
	1000	0.099	
	1100	0.125	
	1200	0.200	
Tarred Fiber	200	0.000	Wheel failed before running 15,000 revolutions under 1200 pounds pressure.
	300	0.026	
	400	0.038	
	500	0.052	
	600	0.071	
	700	0.098	
	800	0.138	
	900	0.182	
	1000	0.250	
	1100	0.295	
	1200		
Leather	350	0.047	Wheel failed before running 15,000 revolutions under 750 pounds pressure.
	450	0.090	
	550	0.015	
	650	0.240	
	750		
Sulphite Fiber	200	0.010	Wheel failed before running 15,000 revolutions under 700 pounds pressure.
	300	0.032	
	400	0.056	
	500	0.088	
	600	0.146	
	700	0.258	

TABLE 31—STRENGTH OF VARIOUS FIBER WHEELS.

WORKING PRESSURE OF CONTACT

The results of these experiments do not furnish an absolute measure of the most satisfactory pressure of contact for service conditions. Other things being equal, the power transmitted will be proportional to this pressure, and hence it is desirable that the value be made as high as practicable. On the other hand, it has been noted as one of the observations of the test that as higher pressures are used, there appears to be a gradual yielding of the structure of the fibrous wheels; and it is reasonable to conclude that the life of a given wheel will in a large measure depend upon the pressure under which it is required to work. After a careful study of the facts involved, it has been determined to base an estimate of the power which may be transmitted upon a pressure of contact which is 20 per cent. of the ultimate resistance of the material as established by the crushing tests already described. This basis gives the following results:

SAFE WORKING PRESSURES OF CONTACT

	PRESSURE
Straw fiber	150
Leather fiber	240
Tarred fiber	240
Sulphite fiber	140
Leather	150

COEFFICIENT OF FRICTION

The coefficient of friction for all wheels tested approaches its maximum value when the slip between driver and driven wheel amounts to 2 per cent. and, within narrow limits, its value is practically independent of the pressure of contact. A summary of maximum results is shown by Table 30. In view of these facts, it is proposed to base a measure of the power which may be transmitted by such friction wheels as those tested upon the frictional qualities developed at a pressure of 150 pounds per inch of width, when operating under a load causing 2 per cent. slip. For safe operation, however, deductions must be made from the observed values. Thus, the results of the experiments disclose the power transmitted from wheel to wheel, while in the ordinary application of friction drives some power will be absorbed by the journals of the driven axle so that the amount of power which can be taken from the driven shaft will be somewhat less than that transmitted to the wheel on said shaft. Again, under the conditions of the laboratory, every precaution was taken to keep the surfaces in contact free of all foreign matter. It was, for example, observed that the accumulation of laboratory dust upon the surfaces of the wheels had a temporary effect upon the frictional qualities of the wheels, and friction wheels

in service are not likely to be as carefully protected as were those in the laboratory. In view of these facts, it has been thought proper to use as the basis from which to determine the amount of power which may be transmitted by such wheels as those tested, a coefficient of friction which shall be 60 per cent. of that developed under the conditions of the laboratory. This basis gives the following results:

COEFFICIENT OF FRICTION WORKING VALUES

	COEFFICIENT OF FRICTION
Straw fiber and iron	0.255
Straw fiber and aluminum	0.273
Straw fiber and type metal	0.186
Leather fiber and iron	0.309
Leather fiber and aluminum	0.297
Leather fiber and type metal	0.183
Tarred fiber and iron	0.150
Tarred fiber and aluminum	0.183
Tarred fiber and type metal	0.165
Sulphite fiber and iron	0.330
Sulphite fiber and aluminum	0.318
Sulphite fiber and type metal	0.309
Leather and iron	0.135
Leather and aluminum	0.216
Leather and type metal	0.246

HORSE-POWER

Having now determined a safe working pressure of contact and a representative value for the coefficeint of friction, it is possible to formulate equations expressing the horse-power which may be transmitted by each combination of wheels tested. Thus, calling d the diameter of the friction wheel in inches, W the width of its face in inches, and N the number of revolutions per minute, the equations become, for combinations of,

	HORSE-POWER
Straw fiber and iron	$0.00030\ dWN$
Straw fiber and aluminum	$0.00033\ dWN$
Straw fiber and type metal	$0.00022\ dWN$
Leather fiber and iron	$0.00059\ dWN$
Leather fiber and aluminum	$0.00057\ dWN$
Leather fiber and type metal	$0.00035\ dWN$
Tarred fiber and iron	$0.00029\ dWN$

	HORSE-POWER
Tarred fiber and aluminum	0.00035 dWN
Tarred fiber and type metal	0.00031 dWN
Sulphite fiber and iron	0.00037 dWN
Sulphite fiber and aluminum	0.00035 dWN
Sulphite fiber and type metal	0.00034 dWN
Leather and iron	0.00016 dWN
Leather and aluminum	0.00026 dWN
Leather and type metal	0.00029 dWN

The accompanying chart gives a convenient means of determining the value of any one of the variable factors in the formula horse-power = 0.0003 dWN for the straw fiber friction wheel working in combination with an iron follower, the remaining factors being known or assumed. To transform values thus found to corresponding ones for the other possible combinations of wheels, it is necessary only to multiply by the proper factor chosen from the table of multipliers given with the chart.

APPLICATION OF RESULTS TO FORMS OTHER THAN THOSE EXPERIMENTED UPON FACE FRICTION GEARING

A fibrous driving wheel, acting upon the face of a metal disk, constitutes a form of friction gear which is serviceable for a variety of purposes. If the driver is so mounted that it may be moved across the face of the disk, the velocity ratio may be varied and the direction of the disk's motion may be reversed. The contact is not one of pure rolling. If the driver is cylindrical in form, the action along its line of contact with the disk is attended by slip, amount of which changes for every different point along the line. The recognition of this fact is essential to a discussion of the power-transmitting capacity of the device.

Experiments involving the spur form of friction wheels already described have shown that slip greatly affects the coefficient of friction; that the coefficient approaches its maximum value when the slip reaches 2 per cent., and that when the slip exceeds 3 per cent., the coefficient diminishes. It is known that reductions in the value of the coefficient with increments of slip beyond 3 per cent. are at first gradual, although the characteristics of the testing machine have not permitted a definition of this relation for slip greater than 4 per cent. The experiments, however, fully justify the statement that for maximum results the slippage should not be less than 2 per cent. nor more than 4 per cent. It is the maximum limit with which we are concerned in considering the amount of power which may be transmitted by face friction gearing.

From the discussion of the previous paragraph, it should be evident that, for best results, the width of face of the friction driver and the distance between the driver and center of disk should always be such that the variations in the velocity of the particles of the disk having contact with the driver will not exceed 4 per cent. A convenient rule which, if followed, will secure this condition is to make the minimum distance between the driver and the center of the driven disk twelve times the width of the face of the driver. For example, a driver having a ¼-inch width of face should be run at a distance of 3 inches or more from the center of the disk. Similarly, drivers having faces ½, 1, or 2 inches in width should be run at a distance from the center of the disk of not less than 6, 12, or 24 inches, respectively. When these conditions are met, all formulas for calculating the power which may be transmitted apply directly to the conditions of face driving.

It may not infrequently happen that friction wheels must be run nearer the center of the disk than the distance specified; there is, of course, no objection to such practice, but it should not be forgotten that as the center of the disk is approached, the coefficient of friction, and consequently the capacity to transmit power, diminishes.

CONDITIONS TO BE OBSERVED IN THE INSTALLATION OF FRICTION DRIVES

Whatever may be the form of the transmission, the fibrous wheel must always be the driver. Neglect of this rule is likely to result in failure which will appear in the unequal wear of the softer wheel, occasioned by slippage.

The rolling surfaces of the wheel should be kept clean. Ordinarily they should not be permitted to collect grease or oil, nor be exposed to excessive moisture. Where this cannot be prevented, a factor of safety should be provided by making the wheels larger than normal for the power to be transmitted.

Since the power transmitted is directly proportional to the pressure of contact, it is a matter of prime importance that the mechanical means employed in maintaining the contact be as nearly as possible inflexible. For example, arrangements of friction wheels which involve the maintenance of contact through the direct action of a spring have been found unsatisfactory, since any defect in the form of either wheel introduces vibrations which tend to impair the value of the arrangement. It is recommended that springs be avoided and that contact be secured through mechanism which is rigid and which when once adjusted shall be incapable of bringing about any release of the pressure to which it is set.

EXPLANATION OF CHART

Chart 15 is plotted for the most common materials used for friction gearing, straw fiber and cast iron, and gives means of determining the variable factors for the fiber wheel in the formula *horse-power* = 0.0003 dWN, in which d is the diameter of the wheel in inches, W its width of face in inches, and N the number of revoultions per minute.

To use the chart for other friction materials multiply the values obtained from the chart by the proper factor selected from the table below:

Straw fiber and aluminum	1.10
Straw fiber and type metal	0.73
Leather fiber and cast iron	1.97
Leather fiber and aluminum	1.90
Leather fiber and type metal	1.17
Tarred fiber and cast iron	0.97
Tarred fiber and aluminum	1.17
Tarred fiber and type metal	1.03
Sulphite fiber and cast iron	1.23
Sulphite fiber and aluminum	1.17
Sulphite fiber and type metal	1.13
Leather and cast iron	0.53
Leather and aluminum	0.87
Leather and type metal	0.97

a. *To find the total horse-power* which can be transmitted by a wheel, having given the diameter of the wheels in inches, the width of its face in inches and the revolutions per minute, locate the intersection of the vertical line representing the given speed with the diagonal line representing the given diameter. Follow the horizontal line passing through this point, to the right or left as the case may be, until it intersects the vertical line representing the given width of face. The diagonal line through this point will give the total horse-power required from the scale so marked.

b. *To find the speed in revolutions per minute* for a wheel, having given its diameter in inches, its width of face in inches, and the total horse-power to be transmitted, locate the intersection of the vertical line representing the width of face with the diagonal line representing the total horse-power to be transmitted. Follow the horizontal line passing through this point, to the right or left as the case may be, until it intersects the diagonal line representing the diameter in inches. The vertical line passing through this point indicates on the scale at the bottom of the chart the speed required.

c. To find the width of face in inches for a wheel, having given the total horse-power to be transmitted, its diameter in inches and its speed in revolutions per minute, locate the intersection of the vertical line representing the given speed with the diagonal line representing the given diameter. Follow

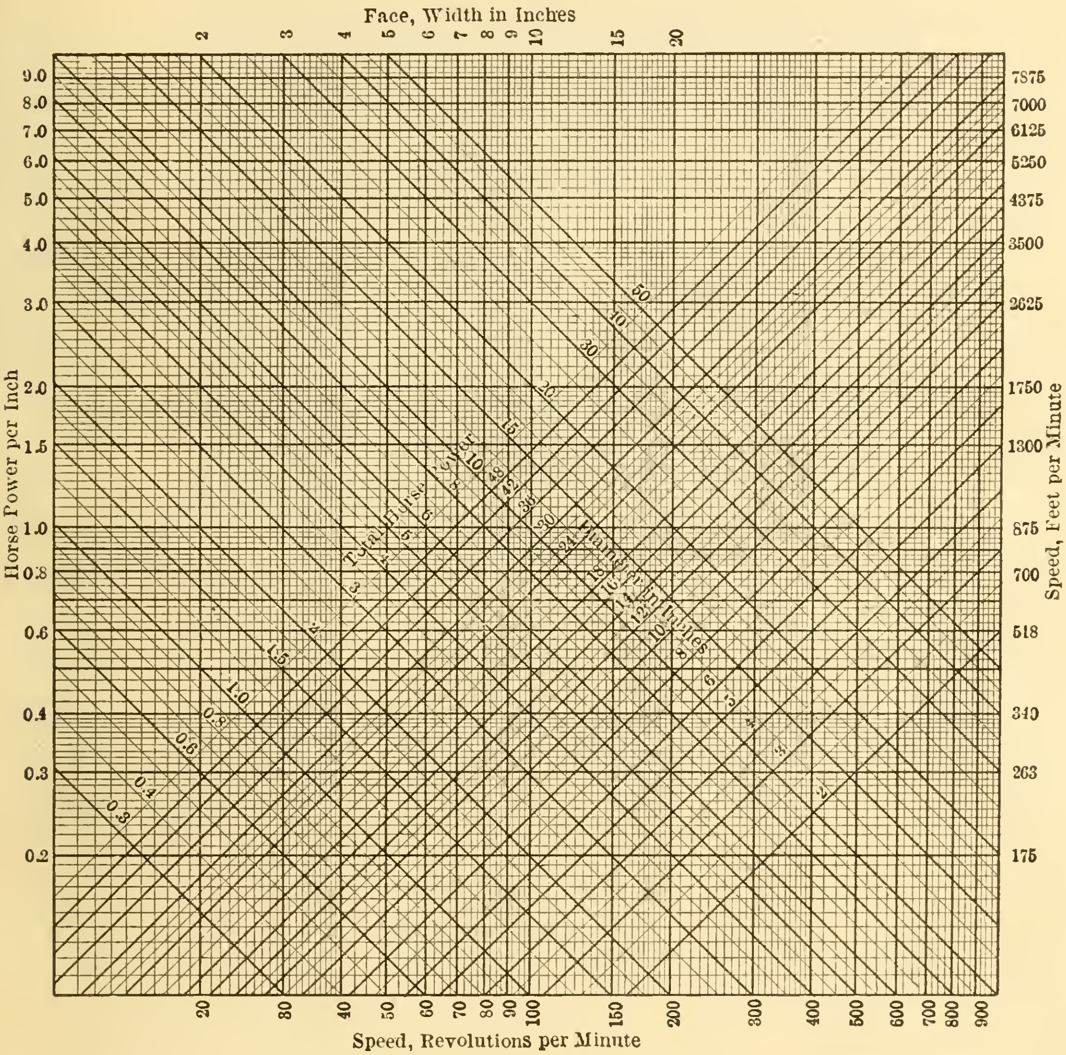


CHART 15. PROPORTIONS OF FIBROUS FRICTION GEARING.

the horizontal line passing through this point, to the right or left as the case may be, until it intersects the diagonal line representing the given total horse-power. The vertical line passing through this point will indicate the width of face required on the scale at the top of the chart.

d. To find the diameter in inches for a wheel, having given the horse-power to be transmitted, its width of face in inches, and its speed in revolutions per

minute, locate the intersection of the vertical line representing the width of face with the diagonal line indicating the total horse-power. Follow the horizontal line passing through this point, to the right or left as the case may be, until it intersects the vertical line representing the speed. The diagonal line passing through this point represents the diameter which is required.

e. To find the surface speed of a wheel, having given its diameter in inches and its speed in revolutions per minute, locate the intersection of the vertical line representing the speed in revolutions per minute with the diagonal line representing the given diameter. The horizontal line passing through this point represents the surface speed in feet per minute which is required, and which is read on the vertical scale at the right of the chart.

f. To find the horse-power per inch of face for a wheel, having given the total horse-power transmitted and the width of the face in inches, locate the intersection of the vertical line representing the width of face with the diagonal line representing the total horse-power. The horizontal line passing through this point represents the horse-power per inch of face required and may be read on the vertical scale at the left of the chart.

FRICTION DRIVE ON A FORTY-FOUR FOOT PIT LATHE*

The machine here described was designed to meet the demands of an establishment manufacturing the heaviest type of electrical machinery. The ever-increasing dimensions of this class of machinery make it particularly desirable that the existing heavy machine tools should be capable of extension of capacity with a view to probable future requirements, and that a pit lathe is peculiarly adapted to such extension will, doubtless, be readily admitted.

The face-plate of this machine measures 30 feet in diameter, and the present dimensions of the pit will admit of swinging 44 feet on centers, with a maximum width of 12 feet. The large face-plate is built up of twelve segments. The rim is of box section, the ends of the rim in each section being finished to make the joint, and the segments being held together at the rim by body-bound bolts. The arms are slotted for bolts, and the space between segments is also shaped to receive the usual square-headed bolts, as the inner end of each segment is fastened to the smaller face-plate by several body-bound bolts.

A feature of interest in connection with this machine is the method of drive adopted, which is a friction roller, 18 inches diameter, made of compressed paper, while the rim of the large face-plate, 15 inches wide, affords the necessary contact surface for driving.

* Extract from a paper presented at the New York meeting of the American Society of Mechanical Engineers by John M. Barney.

Power is supplied by a 75 horse-power motor, quadruple-geared, the use of the multiple voltage system giving the machine a range covering all diameters from six feet to the present capacity, though the gear train is designed to admit of two changes of back gear in addition.

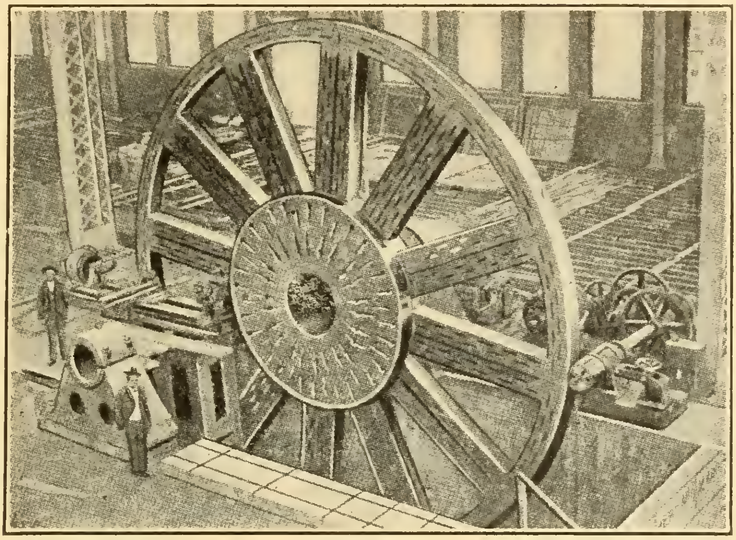


FIG. 246. FRICTION-DRIVEN LATHE.

Fig. 246 shows the assembled pit lathe driven by the friction roller while taking a heavy facing cut, on which occasion four tools were employed. The picture also shows the driving motor with its train of gears and the mechanism employed for adjusting the pressure on the friction roller.

SECTION XIV

Odd Gearing

Under this head are shown a few examples of what has been produced in the way of odd gearing.

THE GRISSON HIGH REDUCTION GEARING

A good deal of interest has been manifested in Germany over the Grisson gearing for use in connection with electric motors, for which use a high ratio of

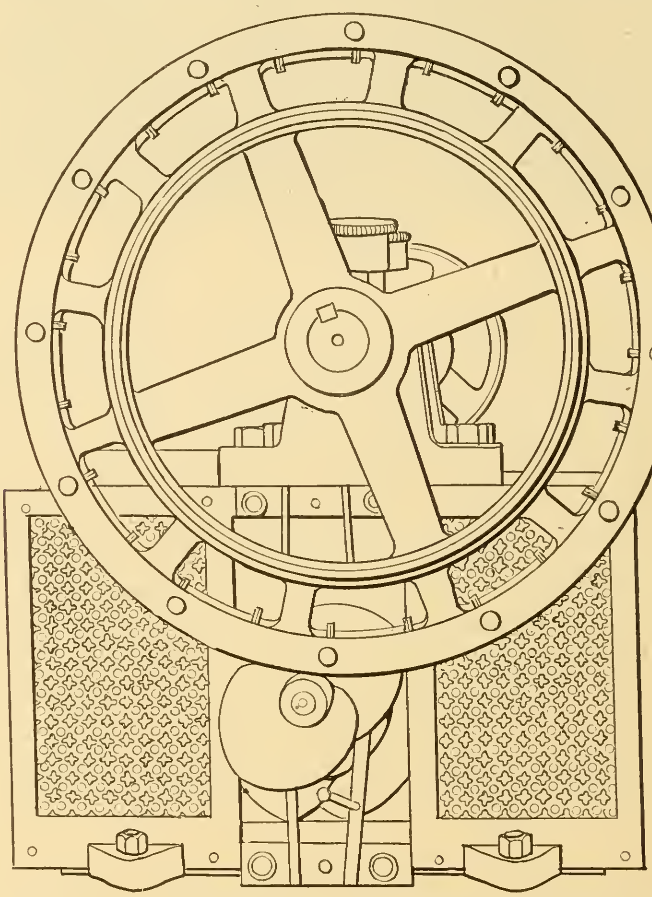


FIG. 247. GRISSON GEARING, USING A THREE-TOOTHED PINION.

speed reduction is very desirable. This gear is a revival in modified form of the two-toothed pinion, having the two teeth in different planes, the modification consisting of the use of roller teeth in the larger gear. The two-toothed

pinion, with sliding contact and radial faces in the larger gear, is shown and discussed in McCord's "Kinematics," where the gear is said to be very satisfactory in operation except for the large amount of sliding which it entails. The use of roller teeth in the Grisson gear is, of course, to do away with this excessive sliding.

Fig. 247 is a side elevation showing its appearance in an actual case. The pinion speed in the construction shown in Fig. 247 was 1200 revolutions per minute and the speed ratio was 12. The teeth of the pinion will be seen to be of the form of heart-shaped cams, each tooth working on its own set of roller teeth in the gear, these roller teeth being placed between appropriate flanges.

The leading feature of the gear is, of course, its high ratio of speeds, together with great compactness and a small center distance. The smallest permissible ratio is said to be one to five and the action of the gears is better with higher ratios. The efficiency also increases with the ratio, experiments being said to have shown an efficiency as high as 95 per cent.

A NOVELTY IN GEARING

The cuts show a somewhat novel style of gearing, the operation of which will be easily understood. The turning of the helical cam or pinion *B* moves the rack *A*, or in Fig 250 a wheel *A* is turned instead of operating the rack. The principle of the device is better shown in Fig. 253, where the oblique continuous lines are avoided by making the gears in sections, each successively a little in advance of the preceding. This shows the driver *B* to consist of a series of eccentrics, and of course the previous figures would show the same eccentricity in any section of the operative portion. The grooves of the rack, or of the driven wheel, should not be perfectly circular, but should be more nearly as shown in section in Fig. 252, with nearly straight lines at the points marked x. The specification of the patent for this gearing states:

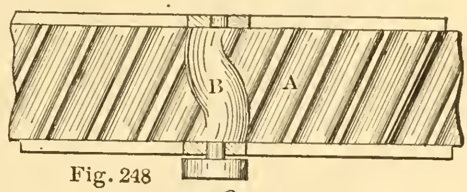


Fig. 248

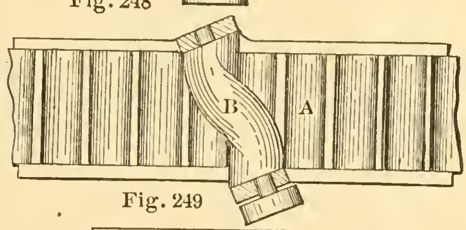


Fig. 249

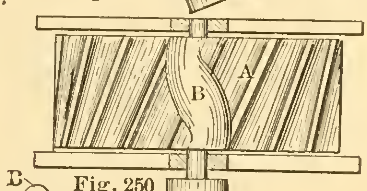


Fig. 250

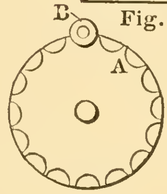


Fig. 251

Fig. 252

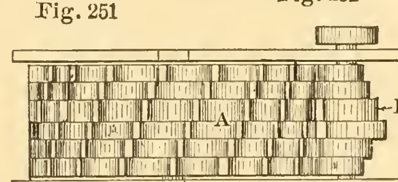


Fig. 253

A NOVELTY IN GEARING.

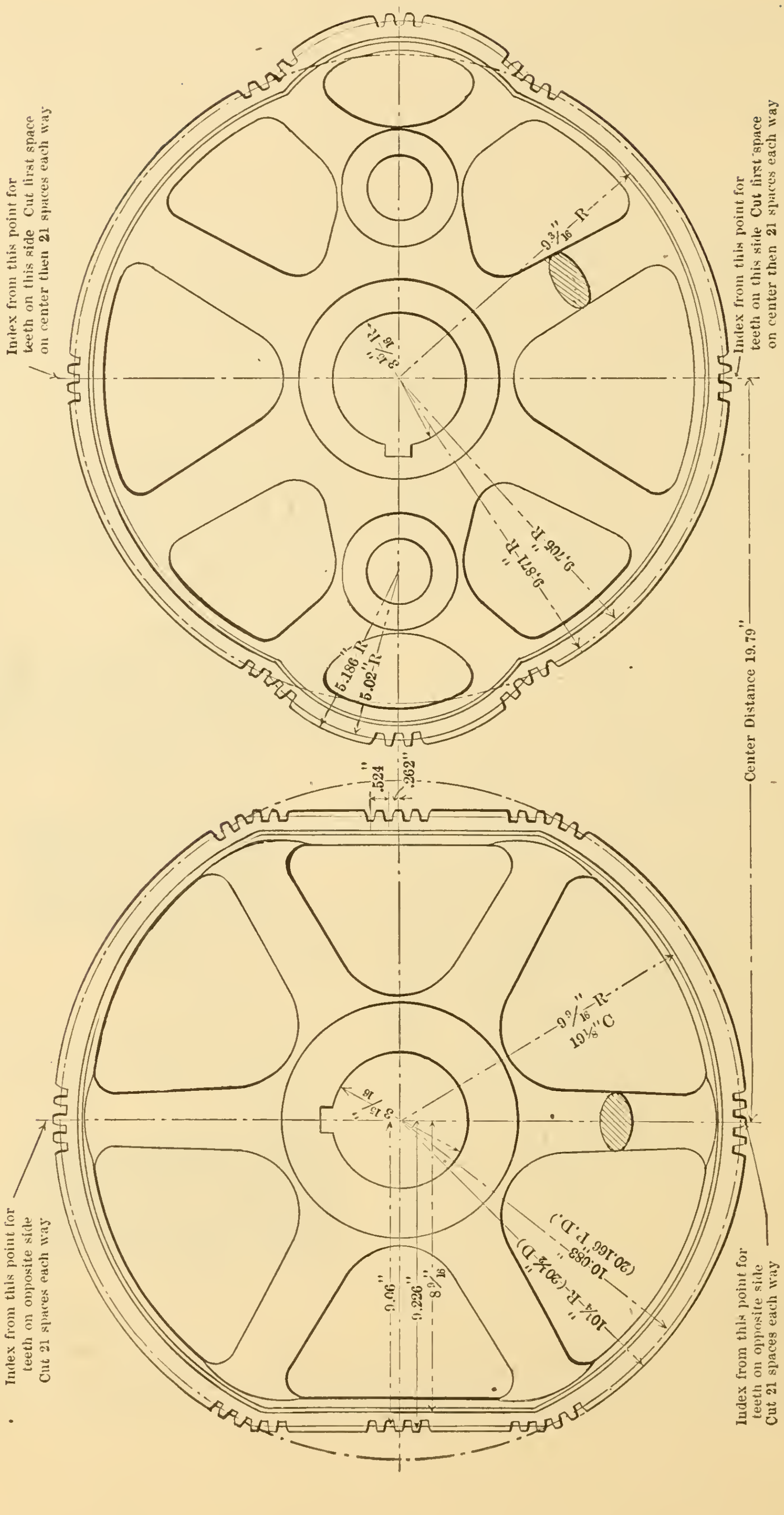


FIG. 254. FIG. 255.

A PAIR OF ODD-SHAPED GEARS.

"The rotation of the member *B* imparts a movement to the member *A*, as a pinion will impart motion to a rack or wheel; but the member *A* is prevented from imparting any movement to the member *B* and is locked thereby when the parts are at rest, *even more positively than a worm-wheel is locked by the worm.*"

Patent was assigned to the Otis Elevator Co., 1901.

TYPE CYLINDER GEAR

Andrew Strom, of the Dayton Pneumatic Tool Company, Dayton, O., designed and cut the odd-shaped gear shown in Figs. 254 and 255, an ordinary milling machine being used.

This gear is practically a round spur gear except for the two flat sides, and while it is not difficult to machine, it combines a spur gear and a rack in a way that is not pleasing to the gear cutting department of a shop.

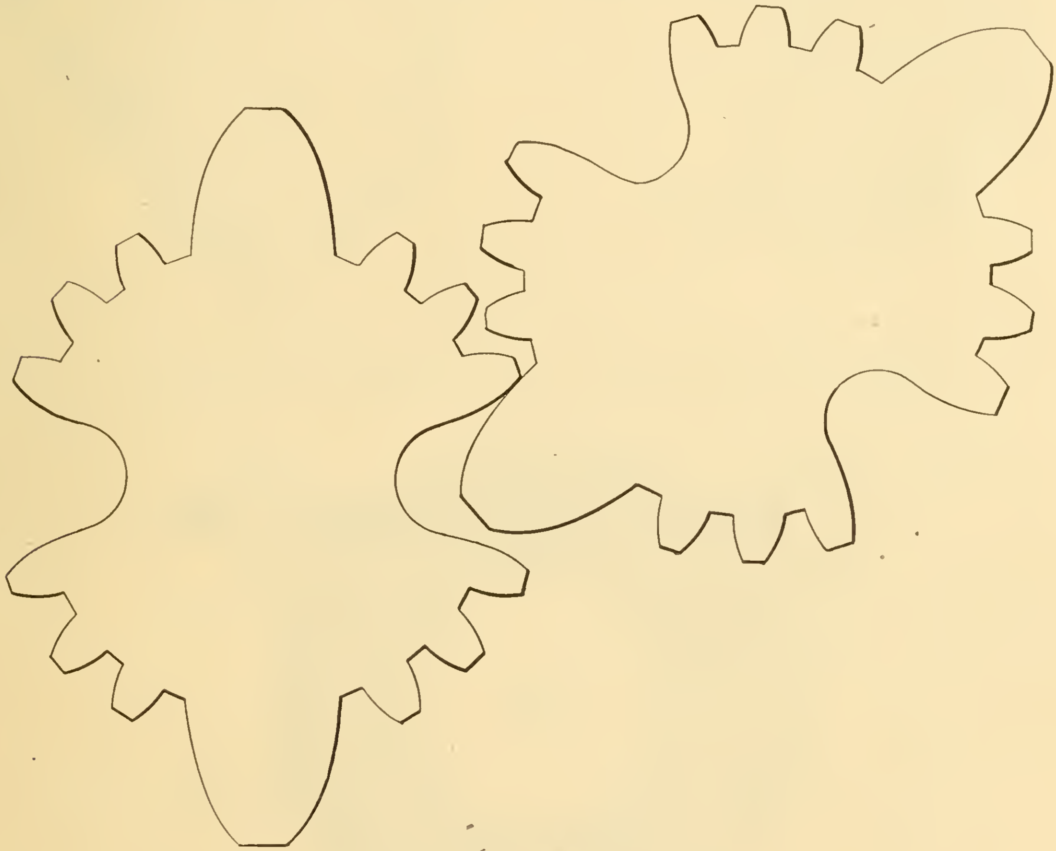


FIG. 256. PUMP GEARS.

The teeth are 0.6 pitch, pitch diameter 20.166, and a total of 120 teeth. The teeth were cut from the four quarters, with a tooth in the center and 21 spaces each side on the round portions, and nine spaces each way on the flats.

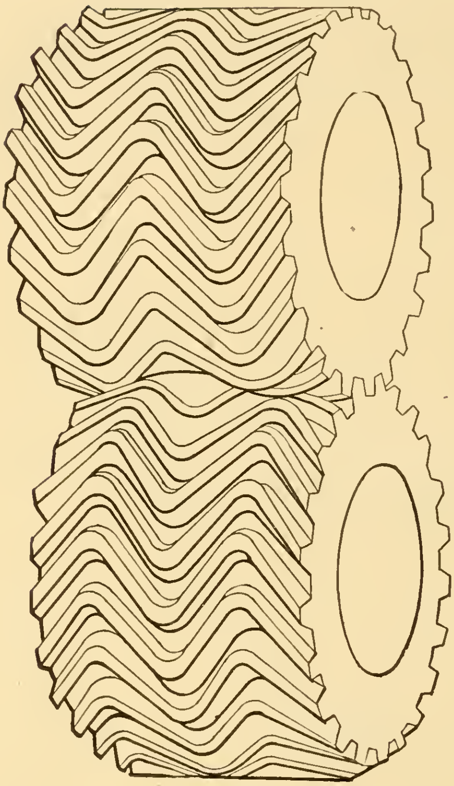


FIG. 257.

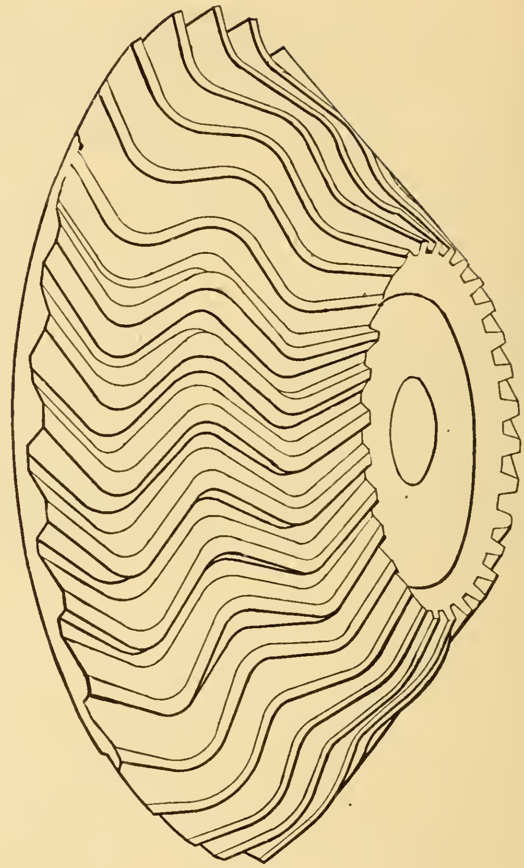


FIG. 258.

SPIRAL-TOOTH GEARS MADE BY CITROEN, HEUSTIN ET CIE, OF PARIS.

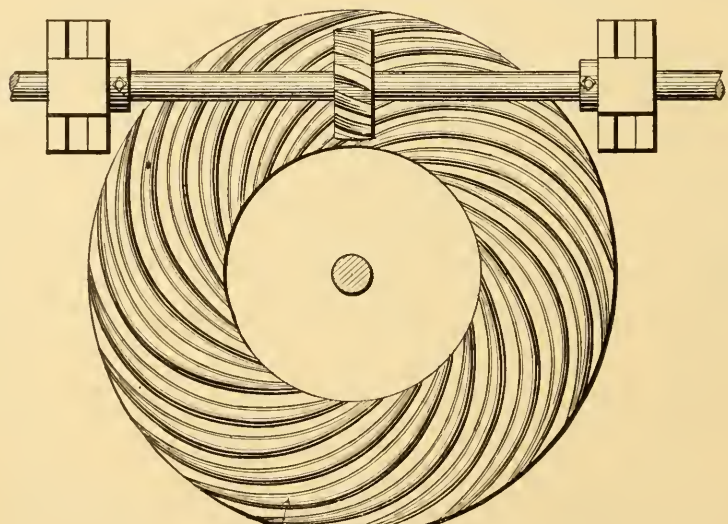


FIG. 259. CROWN SPIRAL GEARS.

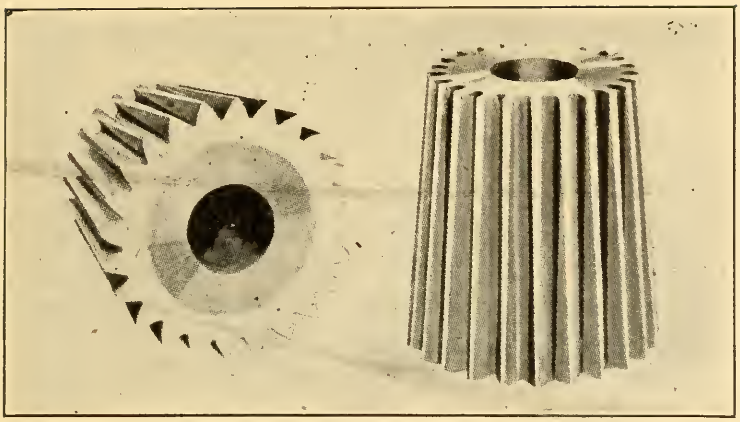


FIG. 260. FELLOWS SPUR BEVELS.

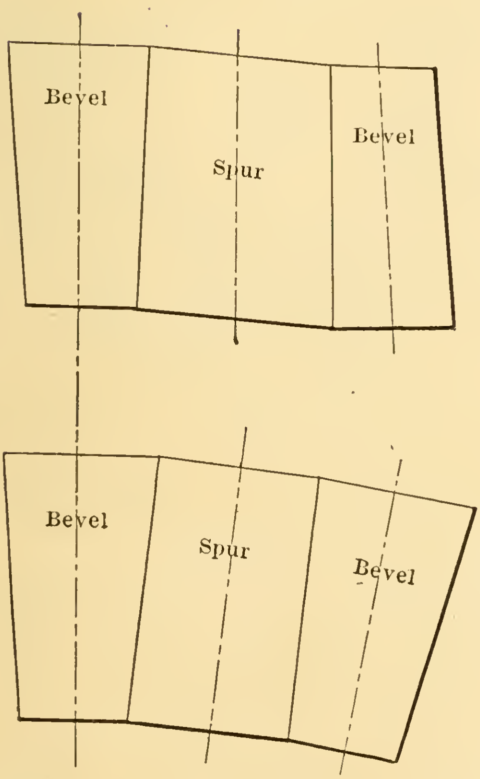


FIG. 261. TWO FELLOWS SPUR-BEVEL GEARS IN COMBINATION WITH A SPUR GEAR.

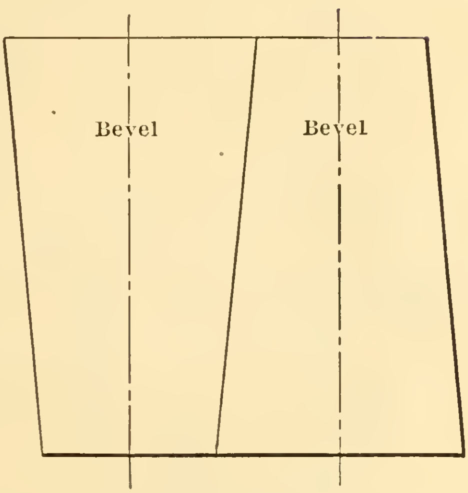


FIG. 262. TWO FELLOWS SPUR-BEVEL GEARS WITH PARALLEL AXES.

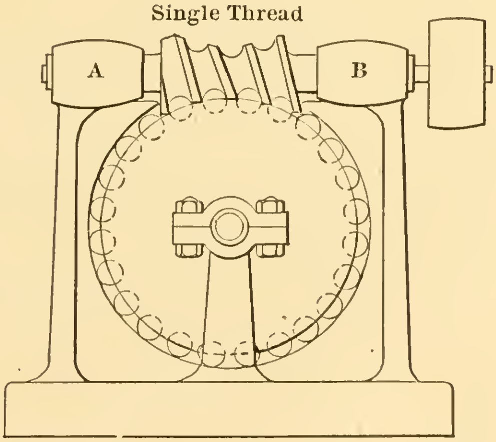


FIG. 263. COLLIER'S BALL-WORM GEAR.

This was cut by compound-index moves, set for 121 teeth, indexing from the two points for the teeth *on the opposite side of the blank.* Using both 47 and 49 circle holes, move crank 1 $\frac{14}{47}$ turns right hand and then backward 15 holes in the 49 circle, or 1 $\frac{14}{47}$ or $-$ $\frac{15}{49}$. Or a special plate of 121 holes could be used, moving 40 holes for each tooth.

In cutting the teeth on the gear for the impression cylinder, Fig. 255, it was necessary to swing it from three centers, the main center for the regular portions of the gear and the two side centers for the smaller portions at each side, and the indexing became quite a difficult proposition.

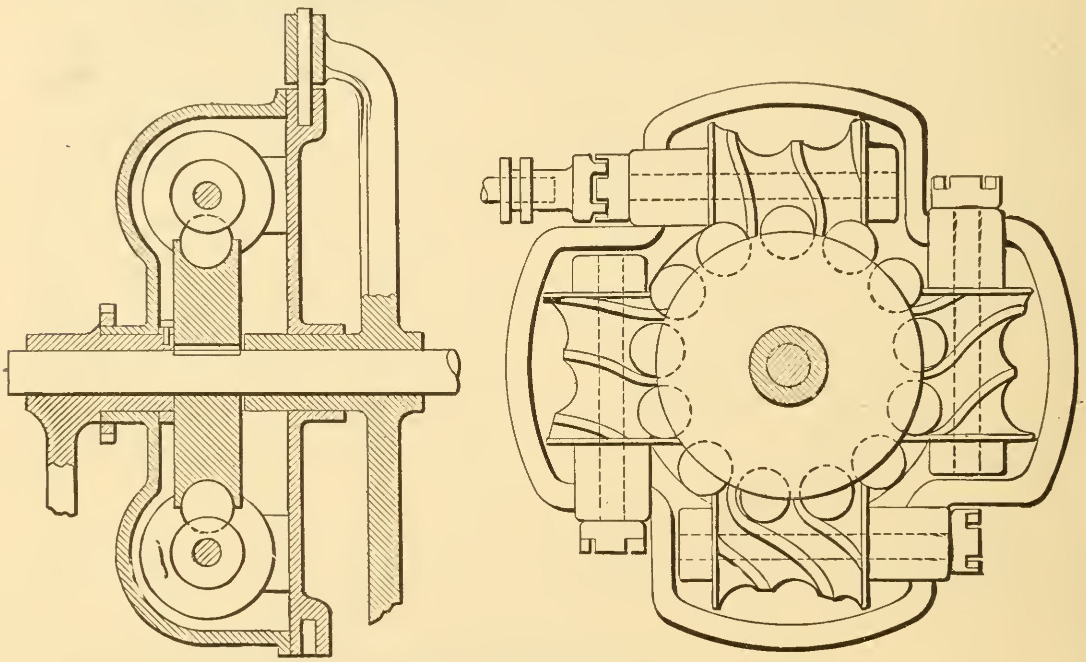


FIG. 264. A COLLIER DRIVE USING FOUR WORMS.

The indexing was not continuous, but was started from four points, at the centers of the four arcs, with the tooth cut in the center, as shown.

It took some careful calculating to find just what to do, but it was finally worked out to index in compound as follows: Three turns + 25 holes in the 47-hole circle, then 12 holes in the 49-hole circle in the same direction, giving 3 $\frac{25}{47}$ + $\frac{12}{49}$ for each tooth.

After the center tooth was cut, 21 teeth were cut on each side of it, bringing the teeth to the beginning of the smaller radius. Then the blank was shifted to either end center and the center tooth was cut on this portion, then eight each way. This meets the other cutting, leaving the sharp-pointed tooth shown.

This indexing is the same as for 60 teeth, but only 17 are cut in all. A No.

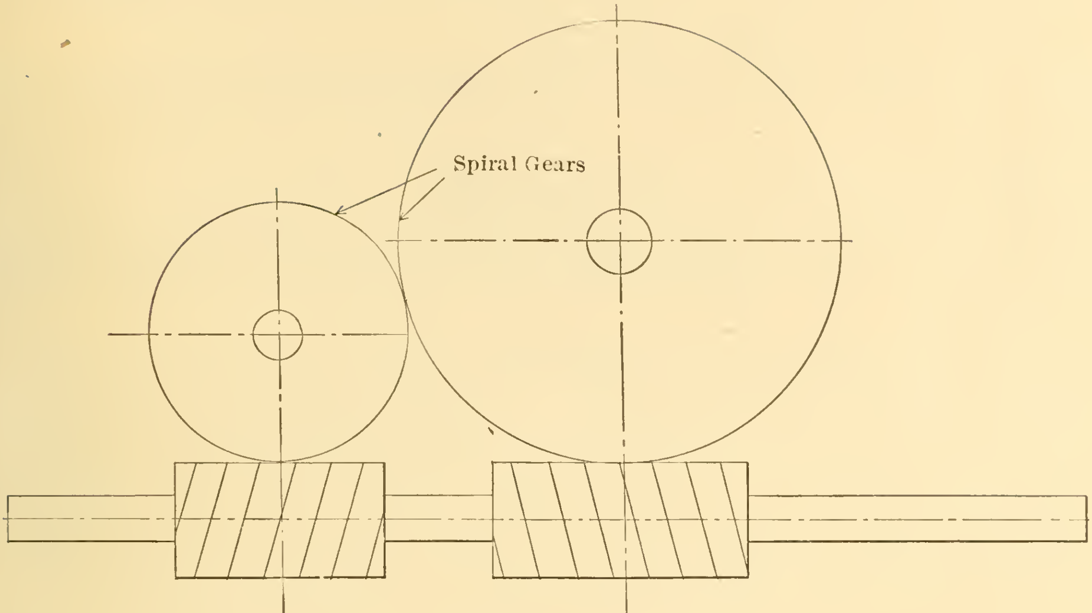


Fig. 265

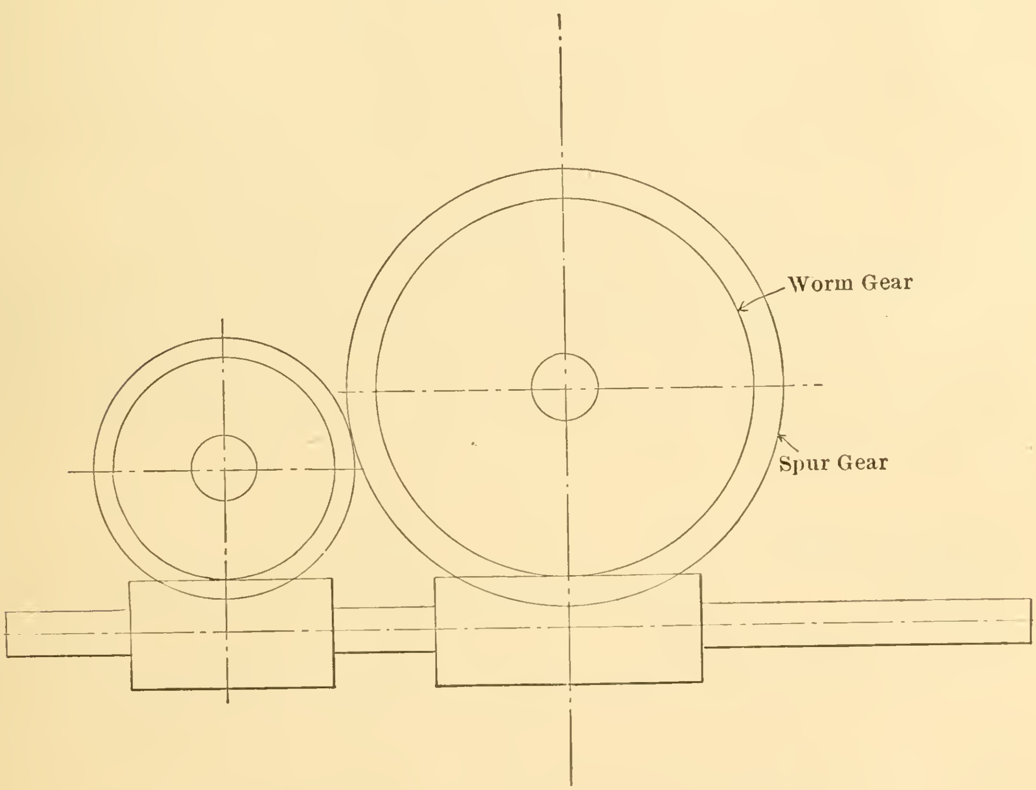


FIG. 266. ARRANGEMENT TO AVOID END THRUST IN A WORM DRIVE.

2 cutter is used, being, of course, the same for all teeth, and is equivalent to a 6-pitch on the normal part of the gear.

The indexing could also be done with a special plate having 233 holes and moving the index pin 80 holes for each cut. This worked out perfectly for this gear.

Figures 256 to 266 are a collection of odd gear drives, giving the reader a faint idea of what it is possible to produce in this line. The arrangement of worm gears in Fig. 266 is often employed to counteract the end thrust of the worm shaft.

SECTION XV

Pattern Work and Molding

The old method of molding gears from pattern is practically replaced by the more modern and accurate plan of machine molding; by which means the mold may be made with the teeth as accurately spaced and formed as if machine cut, the only drawback being the liability of the uneven shrinking of the casting, for which no possible precaution can be taken except careful molding and proper design. The amount of shrinkage in different designs and diameter is a matter of judgment and experience with each particular grade of material.

Patterns are still extensively used for casting cut gear blanks, where quite a little variation in the diameter is then allowable. It is well, however, before making pattern, to ascertain the shrinkage allowance for the foundry to which the pattern will be sent, as there is a greater variation in the shrinkage of castings coming from different foundries, especially in steel.

MOLDING A GEAR WHEEL WITH SWEEPS*

It is not infrequent that jobs like those illustrated in the accompanying drawings are performed in foundries that are engaged in molding heavy work. In point of economy in patternmaking, and also that of molding, the following method has already proved to be by far the most economical and practicable. The casting when finished will be an involute toothed gear wheel with six arms.

It is molded in a round iron flask, and the first operation is to strike off the drag part of the flask, level with its top edge; then the iron *F*, Fig. 267, with a hole for the sweep spindle is located in the center of the drag, and sweep *E* is used to strike a circle which is divided into sixths for the purpose of getting the center line for each arm.

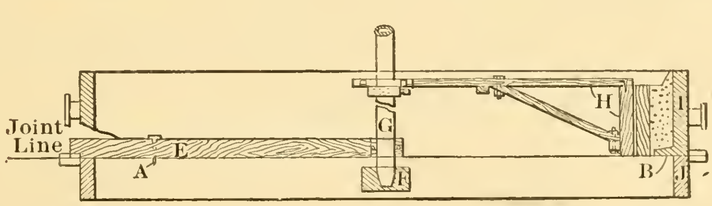


FIG. 267. SHOWING FLASKS WITH SWEEP IN POSITION.

* W. W. Carter, American Machinist.

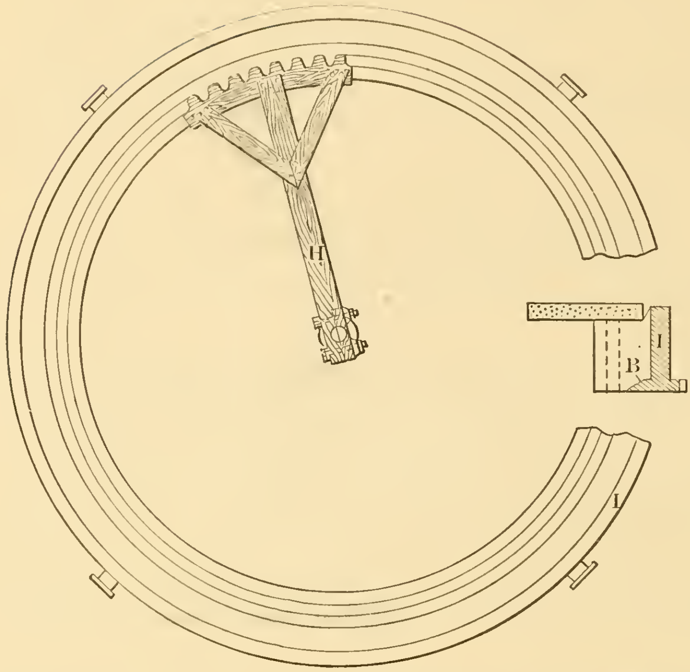


FIG. 268. PLAN VIEW OF CHEEK WITH TEETH SEGMENT IN POSITION.

Sweep *E* is then removed and sweep *H*, which contains the segment of teeth, is placed as shown in Figs. 267 and 268. The top of the drag is next supplied with parting sand and the cheek *I* is put on. This has a flange on the inside, as shown at *B*, to lift the sand surrounding the teeth. The teeth are then rammed up, or rather, in the molder's parlance, the cheek is rammed up. The matter of proceeding to ram up the cheek is, however, accompanied with more or less preliminaries relating to the correct setting of the segment sweep. This being satisfactorily accomplished the teeth on it are subsequently rammed up, each tooth being amply rodded for the purpose of supporting them when the cheek is lifted off, and then the segment is moved around and another set of teeth is rammed up, this operation being repeated until the circle is completed.

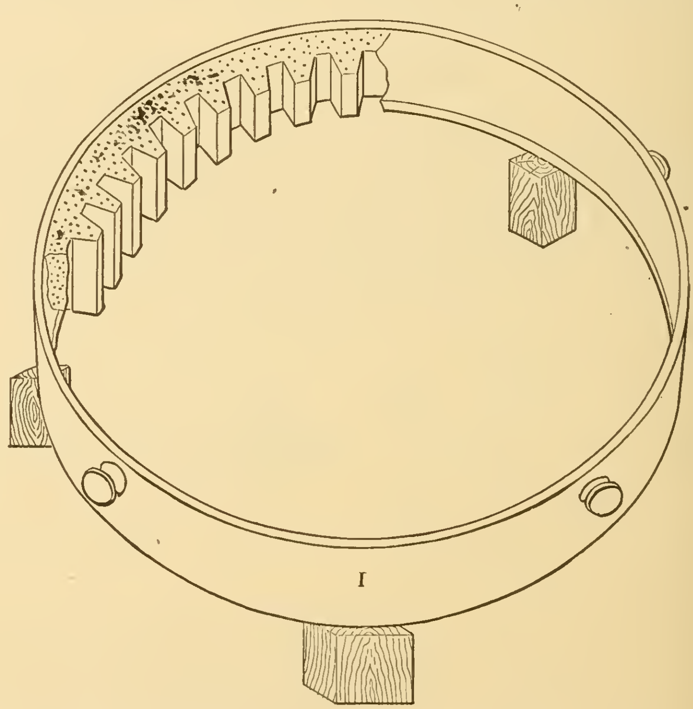


FIG. 269. PERSPECTIVE OF CHEEK RESTING ON BLOCKS.

Incidentally, in most all operations of making large gear wheels the teeth are either nailed or rodded regardless of whether they are to be cheeked off or not. There are, however, some instances where a nail or rod is not needed

at all, owing to the size and the shape of the tooth. When the job is to be cheeked off care should be always exercised regarding the distance from which the nail or rod is set to the bottom joint or flask parting; that is, a very little hanging sand should intervene between the rod and the joint.

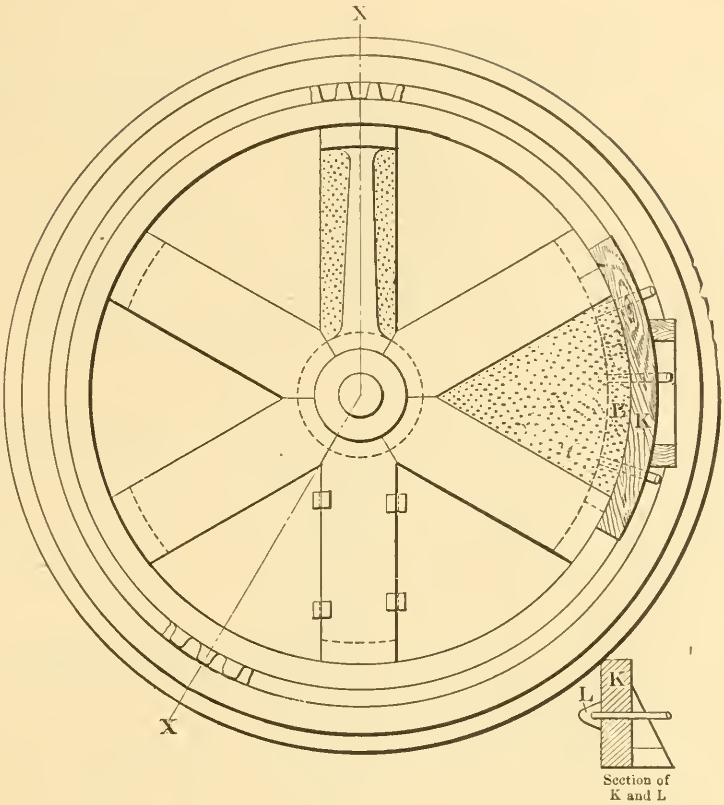


FIG. 270. ARM CORES LOCATED AND BACKING K IN PLACE FOR RAMMING SAND BETWEEN THEM.

When the cheek is rammed up, it should be lifted from the drag and lowered upon three blocks as shown in Fig. 269. The arm cores should then be swung upon the crane and lowered into position, locating them by the center lines made in the sand of the drag. These are shown in their proper location on the drag by Fig. 270. The backing board *K* with the rib *L* is then fastened to the ends of the arm cores and molding sand is rammed in between the cores to form the inside ring of the wheel, as shown in Fig. 270.

Fig. 271 shows a plan and sectional view of the core box for the arms and contains one-half of the arm pattern, two cores being made for each arm. These two cores are strengthened by means of a cast-iron arbor or crab, and when dried away they are matched together to form the arms. It is obvious that each core

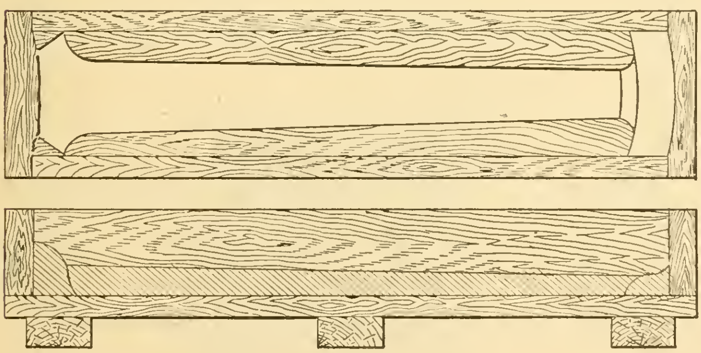


FIG. 271. PLAN AND SECTIONAL SIDE VIEW OF THE ARM CORE BOX.

is only half of an arm, but it is, nevertheless, a whole core and it is made and handled as such; therefore the two cores, being matched and clamped together, are swung up and placed as already described.

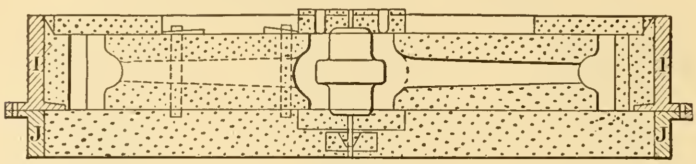


FIG. 272. SECTIONAL VIEW OF MOLD ON LINE X X OF FIG. 270.

When all the intervening spaces between the arm cores are rammed up, the different parts of the mold are given a swabbing of plumbago in solution with syrup water and a subsequent skin drying; then they are assembled, or, as the molder would say, the mold is closed and the covering cores at the hub and periphery are properly set, after which the mold is clamped and made ready to receive the metal. A sectional view of the closed mold, through the line X X of Fig. 270, is shown by Fig. 272.

MAKING A HERRING-BONE PINION PATTERN*

The finished pinion is shown in Fig. 273.

Fig. 274 shows the completed pattern in part section. The crown of the pattern was built up of segments, as shown, with a recessed parting line at the

* G. F. Dodge.

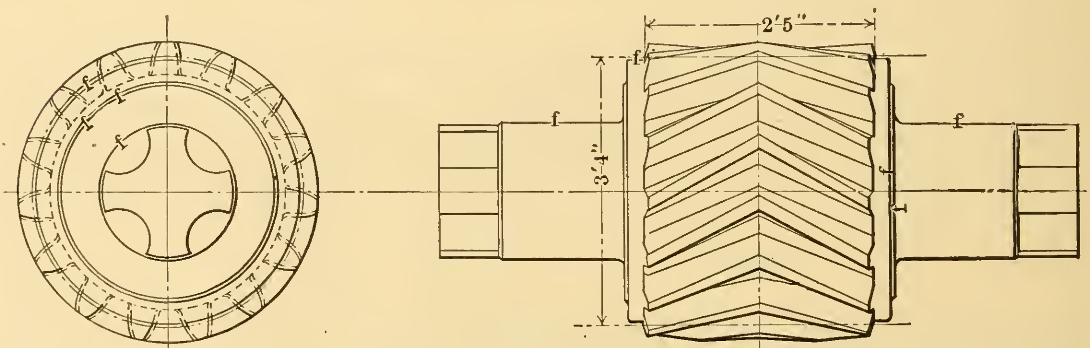


FIG. 273. THE GEAR TO BE MADE.

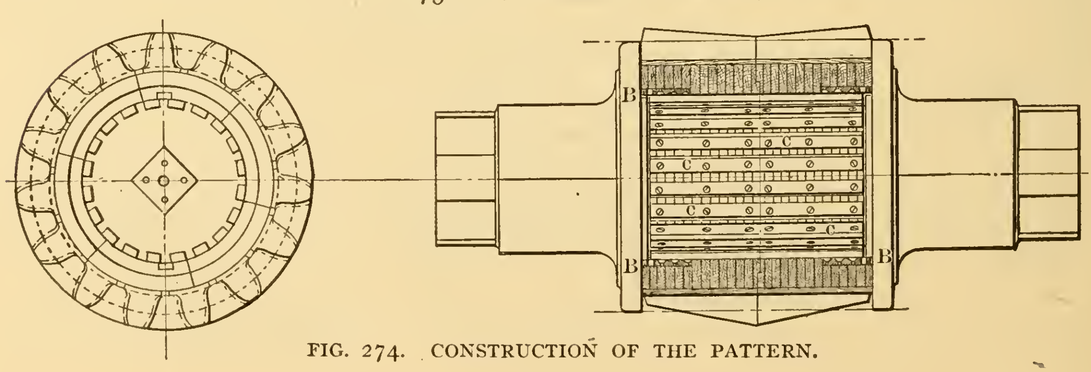


FIG. 274. CONSTRUCTION OF THE PATTERN.

center, and turned one inch smaller in diameter than the root diameter of the pinion. The necks at each end were recessed into the crown, which insured the alignment of the completed pattern. *B B* are lifting eyes for the halves of the crown, and *C C* are hardwood stiffening strips screwed to the inside of the pattern.

Fig. 275 shows the method of fitting the teeth to the crown. The block

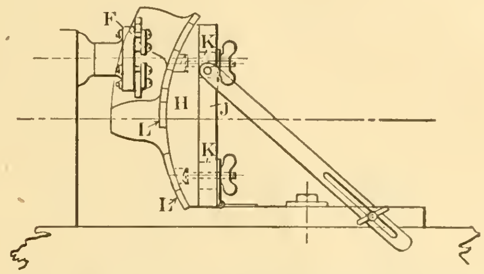


FIG. 276. JIG FOR FORMING TOOTH PROFILE.

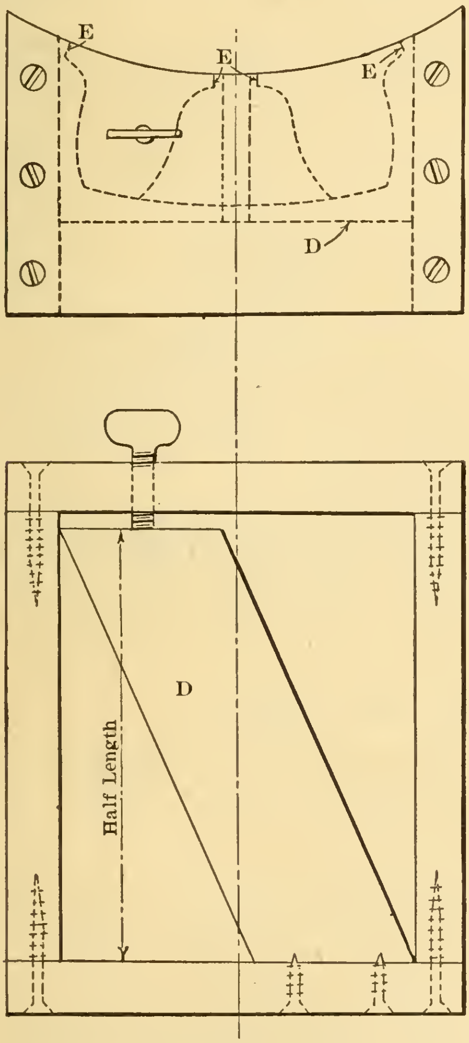


FIG. 275. JIG FOR SAWING SEAL ON CROWN.

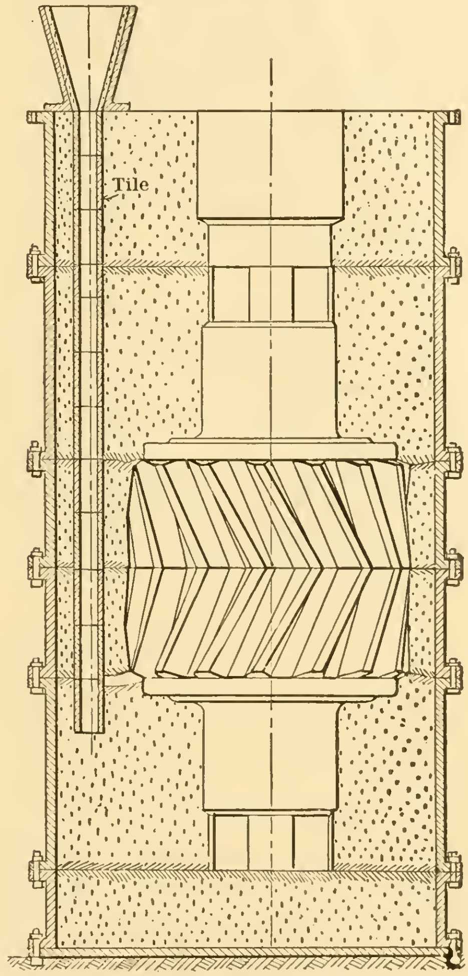


FIG. 277. THE PATTERN IN THE MOLD.

D, of such dimensions as to contain the outline of the tooth (shown in dotted lines), was gotten out to the exact half length of the pattern and placed in the box, where it was held by the thumb-screw and the points of the two screws

at the bottom, while the back was cut to the correct radius on the bandsaw. The edges *E* of the blanks were then carefully laid out and finished by hand.

Fig. 276 shows the most interesting operation; that of forming the outline of the tooth along its winding length. *F* represents the face-plate of the pattern lathe with two fly cutters securely bolted to it. *H* is a segment of an equal crown bolted to the adjustable angle plate *J*, which was in turn bolted to the lathe bed, the crown *H* being adjustable vertically by means of the slots *K K*. Upon this crown were fastened the guide strips *L L* so as to just fit the edges *E E* of the blanks, Fig. 275, and hold them just as they would set on the half crown of the pattern.

With this arrangement it was then only necessary that the tooth outline be carefully laid out on the ends of a tooth and this tooth placed in the guides, adjusting the angle plate and crown segment so that the cutters just came to the outline and clamping tightly. All teeth for that half of the pattern then had a cut taken along them by feeding them along the guide strips past the cutters when a readjustment was made and another cut taken. The whole outline of the tooth was thus finished so close to form that a slight application of sandpaper made a first-class job. The teeth were then screwed to the crown and the spaces between filled in with strips of the right thickness.

It was found after the job was completed that over two weeks' time had been saved by fitting up as per Fig. 276, as it took less than two days to make the fixtures and finish the teeth ready to screw in place.

Fig. 277 shows a sectional view of the completed mold before drawing the pattern. All such pinions are made of open-hearth steel and molded in a similar manner, the flasks being circular in form with an enlargement along one side to accommodate the runner.

MOLDING SPIRAL GEARS AND WORMS *

In textile machinery foundries in this country molding spiral gears is an every-day occurrence, and ordinary worms, single or multiple threaded, also are molded in the same manner.

Spiral gears are rammed up exactly the same as if they were spur gears; but they are drawn from the sand with a twisting motion, and worms, instead of being parted longitudinally, are molded end up and screwed out of the sand as though from a nut. It is not usual to taper the thread or teeth. With the fine sand which is used for "facing," the castings come out almost perfect, and in cases where extreme accuracy is not needed, are quite as good as cut gears.

In several shops for molding the heavier class of gears—which could not,

* F. W. Shaw.

owing to their weight, be steadily drawn—they have an appliance as shown in sketches herewith.

The molding boxes or flasks are planed on the joints, the lugs of the bottom part being flush with the joint, in order that the bracket *A*, Fig. 278, may have a secure foundation. This bracket has fitted into it a screw of the same pitch as that of the spiral gear or the worm, as the case may be, and the nut is of soft metal cast around the screw. The end of the screw is tapered in order that it may be forced into the taper hole in the pattern sufficiently tight to draw the pattern upon turning the hand-wheel.

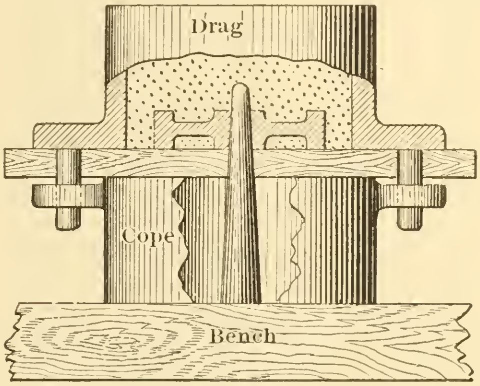


FIG. 279. THE MOLDING OPERATION.

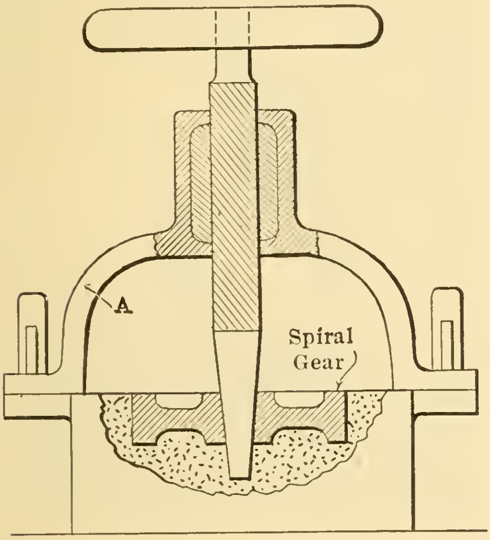


FIG. 278. RIG FOR DRAWING A SPIRAL GEAR PATTERN.

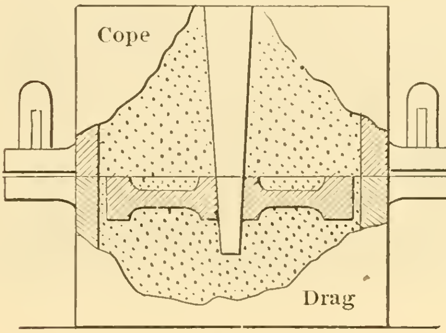


FIG. 280. FINAL OPERATION.

To insure that the drawing tackle be located correctly every time, all the boxes are made interchangeable and the pattern is fixed on a board, as shown at Fig. 279, the taper plug forming a point in addition to being a guide. The whole flask is turned over and the cope rammed up, as in Fig. 280, and the taper plug is withdrawn. After lifting off the top part the appliance for drawing is fixed in position and cottered on after lowering the screw and forcing the taper end into the central hole. It will be noted that a little clearance is allowed in the pin holes to permit of adjustment.

CUTTING BEVEL GEAR TEETH ON THE BAND SAW*

Halftone Fig. 281, which will explain itself, showing the bandsaw as a bevel gear cutter. The cutting edge of the saw runs through the center of the ball, and the center of the ball is at the intersection of the center lines of the gear and pinion shafts. The teeth are laid out upon the outer edge of the gear in the usual way and the operator has only to work to the lines. Gears have

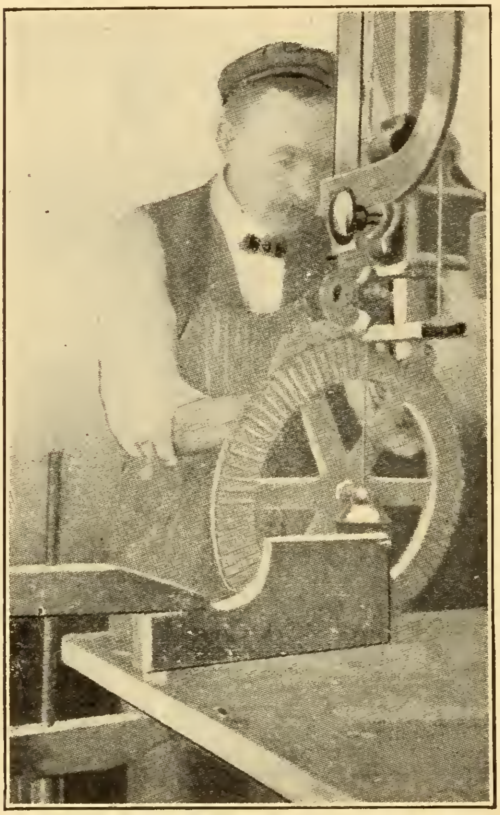


FIG. 281. CUTTING BEVEL GEARS ON A BAND SAW.

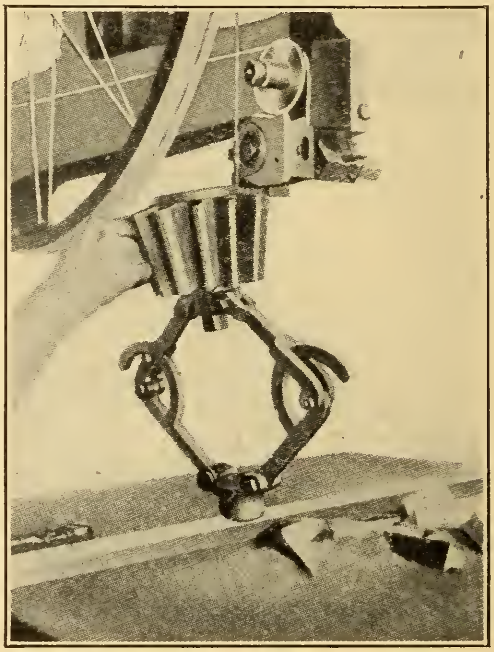


FIG. 282. ADJUSTABLE ATTACHMENT OF SAWING OUT BEVEL GEAR PATTERNS.

been cut up to 36 inches in diameter in this way, so that only sandpapering was required to finish the teeth.

ADJUSTABLE ATTACHMENT FOR SAWING BEVEL GEAR TEETH †

Considerable amount of discussion has taken place in various mechanical journals from time to time on the subject of forming bevel-gear wheel patterns on the band saw by a jig in the form of a ball-and-socket section. Nothing, however, had been said, so far as I know, concerning an adjustable jig, which may be used in sawing out the teeth of wheels of various sizes and angles. In the accompanying halftone, Fig. 282, is presented a method by which this can be successfully accomplished and trust it will be of interest.

* J. L. Gard. † Allen E. Owen.

The method is here shown on the band-saw table in operation. The jig proper is operated on a ball, with the cutting edge of the band saw passing through its center, and can be raised or lowered, as the case may be, to obtain the desired angle by pivot points and set-screws, and is attached to the gear-wheel pattern by a revolving pin.

It is, of course, necessary to use a little care and precision in operating this jig, but by its use not only a quick and accurate job can be turned out, but an appreciable saving in time and material can be effected.

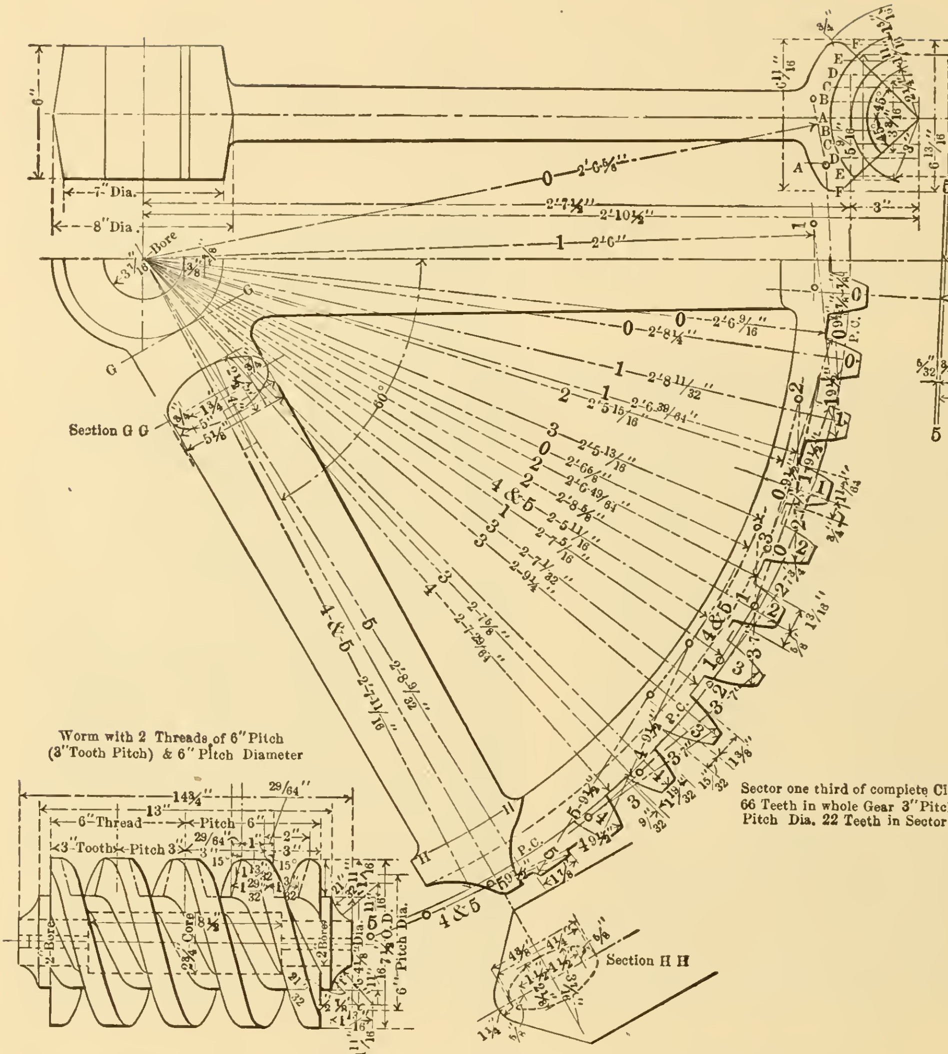
A METHOD OF MAKING A TEMPLET FOR WORM GEAR TOOTH PATTERNS *

Starting with the theory that if a tooth section for one of a pair of mating gears be chosen, a tooth section for the other may be developed from it which will work properly with it, the rest is easy. The theorem is practically at the foundation of all gear designs and does not need to be elaborated here.

A section of the central plane of the worm and gear is chosen at will. In this case the worm-thread side angle was taken as 15 degrees, the dimensions of the thread being as shown in Fig. 283. This being established, the involute tooth to match it is easily obtained by various methods, but in the present case it was obtained in the same way as all the other gear tooth sections.

If we imagine a series of sections parallel to the central plane to be made through the worm and gear, we shall get sections from the worm based on the original tooth section, but varying in increasing degree from it as we get further away from the center. To obtain these sections is a matter of ordinary descriptive geometry, but a brief explanation may not be amiss. In Fig. 285 are shown a series of circles in full lines *A*, *B*, *C*, *D*, in which *A* is the outside diameter of the worm thread and *D* is the base diameter. *B* and *C* are so spaced as to divide the total distance from *A* to *D* into equal intervals. On the longitudinal section, Fig. 286, the right-hand half of the worm-thread section is shown at five spaces, which are the correct tooth spaces of the worm. These are shown in broken and dotted lines. The first one at the left, the right-hand line of the tooth space is also added in the same kind of lines. Parallel to this is drawn a line A_0—D_0, which gives the tooth space as it would be if there were no clearance, this space being $A_0\ D_0\ D_0\ A_0$. It is obvious that we need the tooth space with the clearance *deducted*, so as to generate the tooth to its correct size and not have any corrections for clearance to make after the sections are developed. A series of helices are now drawn corresponding to the circles *A*, *B*, *C*, *D*. These helices are lettered *aa*, *bb*, *cc*, *dd*. To prevent confusion, they are drawn in successive adjacent tooth spaces, and are so

* John J. Smith.



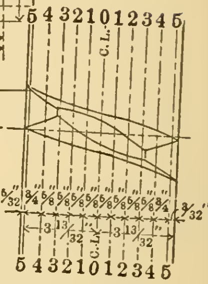


FIG. 283A

FIG. 283. WORM-GEAR SECTOR AND WORM.

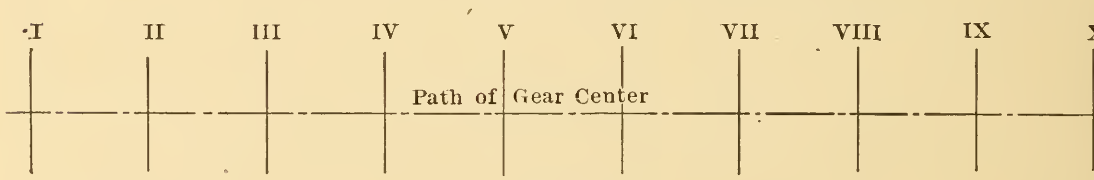
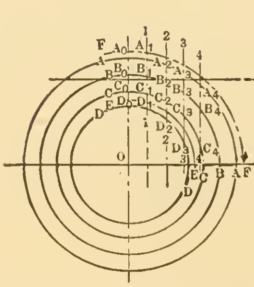


FIG. 284.



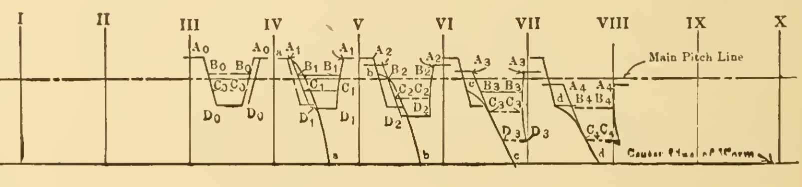


FIG. 285. ASSUMED SECTIONS OF WORM. FIG. 286. OBTAINED SECTIONS OF GEAR.

located axially as to be in the helical side surface of the tooth. In other words, the points a, b, c, d are the tops or vertices of the successive helices corresponding to the circles mentioned and lying *in* the surface of the worm teeth. Parallel sections, chosen equidistant for convenience, are now drawn, beginning with 0—0, the center line, and numbered 1—1, 2—2, 3—3, 4—4. Their intersections with the circles A, B, C, D are lettered A_0, B_0, A_1, B_1, and so on. Denoting the first tooth section from the center by the subscript 1 in each case, the second with the subscript 2, and so on, we have these sections in the different tooth spaces of the worm passing from left to right successively. Their method of development is simple. It will be noticed that center lines are shown which are the centers of the central section of the worm tooth, numbered successively with Roman numerals from I to X; that at the left of the section A_0 D_0 D_0 A_0 being III; that for the section 1, IV, and so on. The intersection A_1, Fig. 285, is projected over the helix aa, and the point A_1, Fig. 286, is obviously the top point of this tooth section. The point B_1 is projected over the helix bb, and the horizontal distance from the center line V to this point is carried with dividers and set off on the same horizontal line, continued from the center line IV. This gives the second point of the No. 1 section denoted by the point B_1 of Fig. 286. The point C_1, Fig. 285, is projected over to the helix cc, and its distance on horizontal line from center line VI is similarly set off on the same horizontal line, prolonged from IV, giving point C_1 of Fig. 286. Point D_1 of Fig. 285 is similarly projected over to helix dd, and its horizontal distance from center line VII is set off on the same horizontal line, prolonged from center line IV, giving point D_1.

As the longitudinal distance parallel to the axis, from one tooth face to another, across the space, is the same on any given radius, the distance B_0—B_0 of the central section is obviously the distance from tooth face to tooth face for *all* the sections on the circle B, and similarly with all the other circles. Therefore from the points derived in the manner just explained, the distances A_0—A_0, B_0—B_0, C_0—C_0, and D_0—D_0 are measured off to the right from the left-hand points A_1, B_1, C_1, D_1, just obtained, and the corresponding section for the right-hand side of the tooth space for the No. 1 section is derived by drawing a smooth curve through them, as, of course, has already been done for the left-hand side. In the same way sections 2, 3, and 4 are developed, corresponding to the section planes bearing the same numbers, there being no points D_4 on the No. 4 section, because No. 4 plane does not cut D circle.

FINDING THE GEAR SECTIONS

We have now a series of sections through the worm, parallel with the central plane, and we can take these as the spaces of teeth and develop the teeth which will mesh with them by the rolling method. This is most easily done by the use of a piece of tracing cloth (Fig. 287). First we draw, parallel with

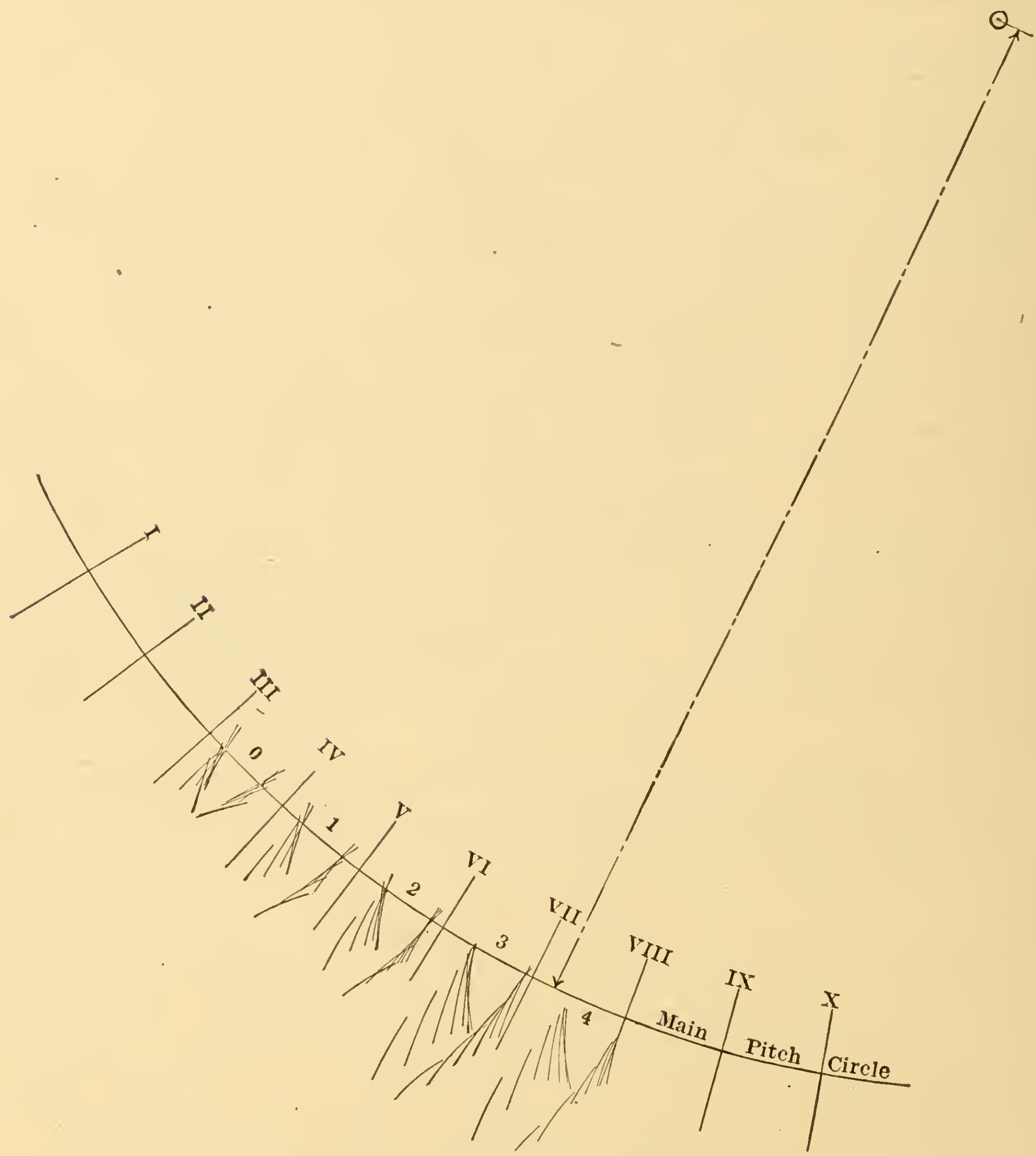


FIG. 287. LAYING OUT THE GEAR SECTIONS.

the center line of the worm, a line marked in Fig. 284, "path of gear center," which is exactly the center distance of the worm and gear from the center line of the worm. On it we project up the tooth centers numbered in Roman

numerals from I to X, those above being numbered to correspond with those below. It has been stated before, but it will do no harm to repeat for emphasis, that these center lines are exactly the pitch distance 3 inches apart. On the tracing cloth (Fig. 287) we put a center and draw a portion of the circle of the same radius as the pitch circle of the worm, marked on Fig. 287 "Main Pitch Circle," and on this we locate a series of points the exact pitch distance apart and draw radial lines through them. These are similarly numbered with Roman numerals from I to X. We now put the point marked on the tracing "Gear Center" on the line marked "path of gear center," Fig. 284, and at the intersection of vertical tooth center I, at the same time putting radial line *I* to correspond with the lower center line *I* on Fig. 286. Then on the tracing cloth draw the side lines of tooth section A_0 D_0 D_0 A_0. This comes just to the right of radial line III on the tracing. Now put gear center (Fig. 287) on line II and bring radial line II to correspond with this below. Draw on the tracing cloth the new position of tooth section A_0 D_0 D_0 A_0, and this time as section No. 1 comes within reasonable distance of the pitch circle we draw lines A_1—D_1, D_1—A_1 also on the tracing at the same setting, this bringing it to the right of tooth center IV. The next time the gear center and radial line III are made to coincide with vertical line III, and the sections 0, 1, and 2 are each drawn on the tracing. The next time section No. 3 is added, and the following time section No. 4. The process is thus continued until the gear center has passed so far to the right that all the tooth sections, if drawn on the tracing cloth, would be completely outside of the tooth spaces on the tracing, which have now taken definite shape. We now have apparently only to fit a curve to the series of lines drawn on Fig. 287 for each of the sections, so as to make what mathematicians call "an envelope," and we shall have what we started out to get, the gear tooth sections corresponding to the worm sections already developed; but just here we come to the necessity for more care than is apparent on the face of the returns.

On Fig. 285 there are drawn two dotted circles *E E* and *F F*, which are respectively the point and base of the gear teeth, and by taking the vertical distances from the line "path of gear center" to the intersections with these circles of planes 1, 2, 3, and 4, we get the tops and bottoms of the tooth sections bearing the corresponding numbers. These are now added at their proper places on the tracing, Fig. 287.

Section *O* is obviously symmetrical, and we now take its center as a reference line and proceed to make a face view of the tooth of which we have developed a series of sections. We can obviously locate a center line corresponding to the one chosen on sections 1, 2, 3, and 4 by stepping off the pitch distance from the center of section *O*. Drawing radial lines through the points

so determined, we get a base line which is equivalent to a plane passing through the center of the central section of the tooth, and through the axis of the worm gear. This, for convenience, we now keep with us. At Fig. 285, *A* is a face view of a tooth shown, based on the center line of the first tooth below the center. The way in which this is made, by simply measuring the distance of the top and bottom of the corresponding sections from the main center line and carrying them with dividers over to Fig. 283*a*, is obvious.

It should be stated for the guidance of others that arcs of the circles were chosen as the envelopes for the curves in the different tooth sections, and these were fitted to the tooth sections before this face view was made, and arcs being prolonged at the base to intersect with the base line of the respective sections. All reasonable care was employed in fitting these arcs to the sections, and they were then assumed to be the proper section; but when the points for Fig. 283*a* were taken from these, smooth curves could by no means be drawn through them; in fact, quite the contrary was the case, and it was necessary to do this work of fitting the arcs to the developed sections a second time with greater care and with due regard for which way the points involved must move in order to give a smooth section. It was found, when this was done, that the radii and base circles for these arcs formed a sort of series in which the progression was quite regular when the face view of the tooth finally became an area of smooth curves. It should be stated that the original angle of contact of the worm teeth was taken as 120 degrees, but this was done almost through inadvertence and subsequently seen to be entirely wrong, being then reduced to 90 degrees, as shown; but before this was done a fifth section plane was used to get the base of the unnecessarily wide teeth given by 120 degrees contact angle. This section, No. 5, was actually not used at all.

LAYING OUT THE TEMPLETS

Having now the corrected series of tooth sections, the question was how to translate them into something that the patternmaker could use to produce the actual teeth, and then how the teeth patterns so produced could be used to the best advantage in molding the gear. The solution is the only real novel portion of this article, and is comparatively simple. The sections 0, 1, 2, 3, and 4 were laid off on tin, with certain reference lines to enable the different tin templets to be properly located with relation to one another, and the individual templets were then stacked up with distance pieces of wood between them at the back end, and bolted together in their proper relative positions. The thickness of the distance pieces was made exactly the same as the distance between consecutive sections, less the thickness of the tin itself. The complete templet is shown in Fig. 288.

Fig. 289. A T-shaped board was made with the top surface flush and smooth. A tin center for the beam compasses was put near the end of the stem of the T. The successive tin blanks, on which the sections were to be laid out, were placed at the other end. They were pieces of tin about 5 by 6 inches, and to hold them in place a small portion of each corner was bent down square about ½ inch and the points so made driven into the soft wood of the layout board. On the board, first of all, the main pitch circle was drawn and radial center lines of two adjacent teeth (the pitch distance apart, of course, on the pitch circle). In addition to these lines a reference circle was drawn 4 or 5 inches larger in radius than the main pitch circle. The successive pieces of tin were fastened down on the board, the main pitch circle drawn on them and the center lines of the two adjacent teeth. Then, from the drawing (Fig. 283), as given herewith, the tooth sections were laid out, sides, top, and bottom. The base circle and the center lines of teeth were plainly scribed on each templet, which was then cut out as shown by the heavy lines in the sketch, with the reference circle as a base at the back and the teeth in front.

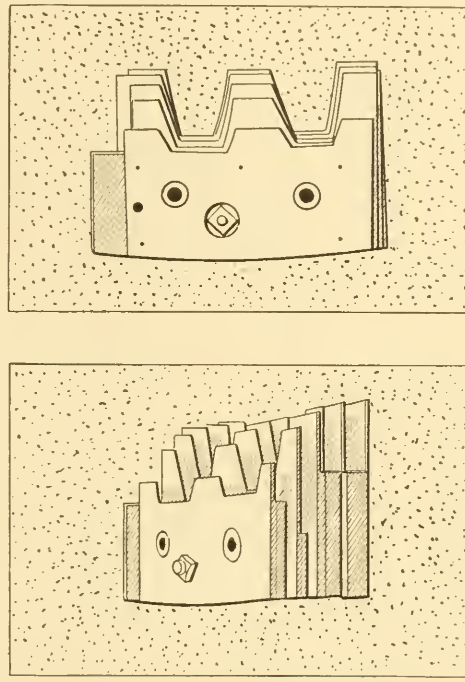


FIG. 288. THE TEMPLETS.

When these were done, it was only necessary to make distance blocks of the required thickness, curve the backs of all of them to the radius of the reference circle, and scribe squarely across the backs thus made two lines whose distance apart was that of the two center lines of teeth measured on the reference circle. Then by stacking the templets and the blocks in their proper order, with the backs and these teeth centers corresponding, the templet was made which translated into solid form the sections whose development has been, I fear, so tediously described.

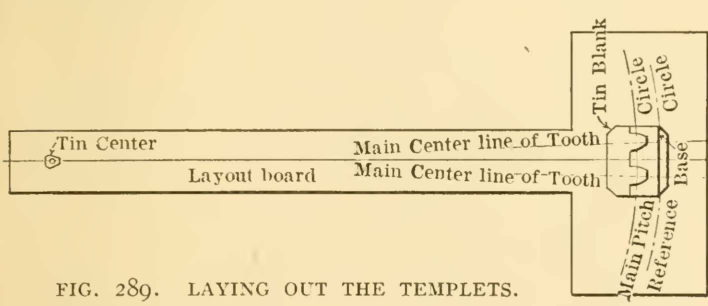


FIG. 289. LAYING OUT THE TEMPLETS.

The teeth were whittled out and shaped to the templet until they fitted all its sections, smooth curves between being secured by eye.

The templet has the great advantage of visibility, all the sections being in sight between the sheets of tin. In spite of this the job was a tedious one, but good when finally done.

Two teeth were laid out in the templet, both as a check and because, when properly located in this way on the templet, the latter could be used to space the teeth around the wheel as well as to get them out. The templet did not give the whole length of the tooth, but only half, from the center out, as it was thought that having the central plane of the templet to work to, would enable the central plane of the teeth to be accurately located in the central plane of the gear, and for the practical reason that fitting anything to a solid templet of this kind is a job whose difficulty increases about with the cube of the size of the templet. As only one gear was to be made and that only a third of a circle, and as the whittling out of these teeth to fit the templet was a fearful job, it was decided to make the pattern so that the teeth would be made in a core box, the sections of core being set around the rim of the gear in a way not unfamiliar in other classes of work. Accordingly, such a core box was made containing two teeth, a photograph of it being shown in Fig. 290. Of course the gear pattern carried a core print in rectangular form projecting out from it, as shown in broken and dotted lines on the axial section of the gear in Fig. 283.

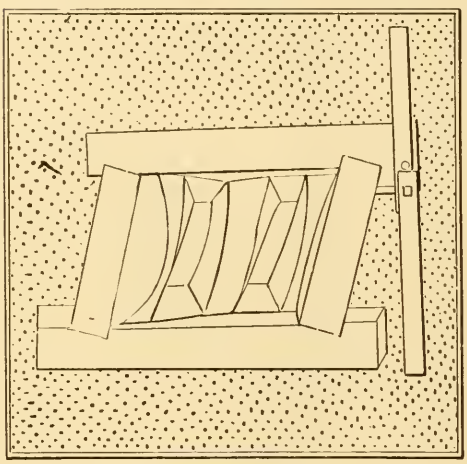


FIG. 290. THE CORE BOX.

It is not to be denied that this process, as a whole, is tedious and requires a great deal of time. Even after the drawing is made, it is not improbable that the draftsman will be required to lay out the tooth templets for the pattern maker. In fact, unless the latter is exceptional, this is much the best way to do. The drafting work itself requires great care and accuracy, if decent results are to be produced, and it is doubtful if the best of draftsmen could put through such a layout complete in less than three or four days and get a satisfactory job of it.

MACHINE-MOLDED TEETH ON LARGE CAST GEARS

These methods and machines herein described were developed and patented by George Mesta, the president of the Mesta Machine Company, and

they are used exclusively at the Homestead works of that company. The idea was to approach as closely as possible to the accuracy of machine-cut gears, but by casting methods.

It may be well to state some of the objections to large wooden gear patterns and thereby show more plainly by contrast the results obtained by these machine-molding methods. As the wooden pattern must have draft, the teeth are of necessity of a varying thickness and cannot mesh with full contact for the entire length of the tooth when two such gears are first put together. For this reason the entire load of the pinion or gear may be carried at one end of the tooth until the teeth are worn to a bearing. Again, wooden patterns in storage or in use, subjected to changes of temperature and humidity, are apt to distort, and thereby is destroyed their accuracy and the accuracy of castings made therefrom. The question of clearance is another important point in connection with gearing and gear teeth, and this must be greater in a gear molded from a solid wooden pattern with the rapping which such a pattern must necessarily undergo, than in a gear machine molded.

As the teeth on a solid wooden gear pattern are individual pieces of wood formed separately and spaced around a central core, it is not fair to assume that there is the degree of accuracy either in the teeth themselves or in their spaces, which we would expect to find if the division of the mold or pattern was made by accurate machine methods. Other minor points in connection with these large wooden gear patterns are that they are expensive, require a great deal of lumber to make, and are unhandy to store because of their size.

The aim of this method is to do away as far as possible with these objections and to produce cast-iron, steel or bronze gears accurate enough to compare favorably with machine-cut gears. No one would attempt to say that these conditions had all been met, yet the results and methods are of interest.

THE MOLDING MACHINES

The machines at first glance call to mind a boring mill. They have a revolving table, housings, cross rail and head, but at this point the direct comparison ends, for they are hand—not power—operated. There are three machines in use. The smallest has a molding capacity from 4 inches in diameter to 48 inches in diameter. The intermediate size has a capacity from 48 inches in diameter to 66 inches in diameter, while the largest machine has a capacity from 66 inches to 156 inches in diameter. These diameters, of course, refer to the size of gear which can be molded. Fig. 291 shows the largest machine at work upon a mold for a cast steel herringbone gear of the following dimensions: 120.97 inches pitch diameter, 4 inches pitch, 12 inches face and 95 teeth.

The machine itself consists of a heavy base carrying a large circular table.

This table is so fixed and has such adjustments that it can be at all times kept in truth with the cross rail. It is revolved by hand through a worm-wheel and worm and a set of change gears. These change gears are so arranged that the machine can be set to divide a circle into any number of parts. This is, of course, the spacing mechanism for the teeth.

The cross rail has a vertical motion on the housings to adjust it for gears of different widths of face. Upon this rail is carried a head having a vertical slide, and to this slide is attached a tooth block or tooth-space pattern for the gears. This head has a horizontal motion on the cross rail, and thus can be adjusted for any diameter of gear within the range of the machine. It also has a vertical motion to adjust it for various widths of face.

THE FLASKS

The flasks are made of cast iron and are plainly shown in place on the machines in Fig. 291. They are made in a variety of sizes, ranging from 14 inches to 160 inches in inside diameter. They are provided with trunnions and clamping lugs, uniformly located and spaced for each size of flask, so that any number of flask sections can be built up as may be required by the length of the gear face or pinion. The faces of these flasks are accurately machined, so that they will fit upon the table of the molding machines or with one another.

METHODS OF MOLDING

In order to describe the methods of molding it is necessary to describe the tooth block or tooth space pattern, which has been referred to. These blocks are shown in Fig. 291, on the lower end of the vertical head slide, but the view is rather unfortunate as but little of the detail shows. It consists of a wooden form or pattern of the space between the teeth of the gear. This is very accurately made, and but one such block having only one tooth space is necessary for each gear. It will be seen that this is a simple pattern to make.

In addition to this tooth block, in order to mold a gear, it is necessary to make a sweep to sweep up the mold to the proper depth for the face of the gear and for the ring of sand between the inside diameter of the flask and the part of the mold which is formed by the tooth block. It is also necessary to provide a core box for the cores to form the arms. The core for the center hole can usually be drawn from stock, for in general they are nothing but straight cylindrical cores. Thus it will be seen that instead of providing a large wooden pattern for a gear or a sector pattern with the necessary sweeps, in this machine-molded method it is only necessary to provide a tooth block, a sweep for the general size of the gear, and a core box for the cores for the arms.

In making a mold such as we see in process in Fig. 291, the flask is placed

upon a ramming plate and the inside of the mold is swept up to the general dimensions of the gear. The lower surface of the ramming plate is provided with grooves, corresponding with tongues or beads on the table of the molding machine. By this means the flask is accurately centered when placed in position ready for the machine-molding operation. The table of each machine is provided with a series of tongues or grooves, in order to accommodate the various sizes of flasks which are adapted to that machine.

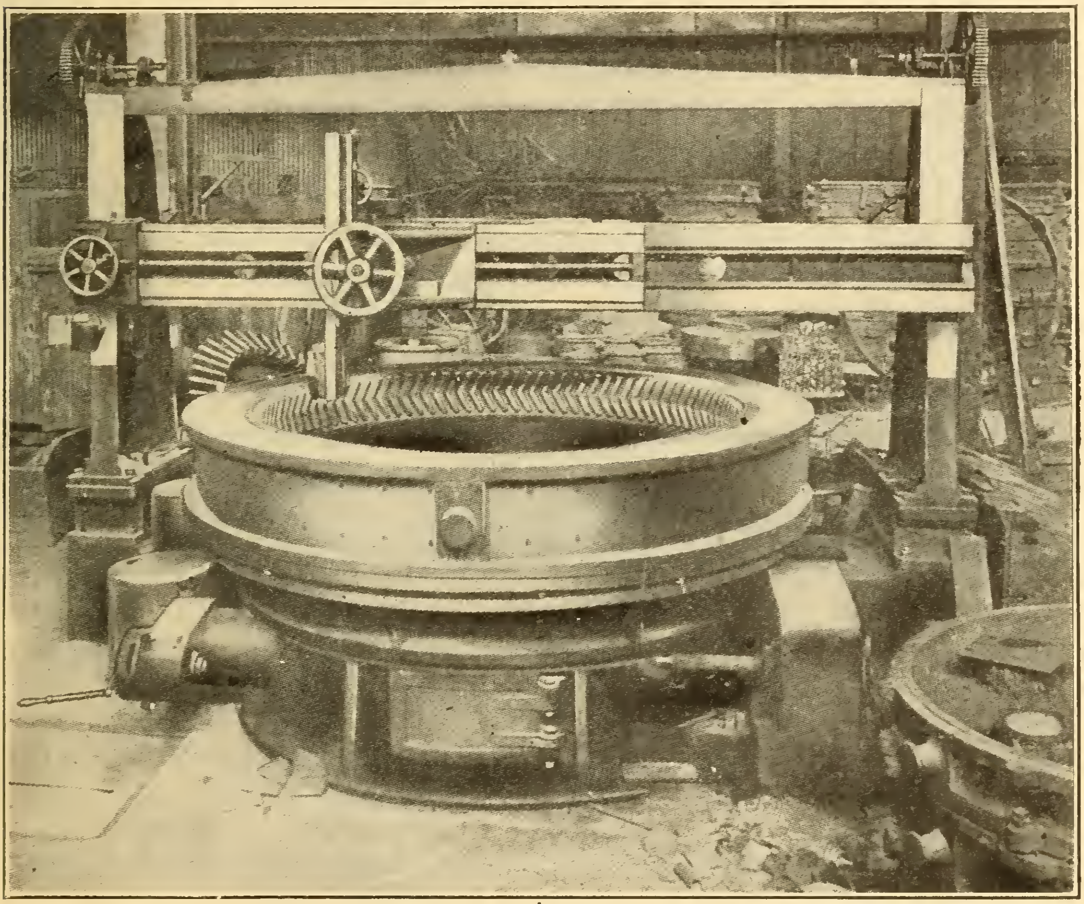


FIG. 291. THE LARGEST GEAR-MOLDING MACHINE.

With the flask in position the cross rail is lowered to a convenient distance above it. The tooth block is properly located and attached to the vertical slide carried by the head. If the mold is to be a spur or worm gear, a stop is placed on the cross rail, so that when the head moves forward to this stop the tooth block or pattern is brought into position with reference to the pitch circle, and is thus accurately and positively located. With this tooth block in its proper position the space in the block is rammed full of sand, thus forming a part of the mold which will form a tooth space in the cast gear. After one

such tooth space is molded, the tooth block is moved back horizontally in order to clear the sand which has just been rammed into position. The table of the machine is now revolved and carries the flask a certain distance, determined by the gearing of the machine. This distance is the pitch of the gear wheel being molded. The tooth block is then again moved forward into the pitch circle and another tooth space molded. These operations are repeated until all the teeth of the gear have been molded. If the gear in process is a bevel or a miter, the tooth block or pattern is raised vertically instead of being moved back horizontally.

It will be seen that as all the teeth are molded from the same pattern or tooth block, they must be alike, and as the spacing or indexing is done mechanically, that also must be accurate. A small card is always before the molder giving him the number of turns of the operating handle, in order to properly index the particular job upon which he is working. These methods are applicable to all kinds of gears, spur, bevel, worm, miter and internal. As the sand is rammed into the tooth block it is supported by means of pieces of wire or nails following usual foundry practice.

All of the gear molds made by this company, whether they are to be used for steel or iron castings, are baked in large furnaces fired by that wonderful gift of nature—natural gas. After the molding operation on the machine the mold is finished by inserting the center core and the corers to form the arms. It is then covered with stock baked cores, known as cover cores.

SECTION XVI

Suggestions for Ordering Gears

DIAMETERS

When diameter is mentioned it is understood to be pitch diameter. The pitch diameter should always be given to check the pitch, as it often happens that diametral and circular pitch are confused. If the pitch diameter cannot be given give the outside diameter.

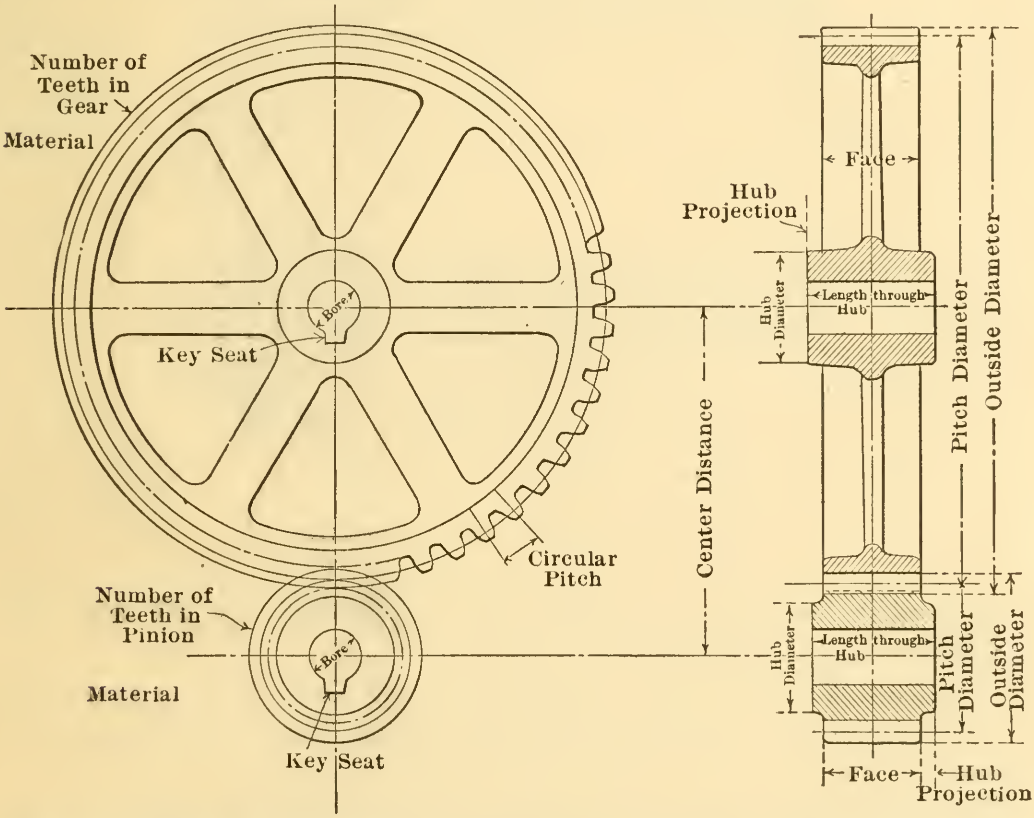


FIG. 292. NECESSARY DIMENSIONS TO COMPLETELY DESCRIBE A PAIR OF SPUR GEARS.

HUBS

Hub length and location should always be specified. If this is not given the usual practice has been to make the ends of hubs flush with face when the face is wide or the gear small in diameter. When the face is narrower and the

gear large, it is common practice to make the hub longer than the face to give it bearing on the shaft. For the heavier class of gears this hub extension is generally made equal on each side of the face; for the smaller class, say 4

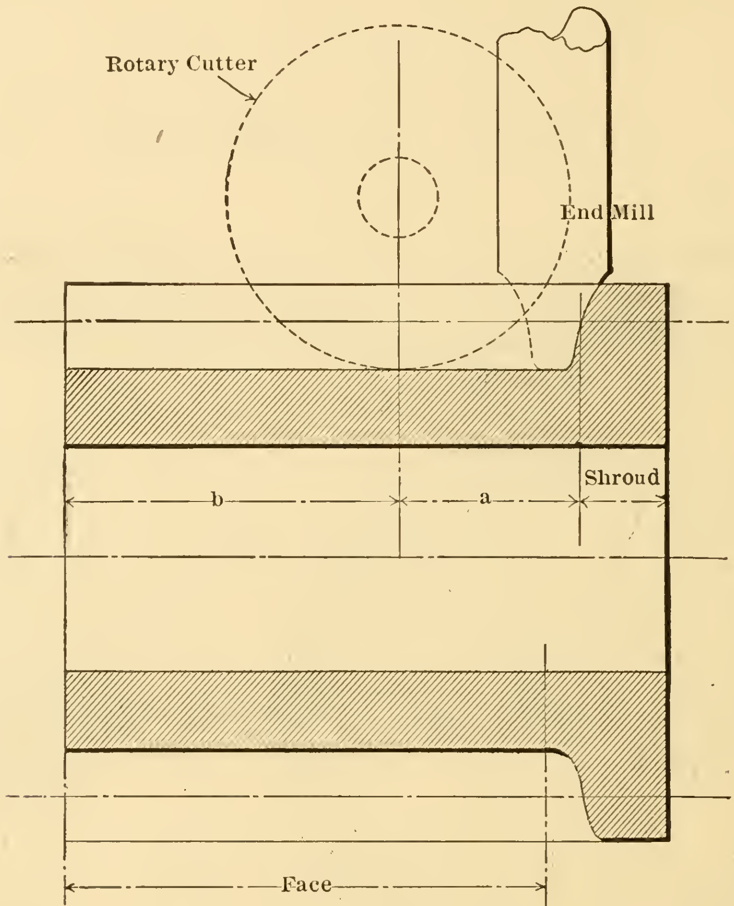


FIG. 293. CUTTING A SHROUDED GEAR.

diametral pitch and under, the hub extension is generally put all on one side to accommodate set screws, etc.

For pinions, especially those made of steel, it is always well to avoid hubs, as these add greatly to the cost, more, in fact, than if the face had been carried the length of the hub.

When preparing blanks to be cut it is important that the ends of the hub be faced true, otherwise the blank will run out when clamped on the arbor.

SHROUDED GEARS

Gear manufacturers are often asked to furnish a shrouded cut gear. This is possible only by employing an end mill, which makes a very expensive operation. Where but one side is required to be shrouded, however, the cost

will be somewhere within reason as the tooth may be milled part way by ordinary methods; the end milling being used for distance *a* in Fig. 293.

Gears are quite often furnished (as shown in Fig. 293) without finishing distance *a*, their face being limited to distance *b*.

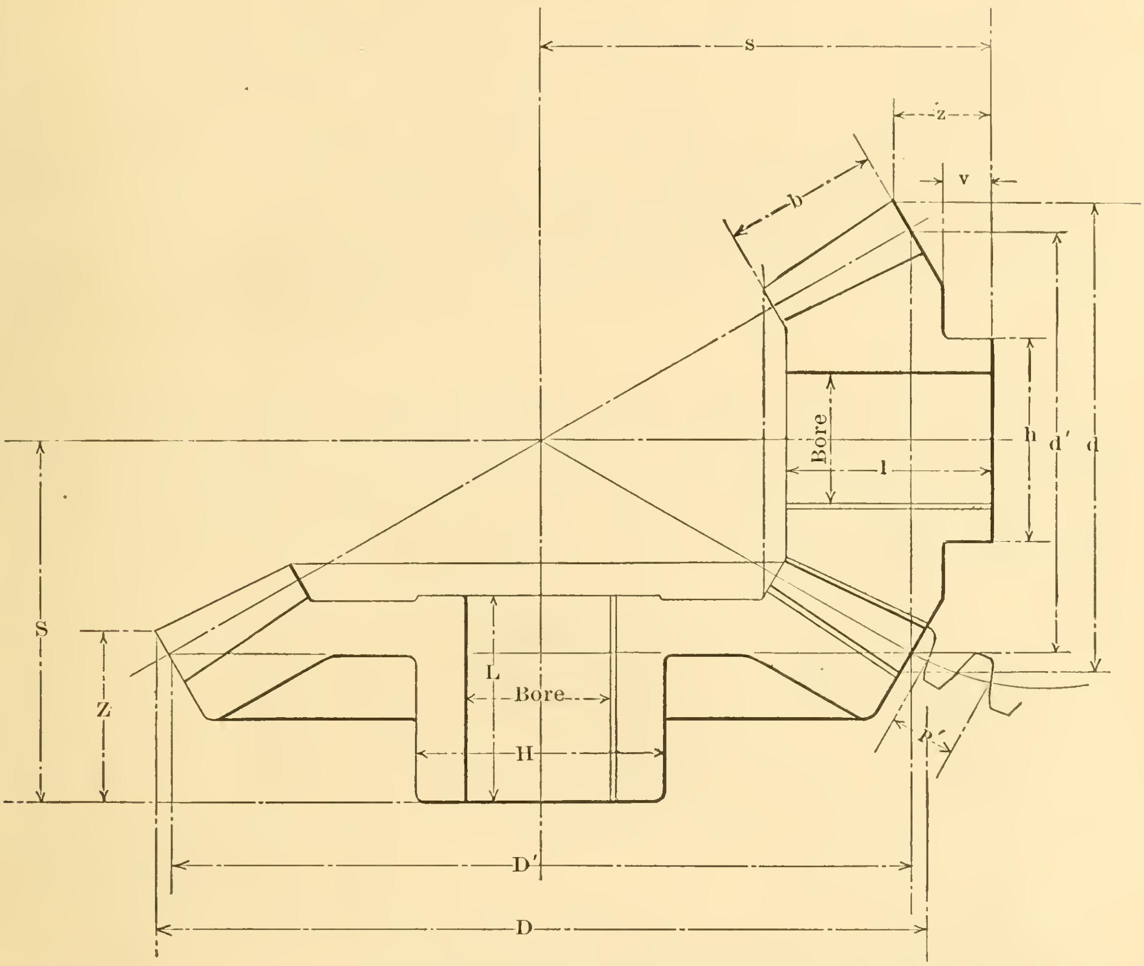


FIG. 294. NECESSARY DIMENSIONS TO COMPLETELY DESCRIBE A PAIR OF BEVEL GEARS.

BORE

The bore of a gear is generally made standard. Any allowance for a fit is understood to be made in the shaft. Gears sent to be cut should be bored, if possible, to some standard and be uniform in size, if extra charges for special bushings is to be avoided; or if the operator cutting the gears is of a saving nature and bushes up with a piece of tin, eccentric gears will be the result.

All allowances for press, shrink, or sliding fits should be specified if you would expect satisfactory results, as there are various ideas on these points.

The taper bore is rapidly coming into favor, as it is easier to machine with proper methods and insures a snug fit on the shaft and a true gear running.

KEYS

Unless otherwise specified, keyways are understood to be straight. If taper, the size given is at the small end. See pages 130 and 134 for standards.

Where hubs are unequal, or in the case of bevel gears, specify the direction from which key is to drive, although it is understood to be from the small end of a bevel gear unless otherwise specified. When two keys are required place them diametrically opposite instead of at 90 degrees.

Avoid the use of a taper keyseat; they will cause the gear to run out of true even if the bore is a snug fit unless extreme care is taken.

BEVEL GEARS

As a great many do not understand what information is necessary in describing a pair of bevel gears, and are unable to make up drawings the following list is given, which, if properly filled out, will be all that is necessary, either for a new transmission or to replace worn gears. Fig. 294 illustrates the corresponding dimensions.

Gear

Number of teeth	N
Pitch	p'
Face	b
Bore	
Pitch diameter	D'
Outside diameter	D
Backing	Z
From point of tooth to center of pinion shaft	S
Length of hub	L
Diameter of hub	H
Keyseat	
Material	

Pinion

Number of teeth	n
Pitch	p'
Face	b
Bore	
Pitch diameter	d'

Outside diameter . . . . *d*
Backing . . . . *z*
From point of tooth to center of gear shaft . . . . *s*
Length of hub . . . . *l*
Diameter of hub . . . . *h*
Keyseat . . . .
Material . . . .

In addition to these dimensions it is advisable to send a paper impression of the teeth, if replacing old gears; this can be made by laying a piece of clean paper over the large end of the teeth, which are first slightly greased or lamp-blacked. Then crease around the edges of the teeth with the end of a pencil. This is very necessary unless the pitch can be given, which is the distance from the center or edge of one tooth to the center or edge of the next—not measured at the points, but on the pitch line.

It is not necessary to make a sketch unless the gears have some special feature, simply give the list of dimensions and the paper templet as described; although a sketch is always best, as many points may come up that would otherwise have been overlooked. When ordering one gear of a pair always give the number of teeth in its mate.

When the ends of the hubs are against a bearing, or collar arranged for that purpose, the distance *S* and *s* should be accurately determined. If, however, there is some distance between the ends of hubs and the bearing, these dimensions are of no special importance. It is good practice to allow an extra ⅛ inch on the ends of the hubs, for fitting when this dimension must be accurate. State whether keyway is straight or tapered, and if tapered, from which side it drives.

It is assumed that the shafts are at right angles unless otherwise specified. When such is not the case the only additional information needed is that shaft angle, although the distance *S* and *s* should be from their point of intersection. These instructions also apply to miter gears.

The pitch diameter D' and d' in Fig. 294 should be given when it is possible. The pitch diameters are generally specified for new work, but Fig. 294 and accompanying instructions were made up in such a manner that they could be used to order replacements, therefore the outside diameter is shown and should be given in case the pitch diameter cannot be determined.

Always give all information possible when ordering replaced bevel gears on account of various corrections now made in the tooth dimensions.

Determine if any corrections in angles will be required before turning blanks to be cut, as some correction is necessary for all generated bevel gears of 14½ or 15 degrees.

It is often just as cheap or cheaper to have a small hub on the back of the bevel gear if made from a casting.

When no backing is specified the manufacturer will make same to suit his ideas, which vary. Backing should always be given so that the gears may seat against bearing; end of bearing should be finished for this purpose.

Avoid putting long hubs on front end of gear as they are often impossible to cut.

If you turn up your own gear, be sure that the distance from the point of the tooth to the end of the hub (backing) is the same in all of them. If this distance is not maintained it adds greatly to the cost of cutting. As some generating machines hold the gear from the small end, it is important, if they are to be cut on this type of machine, that the distance from the point of the tooth to the front end of hub be uniform.

If you do not understand how to turn up bevel gear blanks, send them to the manufacturer as it is expensive experience for you—also for the manufacturer—to attempt to cut gears incorrectly machined. This applies to all types of gears.

If the six arms or ribs are used in bevel gears large enough to be turned on boring mill, it will greatly reduce the turning time if small lugs are cast on the bottom of rim to engage chuck jaws, which are generally four in number. This is especially important if the ribs are extended so that the chuck jaws cannot grip the hub.

When bevel, or for that matter any other type of gears are made from a casting, it pays to add a small hub, if none is required, by which to hold the gears in the chuck; the labor turning off this hub is small in comparison to the time saved.

WORM GEARS

Before designing a worm gear it should first be determined whether a hob is obtainable that will cut the gear. Gear manufacturers will furnish such a list from which one should be selected.

When ordering a worm gear always specify what amount, if any, that centers can be shifted.

Worms should be keyseated before the thread is milled or chased if the cost is of importance. A bronze rim worm gear and a hardened steel worm is a good combination, but a hardened steel worm gear engaging a hardened steel worm is better.

Worm gears should be carefully ground after being assembled with light oil and graphite, otherwise they are very liable to wear quickly.

It is not necessary to turn out the throat radius of worm gear; a single spot in center of throat turned to the throat diameter will answer.

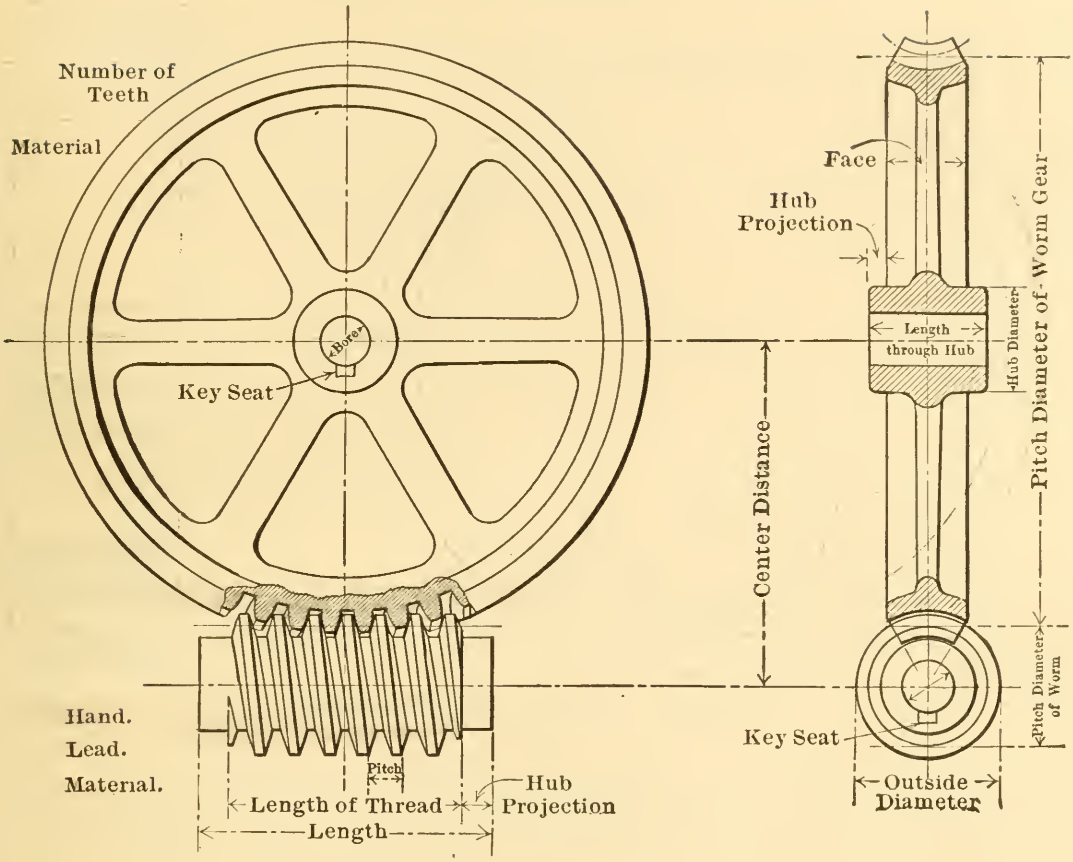


FIG. 295. NECESSARY DIMENSIONS TO DESCRIBE WORM GEARS.

SPIRAL GEARS

Whenever it is possible, have calculations for spiral gear made before the center distance is decided upon. Specify maximum or minimum diameters and center distance.

Pitch mentioned in connection with spiral or helical gear is understood to be the normal pitch, which, in circular pitch, is the shortest distance between two consecutive teeth.

The gear with the greatest angle must be the driver.

RACKS

Avoid ordering racks of an odd thickness as it often requires months to secure special sizes in cold rolled steel. The thickness over all should be in the nearest even sixteenth of an inch; if this is impractical, order rack made of forged steel planed all over. It is not practical to plane up a piece of cold rolled steel. Before designing a spiral rack make sure of the maximum angles that can be cut.

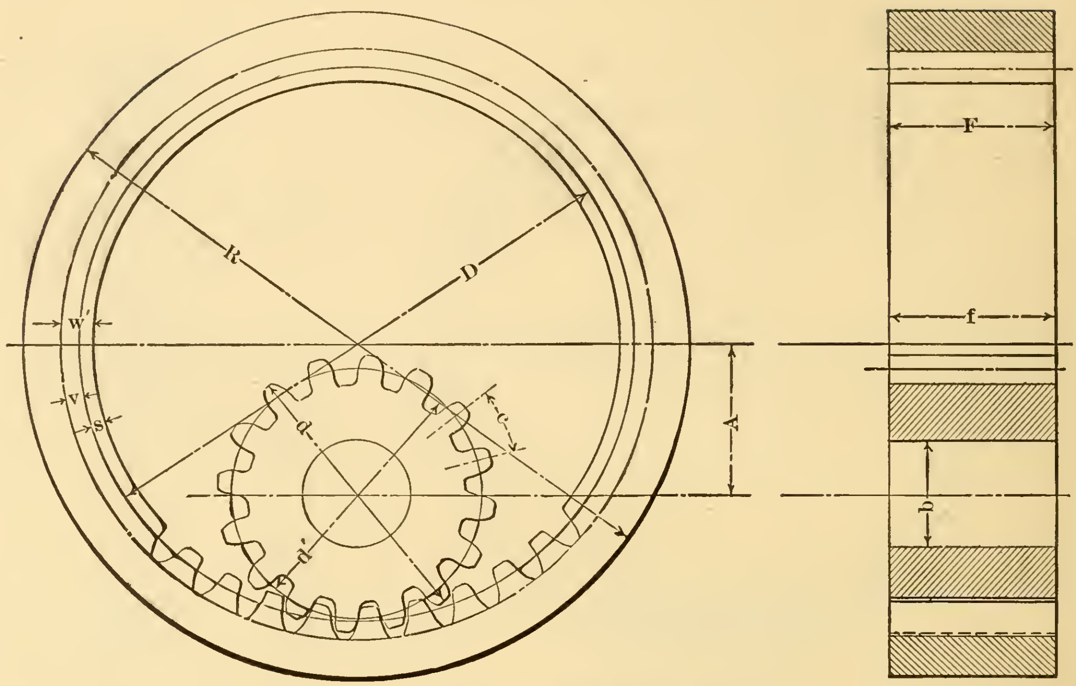


FIG. 296. INTERNAL GEAR AND PINION.

INTERNAL GEARS

If a rotary cutter must be used in cutting internal gear, they must be designed according to Fig. 297, which will be self-explanatory. A special cutter is generally required, but for 4 diametral pitch and finer and for 60 teeth and over, a regular number 1 cutter will make a very satisfactory job.

The Fellows gear shaper will cut internal gear with a minimum amount of

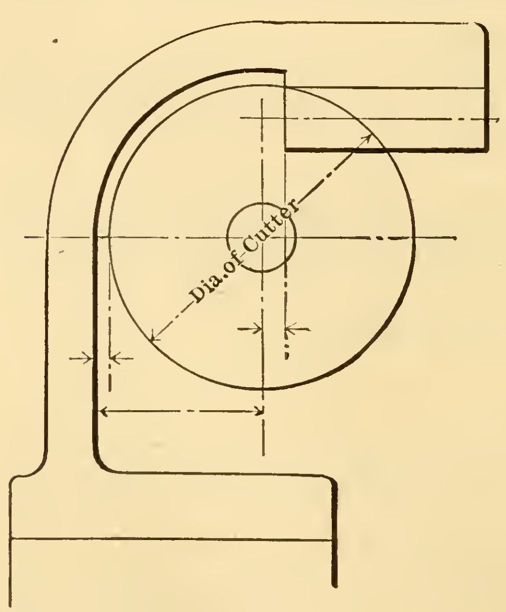


FIG. 297. CUTTING INTERNAL GEAR WITH ROTARY CUTTER.

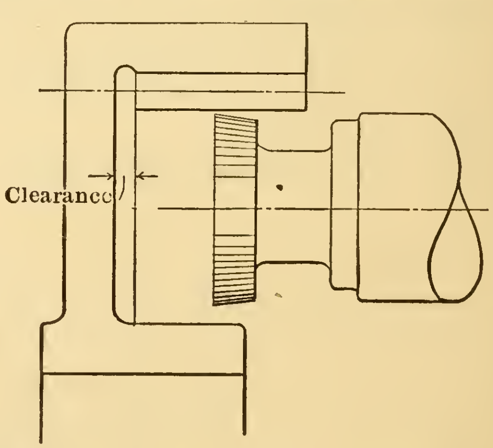


FIG. 298. CLEARANCE FOR INTERNAL GEARS.

clearance ($\frac{3}{16}$-inch). Care must be taken, however, that there is room for the cutter between the inside of the rim and the outside of the hub as illustrated in Fig. 298.

There should be at least 15 teeth difference between the number of teeth in the internal gear and the number of teeth in the engaging spur gear.

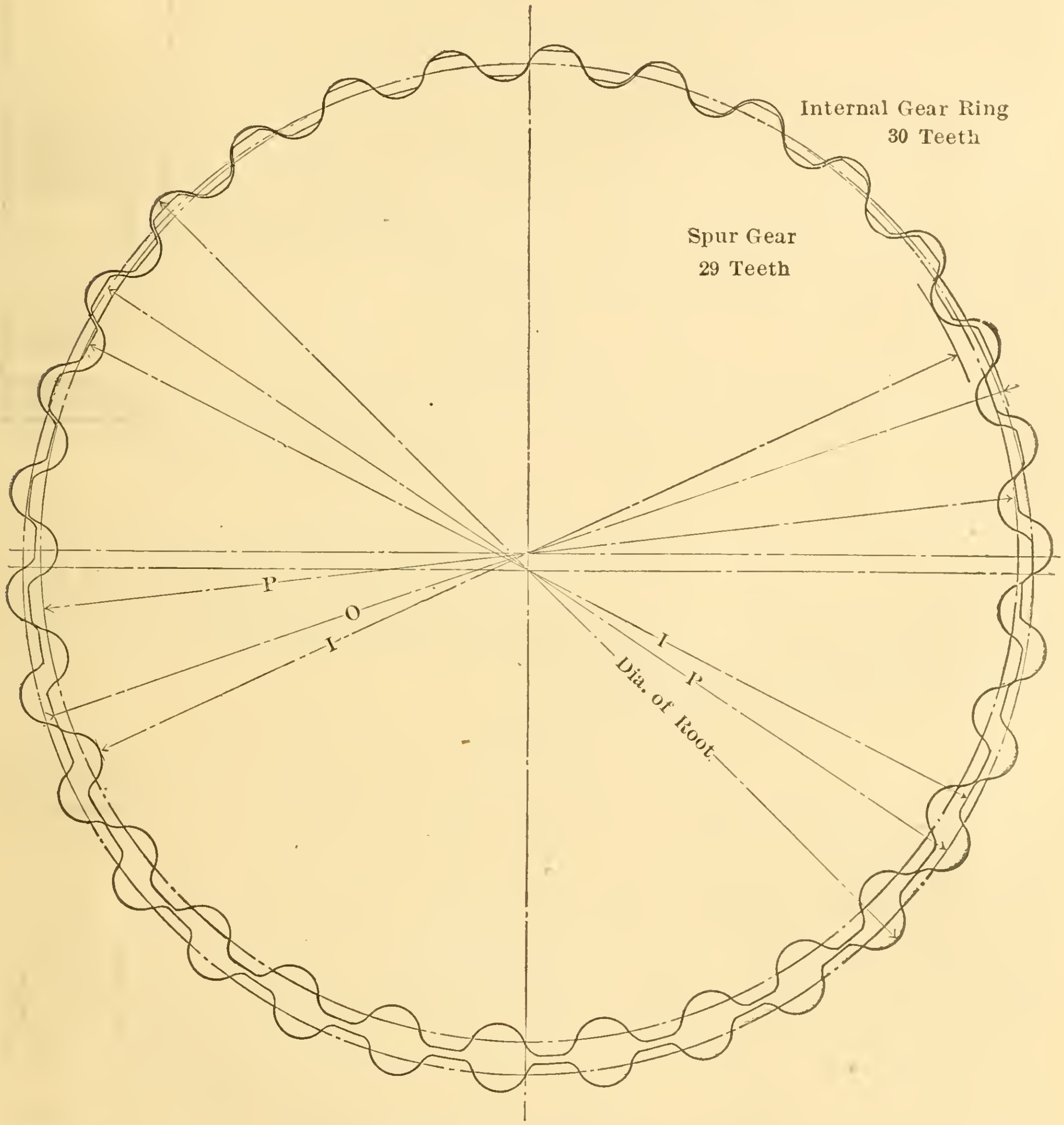


FIG. 299. SPECIAL INTERNAL GEAR DRIVE.

This same rule applies to the relation of the numbers of teeth in the internal gear and the number of teeth in the Fellows cutter.

If the difference between the two is less than 15, the points of the teeth in internal gear will be cut away.

When it is necessary to design an internal gear drive and the difference in the number of teeth is slight the teeth can best be laid out by the cycloidal system, the diameter of the describing circle (see page 3) being made equal to the pitch diameter of one-half the difference in the number of teeth (see Fig. 299).

Fig. 299 shows a case where there is but one tooth difference between the internal gear and the engaging spur gear.

INTERCHANGEABILITY OF PARTS

If any attempt at interchangeability of parts be desired, put limitations on drawings, plus and minus. Mark parts to be left rough, or not necessarily accurate. Give manufacturer idea of how parts are assembled. Send assembled drawing, or, still better, a sample set of gears and engaging parts. Send parts to be fitted, especially if of foreign make.

SET SCREWS

Do not locate the set screw so far under the rim of the gear that it will be necessary to drill hole with a ratchet. If a key is also used, locate set screw over key.

PINS

A taper pin is a poor means of securing the gear, unless used as a safety appliance to prevent a more serious break. It falls out and damages other parts, unless secured by a nut on small end; and shear, unless made unproportionately large—a key is better.

MATERIAL

When no material is specified manufacturers will ordinarily understand that cast iron is wanted.

It is generally advisable to make small pinion of machine steel for obvious reasons; also a steel pinion is often as cheap, if not cheaper, than one of cast iron. This applies also when a large number is required on account of the superior facilities for handling and machining bar steel, especially in automatic machines.

Where machine or forged steel is specified it is understood that a steel approximately 0.30 carbon is wanted. The next higher commercial grade approximates 0.50 carbon. It is not advisable to use a higher carbon content than 0.50 for gears on account of the tendency of the higher carbons to crystallize in service. Although it is sometimes permissible to use high carbon steel or spiral gears (the action of the teeth preventing shocks which tend to crystallize the material), a carbon content as high as 1.20 is often used with success for such gears.

Gears for case hardening are generally made of steel lower than 0.20 carbon,

12 to 14 being recommended by many. If any other grade is required it should be specified. The customer should also state whether cyanide or bone hardening is required. To obtain a harder as well as a tougher gear that will withstand hard usage and heavy loads, it is necessary to resort to an alloy steel which may be tempered. The most commonly used is a steel containing about 0.30 carbon and 3½-per cent. of nickel. Chrome nickel or chrome vanadium steel is much used in high-class automobile gears. There are many grades of alloy steels, some for case hardening, some for oil or water tempering; owing to their great variety and varying values, however, no attempt is made to enter further into this subject.

When gears are to be case hardened or tempered there are several points that should be kept in mind. No machine work is possible on hardened gears, except by grinding; therefore, for parts that must be exact size, it should be specified what allowance be made for this, as the gear is liable to either expand or contract during the process of hardening, due to either the condition of the steel before machining, the general construction of the gear, or to strains set up by the removal of a considerable stock. For this reason it is well to first be sure that the blank is properly annealed, and that if any amount of stock is removed, and it is necessary to maintain accurate sizes in the finished gear, the roughed out blank should again be annealed before finishing; a gear with a square bore is an example of this.

It is always a safe plan to have steel castings and drop forgings annealed. This is especially important with drop forgings, as they are generally allowed to cool from the forging heat on the dirt floor (dies are often hardened this way), and the process of forging is sure to set up internal strains no matter what their shape.

Pin holes through the hub must be drilled before gear is hardened, unless the consumer is to do the hardening. Case hardening gears are often simply carbonized, that is, packed in bone and subjected to heat for the proper time, but not hardened. This extra heating tends to make a better gear than when they are dipped direct from the carbonizing pot; also it offers an opportunity to finish up parts which must be close to size, as the blank has been relieved of strains. If necessary, portions may be machined beyond the depth to where the carbon has penetrated, leaving that surface soft after machining. To do this, however, the carbon content must be below 20 per cent., as the higher carbon steels will harden throughout.

For larger gears the teeth are often cut in a rolled steel rim which is keyed or shrunk on a steel center, as the best steel casting often falls short of meeting the required conditions. This type is used for street railway gears for heavy service (see page 126).

No definite rule can be laid down for the shrinkage of tempered gears, although it is reasonably well established that properly treated alloy steels shrink less than ordinary carbon steel. Each particular grade of steel and style of gear must be tried out separately. An approximate rule, however, would be to allow 0.0005 per inch of diameter. Some gears will require more than this and some less according to their design. A uniform section of material through the gear will tend to minimize these distortions.

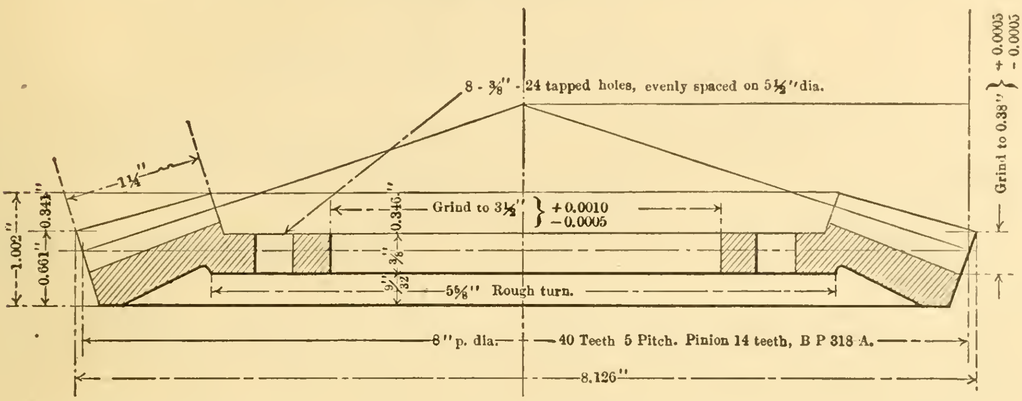
WORKING DRAWINGS FOR GEARS

If it was realized just how much a proper drawing will facilitate production, or rather the delays that are caused by the lack of such drawings, there would be a decided change from the present practice. Many draftsmen seem to see just how many gears they can crowd on to one sheet; the gears are shown in mesh, and working dimensions are conspicuous by their absence. Sometimes a blank sketch is shown with a long list of dimensions covering the balance of the print; the gears listed may be made of three or four different kinds of materials. This is all very nice from the draftsman's standpoint, but how about the workman who has to trace out all of the necessary dimensions? The pitch diameter and backing of bevel gears from the pitch line are necessary dimensions—for the draftsman—but they are of no interest to the workman. Some one must put these drawings in shape before they go into the shop; perhaps new ones must be made. This takes time and means a delay right at the very beginning of the job, and the more difficult the work the longer the delay, for more than one reason.

The revised print finally reaches the shop. Suppose there are a dozen patterns to be made and the gears are all on one print, which also shows several forged steel gears. No work can be done on the forgings until the pattern maker is through with the print; the time taken to construct and check the patterns is lost as far as the steel gears are concerned, which otherwise might have been completed. When the machine work is started the print must follow each gear, in the meantime the balance of the work is waiting unless the foreman is a rapid draftsman. This process must be repeated in each department, and yet the customer wonders "where are his gears." If his draftsman had spent some of the time used in drawing standard gear teeth—to which no attention is paid—on essential dimensions, the work would have been further along. A description of the style of tooth is all that is necessary. Unless a special form of tooth is desired, it is best to show none at all, as this will cause another delay until it is ascertained what *is* required.

A drawing should show all necessary working dimensions. No figuring should be required of the workman into whose hands it is placed.

When special material or special treatments are to be used, always put this information in full on the drawing; also give working limits, and specify surfaces to be ground as well as those upon which no finish is required. It is often advisable to show adjacent parts in dotted lines, so that better judgment



Center and back angles	71° 34′	Addendum	0.250″
Face angle	15° 51′	Whole depth	0.785″
Cutting angle	68° 36′	Thickness	0.380″
Tooth angle	20° 00′		

3¼% nickel steel case hardened in oil.

FIG. 300. DIMENSIONED BEVEL GEAR.

be used in the machine work. And above all other things, put the gears upon separate drawings, especially if made in quantities, as a drawing cannot be in two places in one time; when working upon one gear, a workman is not interested in its mate; besides, this makes the drawing plainer. This applies to all types of gears. Fig. 298 is an illustration.

SECTION XVII

Practical Points in Gear Cutting

The outside diameter of the gear blank should first be measured and any variations noted, so that the proper allowance can be made in the tooth dimensions.

All burs should be removed from the ends of hubs which should be faced true with bore.

When putting several gears together on work arbor they must be faced unusually true or the arbor will be bent. This is especially true when the bore is small and the gears are large in diameter.

Be sure that work arbor has a proper fit in spindle of machine before tightening the draw bolt, also that it runs true before putting on the gear blank.

If a stock bushing does not fit the bore have one made. Do not use paper or tin; and do not use too many bushings.

Be sure that the proper side of rim goes against the back steady rest; this side should be chalked before the gear is taken out of the chuck when turning up the blank.

Use some judgment in tightening the nut on work arbor; that is, do not use or hammer on an 18-inch wrench for a one-inch arbor.

If outer support is used, be sure that it holds work arbor in a horizontal position. Special attention must be given to this point when mounting heavy gears, as they are very likely to draw down the outer end of arbor.

Throw out index worm and make sure that the gear runs true.

When setting steady rest be sure that there are no obstructions on the rim that will strike it and spoil the gear; also oil the surface of rim.

Put on proper feeds and speeds to suit the grade of material being cut and the type of cutter used.

Nick the gear around before starting the cut to prove the indexing.

Be sure that finishing cutter is central.

When dropping cutter for depth of tooth, allow for any error in outside diameter, that the tooth may be of the proper thickness at the pitch line.

Finish just enough of the end of one tooth to be sure of your thickness before proceeding further.

Use chordal tooth parts for measuring the teeth; do not depend upon the shake of the tooth gauge.

If the pitch is not too coarse, two cutters may be employed; one for roughing and one to finish, the roughing cutter being separated from the finishing cutter by a spacing washer.

Care should be taken that the rougher cutter is not too large in diameter or too wide. If the teeth are first roughed out, the roughing cutter should be made to cut the full depth, leaving no stock for the finishing cutter at the bottom of tooth.

Roughing cutter should make central cut in tooth space, otherwise one side of the finishing cutter will wear rapidly.

If the cutter shows signs of distress try cutting down the *speed* before changing the feed *per minute.*

Always use two cutters when possible. It has been argued that this makes inaccurate gears, owing to the change in temperature between the start and finish of the cutting; but such is not the case.

Do not remove gears from the machine until tooth thickness all around has been inspected; this, of course, is impractical for small work.

Use plenty of lubricant when cutting steel. If soda water does not seem to answer use oil; on the other hand, if oil does not give results, try soda water. When cutting bronze try cutting it dry if a lubricant does not give results, and *vice versa.*

Be sure that the cutting edges of finishing cutter are ground radial, otherwise an incorrectly shaped tooth will be the result.

Be sure that the key driving cutter does not bind on the top, otherwise the cutter will be broken. There are generally fillets in the corners of the cutter keyseat, therefore the tops of key should be beveled off to suit.

Do not grind the cutter to suit the arbor; if the arbor runs out of true have it corrected—it will pay in the end.

It pays to use high-speed steel cutters, especially for the finer pitch (4 diametral pitch and under), but put up the speed to at least double that used for carbon cutters, keeping the speed per revolution of cutter the same.

Remember that it is not possible to operate a gear cut absolutely without backlash—there must be some allowance made. When it is known how the gears are to engage it is suggested that the pinions be cut standard; that is, no allowance be made, and that the teeth in engaging gear be made 0.01 of the circular pitch thin on the pitch line. On the other hand, when it is not definitely known how the gears engage cut all teeth 0.005 of the circular pitch thin. This would seem a better rule for general use. When generating bevel gears it is common practice to make the pinion tooth heavier than the gear tooth (according to the ratio), to give additional life to the pinion. Special provision is made on most bevel generating machines to make this setting.

When the number of gears to be cut justifies it, have pair of pins turned to fit the bore of the gears set in a plate at the proper center distance. This plate is to be kept at the machine to test the work, as the cutter must be readjusted as it wears and changes in diameter. Exact centers on their jig may be cheaply attained by turning the stub of one of the pins entering plate eccentric, pinning it securely when adjusted to the proper centers (see Fig. 301).

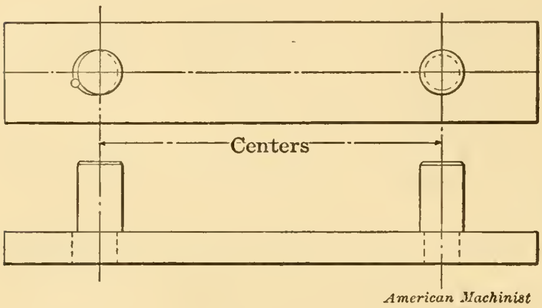


FIG. 301. A SIMPLE TESTING FIXTURE.

After gears are cut there are always slight tool marks, burs, and minor inaccuracies that will prevent their smoother operation when first assembled. If facilities are at hand to give the gears a running test, case-harden one gear out of the lot and run the balance of the gears with it for a short period, in both directions. This will remove tool marks, insure the smooth operating of the gears, and facilitate assembling of parts.

For spur gears it might be suggested that a gear for this purpose be cut on the Fellows shaper, or similar generating machine, enlarging the diameter of the gear until the pitch diameter is near the bottom of the tooth.

$$D' = \frac{N+4}{p}.$$

The teeth of a gear cut in this manner will have no rolling action, the pitch diameters not touching (see Fig. 302). The resulting sliding action is just what is required for the purpose mentioned—that is, polishing the teeth.

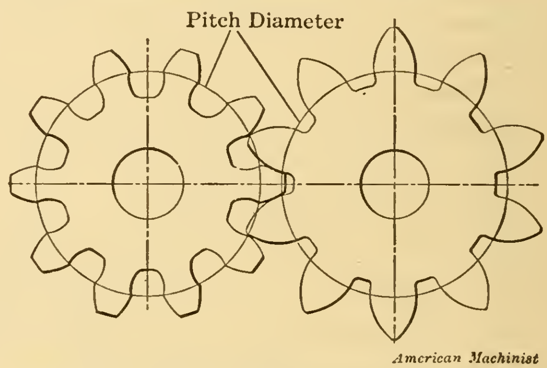


FIG. 302. ARRANGEMENT OF PITCH DIAMETERS TO POLISH TEETH BY SLIDING.

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